A Newsvendor Model with Unreliable Suppliers

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Abstract

We consider the problem of a newsvendor that is served by multiple suppliers, where any given supplier may be unreliable. By unreliable we simply mean that the marginal amount received from a supplier is no more than, and typically is less than, the marginal amount ordered from the supplier. In this setting, the newsvendor needs to determine (1) whether or not to place an order with a given supplier, and (2) if so, then for how much? To address these questions, we develop a general framework in which the newsvendor can diversify its risk of inadequate delivery amounts by spreading its orders among any number and combination of available suppliers that differ in terms of cost and (delivery) reliability. Ultimately, we find that the newsvendor model with unreliable suppliers has the same structural properties as a newsvendor model in which all suppliers are reliable but have limited capacity. Our resulting contribution is two−fold: First, we establish properties of the optimal solution and develop corresponding insights into the trade−off between cost and reliability. Second, we perform comparative statics on the optimal solution, with a particular emphasis on investigating how changes in suppliers cost or reliability affect the newsvendor's ordering decisions and customer service level.

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We consider the problem of a newsvendor that is served by multiple suppliers, where any given supplier may be unreliable. By unreliable we simply mean that the marginal amount received from a supplier is no more than, and typically is less than, the marginal amount ordered from the supplier. In this setting, the newsvendor needs to determine (1) whether or not to place an order with a given supplier, and (2) if so, then for how much? To address these questions, we develop a general framework in which the newsvendor can diversify its risk of inadequate delivery amounts by spreading its orders among any number and combination of available suppliers that differ in terms of cost and (delivery) reliability. Ultimately, we find that the newsvendor model with unreliable suppliers has the same structural properties as a newsvendor model in which all suppliers are reliable but have limited capacity. Our resulting contribution is two-fold: First, we establish properties of the optimal solution and develop corresponding insights into the trade-off between cost and reliability. Second, we perform comparative statics on the optimal solution, with a particular emphasis on investigating how changes in supplier cost or reliability affect the newsvendor's ordering decisions and customer service level.

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1 INTRODUCTION

By definition, a newsvendor faces the challenge of having to decide how much of its product to order from its supplier for its single selling season, prior to observing the random demand for its product. Correspondingly, the newsvendor's sales for the season is constrained both by the demand that materializes and by the quantity that the newsvendor chooses to supply. The economic implications are twofold: On the one hand, if realized demand exceeds the supply, then the newsvendor will sell its entire stock, but at the expense of having excess demand go unsatisfied; on the other hand, if the supply exceeds the realized demand, then the newsvendor will satisfy demand completely, but at the expense of having leftovers. The familiar solution to this trade off indicates that the newsvendor, to maximize expected profit for the selling season, should order just enough supply so that the probability of meeting demand is equal to the ratio of the marginal overstocking cost to the sum of the marginal overstocking plus understocking cost. In this classic setting, the newsvendor's supplier is perfectly reliable in the sense that the quantity provided (by the supplier) is exactly equal to the amount ordered (by the newsvendor). Consequently, a newsvendor needs only to determine how much to order from a single supplier.

In this paper, we explore implications if the newsvendor faced the additional challenge of having to deal with a supply source that did not necessarily deliver the quantity ordered. Such could be the case if a supplier faced, for example, random capacity or a random yield process. In effect, a supplier's delivery quantity in this variation is constrained by its own production output function. Consequently, if the supplier's production output is insufficient to meet the newsvendor's order quantity, then the newsvendor will receive less than what was ordered. In this setting, the newsvendor needs to determine (1) whether or not to place an order with a given supplier, and (2) if so, then for how much?

To address these questions, we develop a modeling framework built around a very general notion of unreliability. Specifically, in our model, we define *unreliable supplier* to mean simply that there exists a positive probability that the marginal quantity delivered by the supplier is less than a marginal unit ordered from the supplier. This framework hinges on the construction of a supplier output function that can be either deterministic or random, as well as either endogenous (in the sense that its parameters can depend on the newsvendor's order quantity) or exogenous. Given this framework, our focus is two-fold: First, we establish properties of the optimal solution and develop corresponding insights into the trade-off between cost and reliability. Second, we perform comparative statics on the optimal solution, with a particular emphasis on investigating how changes in supplier cost or reliability affect the newsvendor's ordering decisions and customer service level.

One contribution of our model is that it unifies a disparate range of reliability constructs interspersed throughout both the newsvendor literature and the random yield literature. (Authoritative reviews of the newsvendor and random yield literatures are provided by Porteus (1990) and by Yano and Lee (1995), respectively. More recent updates include those by Khouja (1999) and Minner (2002), respectively.) Examples of reliability constructs appearing in these literatures include, among others, the case of all-or-nothing delivery (Anupindi and Akella (1993); Gerchak (1996)), the case of random capacity (Ciarallo et al. (1994)), the case of binomial yield (Chen et al. (2001)), the case of stochastic proportional yield (Henig and Gerchak (1990)), and combinations thereof (Wang and Gerchak (1996)). Typically in these papers, the primary focus is on characterizing the structure of the optimal policy, given that each of the supply sources are utilized, so that it may serve as a building block for dynamic inventory models. In contrast, our focus (like, to some degree, the focus of Chen et al. (2001) and Gerchak

(1996)) is on characterizing optimal supplier selection and its implications for the newsvendor, for its suppliers, and for its customer market. Ultimately, we find that the newsvendor model with unreliable suppliers has the same structural properties as a newsvendor model in which all suppliers are reliable but have limited capacity.

Another contribution of our model is the corresponding insights that it yields. In particular, we find that cost generally takes priority over reliability when it comes to supplier selection. Consequently, suppliers with costs that are high relative to other suppliers could be left without a share of the newsvendor's aggregate order, regardless of their relative reliability levels. For these suppliers, reliability improvements would be to no avail; only through improving costs could they break through the threshold to gain a share of the newsvendor's aggregate order. In addition, we find that, when compared to a newsvendor that does not have to deal with an unreliable supply source, a newsvendor that does have unreliable suppliers will order more (in aggregate), but provide a lower customer service level. Moreover, we complement these results with useful comparative statics.

The remainder of this paper is organized as follows. In Section 2, we develop our formal model and demonstrate that, in a reduced form, it yields a structure amenable to a standard newsvendor interpretation. In Section 3, we establish properties of the optimal solution, discuss implications for prioritizing suppliers, and develop corresponding insights. Then, in Section 4 we perform a comparative statics analysis on a two-supplier solution to investigate how the newsvendor should react to changes in fundamental problem parameters. We conclude the paper with Section 5.

2 THE MODEL AND ITS REDUCED FORM

Prior to the beginning of a single selling season, a retailer places orders for its product from among any of N independent suppliers, some or all of which are unreliable. In this context, unreliable means that the marginal amount supplied (i.e., delivered) by a given supplier is less than or equal to the marginal amount ordered from the supplier. Accordingly, let Q_i denote the quantity ordered from supplier i, and let R_i represent the reliability of supplier i, where R_i is an exogenous construct characterized as either fixed or random. Then, $S_i(Q_i,R_i)$, supplier i's supply function, is such that $S_i(Q_i,R_i) \leq Q_i$.

Prevalent supply functions in the literature include the following:

$$S(Q_i,R_i) = \min\{Q_i,R_i\}; \tag{1a}$$

$$S_i(Q_i,R_i) = R_iQ_i; \tag{1b}$$

$$\Pr\{S_i = Q_i\} = 1 - \Pr\{S_i = 0\} \equiv \rho_i. \tag{1c}$$

In general, (1a) characterizes limited-capacity models; (1b) characterizes proportional yield models ; and (1c) characterizes "all-or-nothing" models. Note, however, that (1c) also can be interpreted as a special case of either (1a) or (1b). In particular, if R_i is specified as a Bernoulli random variable that takes on a realized value of either ∞ or 0, then (1a) reduces to (1c); similarly, if R_i is specified as a Bernoulli random variable that takes on a realized value of either 1 or 0, then (1b) reduces to (1c).

Given (1a) – (1c) as well as the applicability of alternative constructs, we model supplier i's supply function more generally by defining the notion of a production output function:

$$S_i(Q_i,R_i) = \min\{Q_i, K_i(Q_i,R_i)\}, \tag{2}$$

where $K_i(.)$ represents the output function. Thus, a given supplier's production output function explicitly constrains the amount that the supplier can deliver (i.e., supply) to the retailer. This specification is useful because it accounts for the possibility that a given supplier's production capability could be either endogenous (in the sense that it depends on the retailer's order-quantity decision) or exogenous, as well as either deterministic or random. Notice, for example, if R_i in (1a) is deterministic, then it corresponds to the case in which $K_i(Q_i,R_i)$ in (2) is exogenous and deterministic. Likewise, if R_i in (1a) is random, then it corresponds to the case in which $K_i(Q_i,R_i)$ in (2) is exogenous and random. Similarly, if R_i in (1b) is random (or, deterministic), then it corresponds to one particular specification of an endogenous and random (or, deterministic) output function.

Let $s_i(Q_i,R_i) \equiv \partial S_i(Q_i,R_i)/\partial Q_i$ represent the marginal quantity supplied by supplier i per marginal unit ordered from that supplier. We assume that $s_i(Q_i,R_i) \leq 1$ for all Q_i and realized values of R_i (or, equivalently, we assume that $\partial K_i(Q_i,R_i)/\partial Q_i \leq 1$). If supplier i is such that $s_i(Q_i,R_i) = 1$ for all Q_i and realized values of R_i, then we say that supplier i is *perfectly reliable*. In contrast, if $s_i(Q_i,R_i) < 1$ for some value of Q_i and realized value of R_i, then we say that supplier i is *unreliable*.

Suppliers are indexed from least to most expensive so that $c_1 < \ldots < c_N$, where c_i denotes the per-unit purchase cost of inventory obtained from supplier i. We assume that the retailer pays supplier i only for the amount actually supplied. That is, we assume that the retailer's procurement cost associated with supplier i is $c_iS_i(Q_i,R_i) \leq c_iQ_i$. One practical implication of this assumption is that our model is most applicable to a procurement environment, where it is reasonable to expect that a retailer pays for what is received rather than for what is ordered. In contrast, this assumption would be less applicable to a production environment in which it is reasonable to expect that a manufacturer must pay not only for the yield of a production run ($S_i(Q_i,R_i)$), but also for the total number of defects that result from the production run (Q_i –

$S_i(Q_i,R_i)$). In such an environment, the total cost associated with "ordering" Q_i units would be c_iQ_i rather than $c_iS_i(Q_i,R_i)$. In Section 5, we discuss some implications if it is assumed that the decision maker pays for what is ordered rather than for what is supplied.

Let D denote the random demand for the retailer's product; and let F and f represent the cumulative distribution function (cdf) and probability density function (pdf) associated with D. Note however, that the assumption of random demand is not necessary. The random demand process D can be replaced by the deterministic process d without affecting the results of our analysis. This is because our analysis centers on the Type-I service level, which is defined as the unconditional probability that demand for the single period does not exceed the supply available for the period, and this is a concept that applies as long as <u>either</u> demand <u>or</u> supply (or both) is random. Because of the central role that service level plays in the interpretation of our results, let $SL(\mathbf{Q})$ denote the retailer's service level, given that $\mathbf{Q} = \{Q_i\}$ is the vector of order quantities placed among the suppliers. Then, by definition, $SL(\mathbf{Q}) = \Pr\{D \leq S_T(\mathbf{Q})\} = E[F(S_T(\mathbf{Q}))]$, where $S_T(\mathbf{Q}) \equiv S_1(Q_1,R_1) + \ldots + S_N(Q_N,R_N)$ represents the total supply available to the retailer for sale during the period. Analogously, let $Q_T \equiv Q_1 + \ldots + Q_N$ denote the total quantity ordered by the retailer.

To complete the specification of our model, assume that the retailer sells its product for the per-unit selling price p; that leftovers are salvaged for the per-unit value v (where a negative v denotes a disposal cost); and that shortages are assessed a per-unit penalty cost π (to signify the cost of lost goodwill). Finally, let $\phi_i \equiv (p + \pi - c)/(p + \pi - v)$ denote the standard newsvendor fractile (i.e., ratio of underage costs to the sum of underage and overage costs) associated with supplier i. This notation, as well as the notation introduced above, is summarized in Table 1.

Table 1. Summary of Notation

N	=	total number of suppliers
p, v, π	=	retailer's per-unit selling price, salvage value, and penalty cost of loss goodwill, respectively
c_i	=	retailer's per-unit purchase cost of supply procured from supplier i
$\phi_i \equiv (p+\pi-c_i)/(p+\pi-v)$	=	newsvendor fractile associated with supplier i
R_i	=	reliability of supplier i
Q_i, $\mathbf{Q} = \{Q_i\}$	=	quantity ordered from supplier i, and order-quantity vector, respectively
$K_i(Q_i,R_i)$	=	supplier i's output function
$S_i(Q_i,R_i) = \min\{Q_i, K_i(Q_i,R_i)\}$	=	quantity supplied (i.e., delivered) by supplier i
$s_i(Q_i,R_i) = \partial S_i/\partial Q_i$	=	marginal quantity supplied by supplier i (per unit ordered)
$Q_T = Q_1 + \ldots + Q_N$	=	total quantity ordered from all suppliers
$S_T(\mathbf{Q}) = S_1 + \ldots + S_N$	=	total quantity supplied by all suppliers
D	=	demand for retailer's product (random or deterministic)
F, f	=	cdf and pdf (or pmf) characterizing D
$SL(\mathbf{Q}) = E[F(S_T(\mathbf{Q}))]$	=	Type-I service level provided by retailer

The retailer's objective is to maximize $\Pi(\mathbf{Q})$, its expected profit for the selling season, where

$$\begin{aligned}\Pi(\mathbf{Q}) &= E\left[\mathrm{p}\cdot\mathrm{sales}(\mathbf{Q}) + \mathrm{v}\cdot\mathrm{Eleftovers}(\mathbf{Q}) - \pi\cdot\mathrm{shortages}(\mathbf{Q}) - \sum_i c_i S_i(Q_i,R_i)\right] \\ &= E\left[\mathrm{pD} + \mathrm{v}\cdot\mathrm{Eleftovers}(\mathbf{Q}) - (\mathrm{p}+\pi)\cdot\mathrm{shortages}(\mathbf{Q}) - \sum_i c_i S_i(Q_i,R_i)\right] \\ &= E\left[\mathrm{pD} + \mathrm{v}\left[\sum_i S_i(Q_i,R_i) - D\right]^+ - (\mathrm{p}+\pi)\left[D - \sum_i S_i(Q_i,R_i)\right]^+ - \sum_i c_i S_i(Q_i,R_i)\right]. \quad (3)\end{aligned}$$

Consider, then, the question of how much to order from supplier i (whether that be zero or otherwise). To that end, define

$$X_i = D - \sum_{j\neq i} S_j(Q_j,R_j). \quad (4)$$

Note that X_i is a random variable representing supplier i's apportioned, or risk-adjusted, demand. In other words, X_i indicates the total (random) demand for the retailer's product, less the total (random) supply provided by all suppliers other than supplier i. As such, X_i corresponds to the random "demand" for the units supplied by supplier i. Accordingly, let

$$\Pr\{X_i \le S_i(Q_i, R_i) \mid R_i\} = \Pr\{D \le S_T(\mathbf{Q}) \mid R_i\} \equiv F_i(S_T(\mathbf{Q})).$$

Then $F_i(S_T(\mathbf{Q}))$ denotes the conditional probability that demand is less than or equal to total supply, given a realized value of R_i. Moreover, the retailer's service level, which, by definition, is the unconditional probability that demand is less than or equal to supply, can be expressed as

$$SL(\mathbf{Q}) \equiv E[F(S_T(\mathbf{Q}))] = E_{R_i}[F_i(S_T(\mathbf{Q}))]. \tag{5}$$

Since X_i is independent of Q_i, one convenient way to express (3) is as follows:

$$\Pi(\mathbf{Q}) = \Pi_i(\mathbf{Q}) - E\left[\sum_{j \ne i} (p - c_j) S_j(Q_j, R_j)\right], \tag{6}$$

where

$$\Pi_i(\mathbf{Q}) = E\left[pX_i + v[S_i(Q_i, R_i) - X_i]^+ - (p + \pi)[X_i - S_i(Q_i, R_i)]^+ - c_i S_i(Q_i, R_i)\right]. \tag{7}$$

The significance of (6) and (7) is as follows: From the perspective of determining how much, if any, to order from supplier i, the second term of (6) can be ignored because it is independent of Q_i. Thus, only $\Pi_i(\mathbf{Q})$ needs to be considered. Notice from (7), however, that for any realized value of R_i, $\Pi_i(\mathbf{Q})$ can be interpreted as a standard newsvendor profit function, where $S_i(Q_i,R_i)$ denotes the resource supply decision while the "exogenous" X_i denotes the random demand for that resource. The one variation here is that the supply decision ($S_i(Q_i,R_i)$) itself might be random. Interestingly, this simple variation creates enough complication that, unlike its standard newsvendor counterpart, (6) is not guaranteed to be concave in general.

Nevertheless, $\Pi_i(\mathbf{Q})$ still can be interpreted as an expectation over two random variables: X_i, which, in general, actually is a convolution of random variables, and R_i, which, in general, is a random variable representing the reliability of supplier i. To make this double expectation explicit, we rewrite (7) as follows:

$$\Pi_i(\mathbf{Q}) = E_{R_i}\left[E_{X_i}\left[pX_i + v[S_i(Q_i,R_i) - X_i]^+ - (p+\pi)[X_i - S_i(Q_i,R_i)]^+ - c_i S_i(Q_i,R_i)\right] \mid R_i\right].$$

Thus, the *conditional* expected profit associated with ordering from supplier i, given a realized value of R_i, is precisely a newsvendor profit function. This implies that the derivative of $\Pi(\mathbf{Q})$ taken with respect to Q_i is

$$M_i(\mathbf{Q}) \equiv \frac{\partial \Pi(\mathbf{Q})}{\partial Q_i} = \frac{\partial \Pi_i(\mathbf{Q})}{\partial Q_i} = (p + \pi - v) E_{R_i}\left[s_i(Q_i,R_i)(\phi_i - F_i(S_T(\mathbf{Q})))\right], \tag{8}$$

where, recall, $\phi_i = (p + \pi - c)/(p + \pi - v)$ and $s_i(Q_i,R_i) = \partial S_i(Q_i,R_i)/\partial Q_i$. Correspondingly, $M_i(\mathbf{Q}) = 0$, which represents the necessary condition for Q_i^*, the optimal quantity to order from supplier i, to be an interior point solution, can be written as

$$E_i\left[\lambda_i(Q_i,R_i) F_i(S_T(\mathbf{Q}))\right] = \phi_i\,, \tag{9}$$

where

$$\lambda_i(Q_i,R_i) \equiv \frac{s_i(Q_i,R_i)}{E_i[s_i(Q_i,R_i)]}$$

and, for notational simplicity, we now use $E_i[.]$ in lieu of $E_{Ri}[.]$ to indicate an expectation over the random variable R_i.

As in the classic newsvendor problem, the right hand side (RHS) of (9) is the familiar critical fractile associated with supplier i, and the left hand side (LHS) has a service level interpretation. In particular, the LHS can be interpreted as the *weighted conditional service level given that Q_i matters*, where $\lambda_i(Q_i,R_i)$ denotes the weight for any given value of R_i. In this context, "Q_i matters" if, for a given value of R_i, the supply function depends on Q_i. That is, Q_i matters for realized values of R_i that are such that $s_i(Q_i,R_i) > 0$.

To better illustrate this notion, consider the application of (9) to the case in which supplier i's output function is exogenous and random (note that, for the single-supplier version of this case, Ciarallo et al. (1994) demonstrate that the retailer's profit function is unimodal, but not concave):

Illustration. Let $S_i(Q_i,R_i) = \min\{Q_i,R_i\}$, where R_i is a random variable characterized by a known probability distribution. Then, $S_i(Q_i,R_i)$ depends on Q_i if and only if the realized value of R_i meets or exceeds Q_i. Hence, for any given realized value of $R_i = r$, Q_i "matters" if and only if $r \geq Q_i$; and Q_i does not matter if $r < Q_i$. Moreover, for realized values of R_i in which Q_i does matter, $s_i(Q_i,R_i) = 1$. Consequently, $E_i[s_i(Q_i,R_i)] = \Pr\{R_i \geq Q_i\}$. Correspondingly, the conditional weighted service level given that Q_i matters is

$$E_i\left[\lambda_i(Q_i,R_i)F_i(S_T(\mathbf{Q}))\right] = \frac{0 \cdot E_i\left[F_i(S_T(\mathbf{Q})) \mid R_i < Q_i\right] + 1 \cdot E_i\left[F_i(S_T(\mathbf{Q})) \mid R_i \geq Q_i\right]}{\Pr\{R_i \geq Q_i\}}$$

$$= \frac{E_i\left[\Pr\{D \leq Q_i + \sum_{k\neq i} S_k(Q_k,R_k)\}\right]\Pr\{R_i \geq Q_i\}}{\Pr\{R_i \geq Q_i\}} = \Pr\{D \leq Q_i + \sum_{k\neq i} S_k(Q_k,R_k)\}.$$

Therefore, from (9), the optimality condition for Q_i^* to be an interior point solution when supplier i's output function is exogenous and random is

$$\Pr\{D \leq Q_i + \sum_{k\neq i} S_k(Q_k,R_k)\} = \phi_i\,, \tag{10}$$

which is consistent with the optimality condition established in Ciarallo et al. (1994).

To conclude this section, we note that the model developed here does not require the supply function of each supplier to be of the same functional form. Consequently, given (2), what distinguishes supplier i from other suppliers is its specific combination of cost (c_i), reliability (R_i), and output function ($K_i(Q_i,R_i)$).

3 SUPPLIER SELECTION: COST VERSUS RELIABILITY

Since suppliers differ on cost and reliability, in this section we examine how these two attributes affect the outcome of the retailer's supplier selection decision, which refers to the process of choosing suppliers with which to place orders. To characterize these results, we define a supplier to be *active* if it is optimal for the retailer to place an order with that supplier. Conversely, we define a supplier to be *inactive* if it is optimal for the retailer not to place an order with that supplier. Given these definitions, we find that, in general, cost takes precedence over reliability when it comes to choosing suppliers. In particular, we establish and discuss the following properties of optimal selection:

1. If a given supplier is inactive, then all more expensive suppliers will be inactive.

2. If a given supplier is perfectly reliable, then all suppliers more expensive than the perfectly reliable supplier will be inactive. Hence, no more than one perfectly reliable supplier will be active. Moreover, if a perfectly reliable supplier *is* active, then the retailer's optimal service level will be identically equal to the newsvendor fractile associated with that supplier.

3. Unless the marginal benefit associated with selling a unit *purchased* from a given supplier ($p + \pi - c_i$), is strictly less than the *expected* marginal benefit associated with selling a unit *ordered* from <u>each</u> less expensive supplier, the supplier will be inactive.

4. For problem specifications in which the Karush-Kuhn-Tucker (KKT) conditions are sufficient for determining an optimal solution, if ordering optimally from the n least expensive suppliers would yield a service level that is greater than the newsvendor fractile associated with supplier n+1, then supplier n+1 and all more expensive suppliers will be inactive.

5. The optimal total quantity ordered from all active suppliers is at least as large as the total quantity ordered in an otherwise equivalent problem in which all suppliers are perfectly reliable. However, the resulting optimal service level is no greater than the resulting service level in the problem with only perfectly reliable suppliers.

To develop these results, first note that $\phi_1 > \ldots > \phi_N$ since $\phi_i = (p + \pi - c_i)/(p + \pi - v)$ and suppliers are indexed such that $c_1 < \ldots < c_N$. Next, note that any optimal solution $\mathbf{Q}^*$ must satisfy the following KKT optimality conditions for all i:

$$M_i\left(\mathbf{Q}^*\right) \le 0\,; \tag{11}$$

$$Q_i^* M_i\left(\mathbf{Q}^*\right) = 0\,, \tag{12}$$

where $M_i(\mathbf{Q})$ is given by (8). Finally, consider the following 3 lemmas, which serve as building blocks for the technical analysis of this section (proofs of these lemmas are provided in the appendix):

Lemma 1. *For any given $\boldsymbol{Q}$, $M_i(\boldsymbol{Q}) \le (p+\pi-v)E_i\left[s_i(Q_i,R_i)\right]\left[\phi_i - SL(\boldsymbol{Q})\right]$.*

Lemma 2. *If $Q_i^* = 0$, then $SL\left(\boldsymbol{Q}^*\right) \ge \phi_i$.*

Lemma 3. *If $Q_i^* > 0$, then $SL\left(\boldsymbol{Q}^*\right) \ge E_i\left[s_i\left(Q_i^*,R_i\right)\right]\phi_i$.*

We now formally establish and discuss the results of this section.

Proposition 1. *For i = 1,…,N-1, if $Q_i^* = 0$, then $Q_{i+1}^* = 0$.*

Proof. Assume that, in an optimal solution, $Q_i^* = 0$. Then, from Lemma 2 and the indexing of suppliers, $SL\left(\mathbf{Q}^*\right) \ge \phi_i > \phi_{i+1}$. Thus, from Lemma 1,

$$M_{i+1}\left(\mathbf{Q}^*\right) \le (p+\pi-v)E_{i+1}\left[s_{i+1}\left(Q_{i+1}^*,R_{i+1}\right)\right]\left[\phi_{i+1} - SL\left(\mathbf{Q}^*\right)\right] < 0\,.$$

But, from (12), $M_{i+1}\left(\mathbf{Q}^*\right) < 0$ implies that $Q_{i+1}^* = 0$. In other words, if $Q_i^* = 0$, then $Q_{i+1}^* = 0$. □

Proposition 1, in effect, establishes a precedence ranking among the suppliers based only on each supplier's cost. Basically, in terms of selecting suppliers, it is optimal for the retailer to start by choosing the least expensive supplier, and then to add suppliers to its selection set one by one, according to how inexpensive the supplier is. Consequently, the optimal number of active suppliers, say N^*, will be such that supplier i is active if and only if $c_i \le c_{N^*}$.

Proposition 2. *For i = 1,…,N-1, if supplier i is perfectly reliable, then* $Q_j^* = 0$ *for j = i+1,…,N.*

Proof. Proposition 1 implies that if $Q_{i+1}^* = 0$, then $Q_j^* = 0$ for all j > i+1. Therefore, to establish this proposition, it suffices to show that that $Q_{i+1}^* = 0$ if supplier i is perfectly reliable. Assume, then, that supplier i is perfectly reliable (that is, $s_i(Q_i,R_i) = 1$ for all Q_i and realized values of R_i). There are two possible cases. On the one hand, if $Q_i^* = 0$, then $Q_{i+1}^* = 0$ directly from Proposition 1. On the other hand, if $Q_i^* > 0$, then (12) implies that $M_i(\mathbf{Q}^*) = 0$. Thus, from (8),

$$0 = E_i\left[s_i\left(Q_i^*, R_i\right)\left(\phi_i - F_i\left(SL\left(\mathbf{Q}^*\right)\right)\right)\right] = \phi_i - SL\left(\mathbf{Q}^*\right).$$

That is, $SL\left(\mathbf{Q}^*\right) = \phi_i > \phi_{i+1}$. But, from Lemma 1, if $\phi_{i+1} < SL\left(\mathbf{Q}^*\right)$, then $M_{i+1}(\mathbf{Q}^*) < 0$. And, from (12), if $M_{i+1}(\mathbf{Q}^*) < 0$, then $Q_{i+1}^* = 0$. Therefore, if supplier i is perfectly reliable, then $Q_{i+1}^* = 0$. □

Proposition 2 indicates that there can be at most one perfectly reliable supplier that becomes active. Accordingly, given Proposition 1, if there exist two or more perfectly reliable suppliers, the only one that is even eligible to become active is the least expensive one. An intuitive argument for why more expensive suppliers will not become active when a perfectly reliable supplier exists is as follows: Suppose the retailer is considering ordering a unit from a more expensive supplier when a less expensive supplier that is perfectly reliable exists. If the retailer instead diverts the unit to the less expensive, perfectly reliable supplier, then the retailer can save on procurement cost given that the unit is received. Moreover, since diverting the unit to the perfectly reliable supplier also reduces uncertainty, the retailer can order less units in the aggregate, thereby further reducing expected procurement cost.

Even though a perfectly reliable supplier can render all more expensive suppliers inactive, the following proposition indicates that perfect reliability is hardly sufficient for a given supplier to become active itself.

Proposition 3. *If, in an optimal solution,* $\phi_j < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$ *for* <u>*any*</u> *i < j, then* $Q_j^* = 0$.

Proof. Assume that, in an optimal solution, $Q_j^* > 0$. Then, from (12), $M_j(\mathbf{Q}^*) = 0$. Consequently, given the proposition assumption that there exists an i < j that is such that $\phi_j < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$, Lemma 1 implies that $SL\left(\mathbf{Q}^*\right) \le \phi_j < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$. But, according to Lemma 3, if $SL\left(\mathbf{Q}^*\right) < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$, then $Q_i^* = 0$. And, from Proposition 1, if $Q_i^* = 0$ for i < j, then $Q_j^* = 0$, which contradicts the original assumption that $Q_j^* > 0$. Therefore, if there exists an i < j that is such $\phi_j < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$, $Q_j^* > 0$ is an invalid assumption, which implies that $Q_j^* = 0$. □

To interpret Proposition 3, note that the condition $\phi_j < E_i\left[s_i\left(Q_i^*, R_i\right)\right]\phi_i$ is equivalent to $\left(p + \pi - c_j\right) < E\left[s_i\left(Q_i^*, R_i\right)\left(p + \pi - c_i\right)\right]$. In this expression, note that $(p + \pi - c_j)$ represents the *conditional* marginal benefit associated with selling a unit ordered from supplier j (i.e., the underage cost), *given* that supplier j actually delivers the unit. And, $E\left[s_i\left(Q_i^*, R_i\right)\left(p + \pi - c_i\right)\right]$ represents the *expected* marginal benefit associated with selling a unit ordered from supplier i, which is an expectation conditioned over the likelihood that supplier i actually delivers the unit. Thus, Proposition 3 indicates that the expected marginal benefit associated with ordering from less expensive suppliers is pivotal for determining whether or not to place orders with more expensive suppliers, regardless of the more expensive supplier's reliability. As a result, *any* less expensive supplier can effectively render a more expensive supplier inactive, even if the more expensive supplier is perfectly reliable.

From a technical standpoint, Proposition 3 is useful because many typical supply functions are such that $s_i(Q_i,R_i)$ actually is independent of Q_i. Indeed, $s_i(Q_i,R_i)$ reduces to $s_i(R_i)$ for each of the three prevalent supply functions noted in Section 2. As a result, for these suppliers,

$E_i\left[s_i\left(Q_i^*, R_i\right)\right]$ reduces to the constant $E_i[s_i(R_i)]$; thus, the eligibility test provided by Proposition 3 can be applied immediately to reduce the set of eligible suppliers *before* attempting to solve the associated optimization problem.

Although Propositions 2 and 3 establish that perfect reliability is no guarantee for becoming active, it is interesting to note the following corollary, which indicates that *if* a perfectly reliable supplier becomes active, then the retailer's optimal solution can be interpreted as a *base service level* policy.

Corollary. *If supplier i is perfectly reliable and active, then* $SL\left(\boldsymbol{Q}^*\right) = \phi_i$.

Proof. If supplier i is perfectly reliable, then, by definition, $s_i(Q_i,R_i) = 1$ for all Q_i and realized values of R_i). Moreover, if supplier i is active, then, also by definition, $Q_i^* > 0$. Therefore, from (12) and (9),

$$0 = E_i\left[s_i\left(Q_i^*, R_i\right)\left(\phi_i - F_i\left(SL\left(\mathbf{Q}^*\right)\right)\right)\right] = \phi_i - SL\left(\mathbf{Q}^*\right).$$

That is, $SL\left(\mathbf{Q}^*\right) = \phi_i$. □

According to this corollary, if it is optimal for the retailer to order from a perfectly reliable supplier, then the retailer's optimal service level is independent of any other active supplier's cost and reliability. Consequently, if a less expensive supplier's cost or reliability were to change, for whatever reason, then the change would not affect the retailer's optimal service level. The change would only affect the allocation scheme that the retailer uses to achieve the optimal service level. Thus, if a perfectly reliable supplier is active, then that supplier's role is to complement the combined orders placed with all less expensive suppliers in a very special way: regardless of how the retailer spreads its orders among less expensive suppliers, the purpose of the perfectly reliable supplier is to deliver the remaining quantity required to bring the retailer's service level up to the base level ϕ_i. For this reason, we refer to a perfectly reliable supplier that becomes active as an *anchor supplier*.

Generally speaking, Propositions 1 – 3 establish cost as a priority over reliability when it comes to selecting suppliers with which to place orders. In particular, Propositions 1 –3 all establish conditions under which more expensive suppliers will be rendered inactive, *regardless of the reliability levels of those suppliers*. Thus, if a particular unreliable supplier is rendered inactive by one or more of these propositions, then improving reliability will be to no avail; only through improving costs can the supplier break through these thresholds to become active.

From a technical standpoint, Propositions 1 – 3 are useful for simplifying the solution procedure. In particular, Propositions 2 and 3 can be used to immediately pare the original set of N suppliers to a potentially smaller set of eligible suppliers. Then, given the resulting set of eligible suppliers (say N_E, where $N_E \leq N$), Proposition 1 can be exploited as follows: Define the n-supplier subproblem as the subproblem in which the retailer sets $Q_j = 0$ for j = n+1,…,N_E; and solves for Q_i^* for i = 1,…,n. Then, the retailer can be guaranteed to find the optimal solution by iteratively solving n-supplier subproblems, beginning with n = 1 and ending with N_E.

Alternatively, if a problem instance is appropriately-well behaved so that the KKT conditions are guaranteed to have a unique solution, then the optimal solution can be obtained directly from (11) and (12). Or, as indicated below, one can use the same iterative procedure as above, but end with the first subproblem that yields a solution that satisfies the service-level condition indicated by Proposition 4.

Proposition 4. *Suppose that the KKT conditions are sufficient for establishing optimality. Moreover, define the n-supplier subproblem as the constrained problem in which Q_j is set equal to 0 for j = n+1,...,N; and let $\mathbf{Q^n} \equiv \{Q_i^n\}$ denote the optimal solution to the n-supplier subproblem. If $\mathbf{Q^n}$ is such that $SL(\mathbf{Q^n}) > \phi_{n+1}$, then n is the optimal number of active suppliers for the N-supplier problem.*

Proof. Let $\mathbf{Q}^n = \{Q_1^n,...,Q_n^n,0,...,0\}$ denote any optimal solution to the n-supplier problem. Then, by the definition of optimality, $\mathbf{Q}^n$ satisfies the system of equations given by (11) and (12). Specifically, for i = 1,…,n: $M_i(\mathbf{Q}^n) \le 0$ and $Q_i^n M_i(\mathbf{Q}^n) = 0$.

Consider, then, the general N-supplier problem. Given that the KKT conditions are sufficient for establishing optimality, (11) and (12) have a unique solution. Moreover, that unique solution corresponds to $\mathbf{Q}^*$, the optimal solution to the general problem. Thus, to complete the proof, it suffices to show that the feasible candidate solution $\mathbf{Q}^n$ indeed satisfies (11) and (12) for i = 1,…,N if $SL(\mathbf{Q}^n) > \phi_{n+1}$.

For i = 1,…,n, the candidate solution is $Q_i = Q_i^n$, where, recall, Q_i^n is defined such that $M_i(\mathbf{Q}^n) \le 0$ and $Q_i^n M_i(\mathbf{Q}^n) = 0$. Thus, for i = 1,…,n, the candidate solution satisfies (11) and (12) trivially. For i = n+1,…,N, the candidate solution is $Q_i = 0$. Thus, for i = n+1,…N, the candidate solution also satisfies (12) trivially. Therefore, to complete the proof, consider (11) for i = n+1,…N. From (8),

$$M_i(\mathbf{Q}^n) = (p + \pi - v)E_i\left[s_i(0,R_i)\left(\phi_i - F_i\left(\textstyle\sum_1^n S_k(Q_k^n, R_k)\right)\right)\right]. \tag{13}$$

Note, however, that each of the Q_k^n's in the summation of this expression is independent of R_i (for i = n+1,…,N) because these Q_k^n's are defined as any solution to the system of equations given by (11) and (12) for k = 1,…,n (which, given that $Q_i = 0$ for i = n+1,…,N, is a system of equations independent of R_i for i = 1,…,n). Thus, given that i = n+1,…,N, (13) simplifies to

$$M_i(\mathbf{Q}^n) = (p + \pi - v)E_i[s_i(0,R_i)]\left[\phi_i - F_i\left(\textstyle\sum_1^n S_k(Q_k^n, R_k)\right)\right] = (p + \pi - v)E_i[s_i(0,R_i)][\phi_i - SL(\mathbf{Q}^n)] < 0,$$

where the inequality follows because $SL(\mathbf{Q}^n) > \phi_{n+1}$ by the assumption of the proposition, and because $\phi_{n+1} > \phi_i$ for i = n+2,…,N by the indexing of suppliers. Thus, if $SL(\mathbf{Q}^n) > \phi_{n+1}$, then the candidate solution $\mathbf{Q}^n$ satisfies (11) and (12) for all i, which implies that $\mathbf{Q}^n$ is the optimal solution to the general problem. □

The structure of the optimal policy, which boils down to finding the N^* cheapest active suppliers, is essentially identical to the solution to a newsvendor problem in which each supplier in a given set has limited capacity. In other words, if a given set of suppliers each have a supply function given by (1a), where R_i is deterministic, then it is well-known that the corresponding newsvendor problem is concave and that the resulting optimal solution dictates ordering from a more expensive supplier only after exhausting the capacities of all less expensive suppliers (Porteus, 1990). Thus, Propositions 1 – 4 effectively establish that, although concavity might not be preserved when expanding the definition of "unreliability" beyond the notion of limited capacity to include the more general construct applied in this paper, the basic criteria for selecting suppliers *is* preserved.

We conclude this section by offering insights on how the existence of unreliable suppliers affects a retailer's optimal ordering policy. The insights come from comparing Q_T^* and $SL(\mathbf{Q}^*)$, the retailer's optimal total order quantity and service level, respectively, against the optimal Q and SL for a newsvendor problem, which serves as the benchmark for comparison.

Proposition 5. *Let $N^* \le N$ denote the most expensive active supplier in an optimal solution. Then $\phi_{N^*+1} \le SL(\boldsymbol{Q}^*) \le \phi_{N^*} < \phi_{N^*-1} < \ldots < \phi_1 \le F(Q_T^*)$.*

Proof. Given the definition of N^*, $Q_i^* > 0$ if and only if $i \le N^*$. Thus, from Lemma 2, $\phi_{N^*+1} \le SL(\mathbf{Q}^*)$. And, from (12), $M_{N^*}(\mathbf{Q}^*) = 0$. Therefore, from Lemma 1,

$$0 \le \phi_{N^*} - SL(\mathbf{Q}^*),$$

which, given the indexing of suppliers, implies that $SL(\mathbf{Q}^*) \le \phi_{N^*} < \ldots < \phi_1$.

Next, from (8), for any given **Q**,

$$M_1(\mathbf{Q}) = (p + \pi - v)\{E_1[s_1(Q_1, R_1)]\phi_1 - E_1[s_1(Q_1, R_1)F_1(S_T(\mathbf{Q}))]\}.$$

Note, however, that $S_T(\mathbf{Q}) \equiv \sum_1^N S_k(Q_k, R_k) \le \sum_1^N Q_k \equiv Q_T$ because $S_k(Q,R_k) \le Q$ for all k, by the definition of unreliability. Therefore, $F_1(S_T(\mathbf{Q})) \le F_1(Q_T)$; hence, for any given **Q**,

$$M_1(\mathbf{Q}) \ge (p + \pi - v)E_1[s_1(Q_1, R_1)][\phi_1 - F_1(S_T(\mathbf{Q}))].$$

But, since supplier 1 is an active supplier, (12) implies that $M_1(\mathbf{Q}^*) = 0$. Therefore, $F_1(S_T(\mathbf{Q}^*)) \ge \phi_1$. □

To interpret Proposition 5, consider the classic newsvendor problem as a benchmark. In the context of our model, the newsvendor problem represents the special case scenario in which all N suppliers are perfectly reliable. Accordingly, by Propositions 2 and 3, only supplier 1 will be active in the newsvendor's optimal solution. Moreover, by the corollary to Propositions 2 and 3, the newvendor's optimal quantity to order from supplier 1 is the quantity Q_b that satisfies $\phi_1 = SL(Q_b) \equiv F(Q_b)$, which is the familiar result. Therefore, Proposition 5 implies that $SL(\mathbf{Q}^*) \le SL(Q_b) = F(Q_b) \le F(Q_T^*)$. In other words, when compared to an otherwise equivalent retailer that does *not* have to deal with unreliable suppliers (i.e., a newsvendor), a retailer that *does* have to deal with unreliable suppliers will order more ($Q_T^* \ge Q_b$) but provide a lower level of service to its customers.

4 Comparative Statics: The Case of Two Active Suppliers

In Section 3, we developed properties characterizing the retailer's optimal solution, generally establishing that cost takes priority over reliability when it comes to selecting suppliers; and, as a result, only the N^* least expensive suppliers will be active, where N^* is such that $\phi_{N^*+1} \leq SL(\mathbf{Q}^*)$ $\leq \phi_{N^*}$. In this section, we develop additional insights by analyzing in further detail the case in which $N^* = 2$. Specifically, we develop some intuition with regard to the following questions:

1. What is the strategic relationship between Q_1^* and Q_2^*? In the sense of Bulow et al. (1985), are they strategic complements or strategic substitutes? Moreover, how do changes in supplier cost affect Q_1^* and Q_2^*?

2. How do changes in supplier cost affect Q_T^* and $SL(\mathbf{Q}^*)$?

3. Similarly, how do changes in supplier reliability affect Q_1^*, Q_2^*, Q_T^*, and $SL(\mathbf{Q}^*)$?

Although we are able to answer the first of these questions for the general case in which each supplier's supply function is characterized by (2), we require additional problem structure to develop corresponding insight with respect to the second two questions. Consequently, we approach these issues by tailoring our analysis to the family of problems involving an exogenous output function so that each supplier's supply function can be characterized by (1a).

We begin addressing the questions of this section with the following proposition, which effectively validates the intuition that the retailer's order quantities not only are strategic substitutes, but also are such that a decrease in one supplier's cost will result in an increase in that supplier's order quantity while yielding a decrease in the other supplier's order quantity.

Proposition 6. *For $j \neq i$, let $Q_j^*(Q_i;\phi_i)$ denote the optimal value of Q_j as a function of Q_i. Then, $Q_j^*(Q_i;\phi_i)$ is decreasing in Q_i. Moreover, Q_i^* is increasing in ϕ_i, and $Q_j^* = Q_j^*(Q_i^*;\phi_i)$ is decreasing in ϕ_i.*

Proof. The first part of this result follows directly from (8) by taking the cross-partial derivative:

$$\frac{\partial M_j(\mathbf{Q})}{\partial Q_i} = -(p+\pi-v)E_j\left[E_i\left[s_i(Q_i,R_i)s_j(Q_j,R_j)f_i(S_T(\mathbf{Q})\,|\,R_j)\right]\right] < 0,$$

which is sufficient for establishing that $\partial Q_j^*(Q_i;\phi_i)/\partial Q_i < 0$ (Topkis, 1998). Note also, from (8), that $\partial M_j(\mathbf{Q})/\partial \phi_i = 0$, which implies that $\partial Q_j^*(Q_i;\phi_i)/\partial \phi_i = 0$.

Next, substitute $Q_j^*(Q_i;\phi_j)$ into $M_i(\mathbf{Q})$ to reduce $M_i(\mathbf{Q})$ to a function of the single variable Q_i, given the parameter ϕ_i:

$$M_i(Q_i;\phi_i) \equiv M_i(\mathbf{Q};\phi_i) = M_i(\{Q_i, Q_j^*(Q_i;\phi_i)\}; \phi_i) = (p+\pi-v)E_i[s_i(Q_i,R_i)(\phi_i - F_i(S_T(\mathbf{Q})))].$$

Accordingly, since $\partial Q_j^*(Q_i;\phi_i)/\partial \phi_i = 0$,

$$\frac{\partial M_i(Q_i;\phi_i)}{\partial \phi_i} = \frac{\partial M_i(\{Q_i, Q_j^*(Q_i;\phi_i)\}; \phi_i)}{\partial \phi_i} + \frac{\partial M_i(\{Q_i, Q_j^*(Q_i;\phi_i)\}; \phi_i)}{\partial Q_j^*(Q_i;\phi_i)} \frac{\partial Q_j^*(Q_i;\phi_i)}{\partial \phi_i}$$

$$= (p+\pi-v)E_i[s_i(Q_i,R_i)] > 0 .$$

This implies that $dQ_i^*/d\phi_i > 0$. Therefore, since $\partial Q_j^*(Q_i;\phi_i)/\partial Q_i < 0$,

$$\frac{dQ_j^*}{d\phi_i} = \frac{\partial Q_j^*(Q_i^*;\phi_i)}{\partial Q_i^*}\frac{dQ_i^*}{d\phi_i} + \frac{\partial Q_j^*(Q_i^*;\phi_i)}{\partial \phi_i} = \frac{\partial Q_j^*(Q_i^*;\phi_i)}{\partial Q_i^*}\frac{dQ_i^*}{d\phi_i} < 0 .$$ □

Since the retailer will react to a change in one supplier's cost by increasing one order quantity while decreasing the other, it would be interesting to investigate the combined effect. However, a definitive resolution of this issue is difficult to obtain in general because of the implicit tug of war occurring between the two order quantities. Therefore, we develop insight by analyzing a tractable variation of the general problem. Specifically, if we assume that the output function of each supplier is exogenous, then we can ascertain that the increase in Q_i^* resulting from a decrease in supplier i's cost more than compensates for the corresponding decrease in Q_j^*. Consequently, we find that a decrease in supplier i's cost (as reflected by an increase in ϕ_i) results in a higher overall total quantity ordered by the retailer. Moreover, this increase in the retailer's total order quantity translates into a higher service level. Thus, the ultimate benefactor of a decrease in supplier cost is the retailer's customer market. We summarize these results with the following proposition. (See Appendix for formal proof.)

Proposition 7. *If supplier 1 and supplier 2 each have an exogenous output function, then* Q_T^* *and* $SL(\boldsymbol{Q}^*)$ *are increasing functions both of* ϕ_1 *and of* ϕ_2.

To perform an analogous comparative statics investigation with respect to supplier reliability, we continue to study the case of exogenous output functions, but we now need to be more precise with our notion of "more reliable." Consequently, let $R_i = R_i(\rho_i,\varepsilon_i)$, where ε_i is an independent random variable representing possible uncertainty in supplier i's output function, and ρ_i is a fixed parameter representing supplier i's reliability level. Then define $\partial R_i(\rho_i,\varepsilon_i)/\partial\rho_i > 0$ so that the higher is ρ_i, the more reliable is supplier i. Finally, to operationalize this construct, assume that $\varepsilon_i \sim G_i(u)$, where $G_i(u)$ is a continuous probability distribution; and that ε_i is such that, for any given value of ρ_i, a single value of ε_i, namely $z_i(Q_i,\rho_i)$, satisfies $R_i(\rho_i, z_i(Q_i,\rho_i)) = Q_i$. The significance of $z_i(Q_i,\rho_i)$ is that it partitions the probability state space characterizing ε_i into two regions: one region is such that $R_i(\rho_i,\varepsilon_i) < Q_i$ for all ε_i in the region, and the other region is such that $R_i(\rho_i,\varepsilon_i) \geq Q_i$ for all ε_i in the region. Without loss of generality, we construct the state space so that $R_i(\rho_i,\varepsilon_i) < Q_i$ if and only if $\varepsilon_i < z_i(Q_i,\rho_i)$. Suppose, for example, that $R_i(\rho_i,\varepsilon_i) = a_i(\rho_i) + b_i(\rho_i)\varepsilon_i$. Then, $z_i(Q_i,\rho_i) = (Q_i - a_i(\rho_i))/b_i(\rho_i)$. Proposition 8 characterizes the relevant findings for this problem specification. (See Appendix for proof.)

Proposition 8. *If both suppliers have exogenous output functions such that $K_i(R_i) = R_i(\rho_i, \varepsilon_i)$, where $\partial R_i(\rho_i, \varepsilon_i)/\partial \rho_i > 0$, ε_i is a random variable, and $z_i(Q_i, \rho_i)$ is defined such that $R_i(\rho_i, z_i(Q_i, \rho_i)) < Q_i$ if and only if $\varepsilon_i < z_i(Q_i, \rho_i)$, then:*

(a) the KKT conditions are sufficient for determining Q_i^* and Q_j^*;

(b) *Q_i^* is increasing in ρ_i, while Q_j^* is decreasing in ρ_i;*

(c) *$SL(\boldsymbol{Q}^*)$ is increasing both in ρ_1 and in ρ_2; but*

(d) *Q_T^* is <u>decreasing</u> both in ρ_1 and in ρ_2.*

We conclude this section by noting two observations about Proposition 8 that warrant additional highlighting. First, recall from Section 2 that, in general, we have no guarantee that the retailer's general profit function is significantly well behaved so as to ensure that the KKT conditions will be sufficient for computing the optimal solution. Thus, although the KKT conditions are useful for *characterizing* the optimal solution in general, their applicability for actually *producing* the optimal solution should be verified on a case-by-case basis. Part (a) of Proposition 8 does just that, establishing, in effect, that the retailer's profit function is unimodal for this specification of the problem.

Second, note that, for the most part, the qualitative results of Proposition 8 directly parallel their counterparts from Propositions 6 and 7. Thus, in general, an increase in an active supplier's reliability will yield effects similar to a decrease in an active supplier's cost, as might be expected. However, the one notable exception to this general rule is the effect on Q_T^*: a more favorable supplier reliability results in a *decrease* in the retailer's total quantity ordered, whereas a more favorable supplier cost results in an *increase* in the retailer's total quantity ordered. One intuitive explanation for this contrast is as follows: The ultimate effect resulting from an improvement in an active supplier's cost or reliability is an increase in the retailer's service level. But, service level depends not on what the retailer orders in the aggregate, it depends on what the retailer receives in the aggregate, which is an amount characterized by the output functions of the suppliers. These output functions, in turn, depend on the retailer's order quantities and the suppliers' reliabilities, but they do not depend on the suppliers' costs. Hence, it is natural that the retailer would need to increase its aggregate order quantity in order to increase its service level when the service level increase is in response to an improvement in supplier cost since, in such a case, order quantities are the retailer's primary means by which to execute the desired effect. In contrast, the retailer need not necessarily increase its aggregate order quantity to achieve an increased service level when the desired service level increase is in response to an improvement in supplier reliability since, in such a case, the retailer reaps the side benefit of an improved yield.

5 Conclusions

We have considered the problem of a newsvendor that is served by multiple suppliers, where any given supplier may be unreliable. By unreliable we mean that the marginal amount received from a supplier is no more than, and typically is less than, the marginal amount ordered. Our results indicate that, in general, although reliability may influence how much is ordered from an active supplier, cost takes priority over reliability when it comes to supplier selection. Even perfect reliability is no guarantee for selection since, in an optimal solution, a given supplier can be active only if all less expensive suppliers are active, regardless of that supplier's reliability level. This appealing result is all the more attractive because our notion of reliability is very general. In particular, our construction hinges on the notion of an output function that can be either endogenous, in the sense that its parameters depend on the order quantity, or exogenous. Moreover, the output function can be either deterministic or random.

Although our model is developed with a newsvendor, procurement environment in mind, there is a natural connection with the literature on multi-period manufacturing lot sizing with random yield. This is because, in multi-period settings of the lot sizing model, the single-period setting of the newsvendor is a basic building block. To make the connection, compare our single-period N-supplier newsvendor model with a single-supplier N-period model in which an order is placed at the start of each period after observing the inventory position. If we associate each supplier i in the multiple supplier model with each period i in the multiple period model, then the multiple supplier model boils down to a variation of the multiple period problem in which the decision maker must choose at the start of the *first* period how much to order for each of the next N periods. The significance of this correspondence is that it may directly yield unimodality in some of our problem settings when the corresponding multiple period random yield problem is unimodal.

While we have appropriately assumed that the newsvendor only pays for those "good" units it receives, in many manufacturing settings it might be more appropriate to pay for what is ordered. Since it follows from Van Meighem and Rudi (2002) that in such a newsvendor setting the objective function becomes concave, more traditional optimization techniques can be exploited to produce an optimal solution. Interestingly, however, unimodality appears to be attained at the expense of a loss of some structural property: basically, in this variation, it no longer is the case that cost necessarily takes priority over reliability. Consequently, there is a basic change in the nature of risk sharing in the supply channel. In effect, when the retailer pays only for those units received, the retailer bears some risk of underproduction, but the supplier bears the risk of defective or inefficient output. Conversely, when the retailer pays for the entire manufacturing lot, the supplier effectively is indemnified of all risk since the retailer ends up burdening not only the risk of underproduction, but also the risk of defective or inefficient output.

APPENDIX

Proof of Lemma 1. From (8),

$$M_i(\mathbf{Q}) = (p + \pi - v)\{E_i[s_i(Q_i, R_i)]\phi_i - E_i[s_i(Q_i, R_i)F_i(S_T(\mathbf{Q}))]\} \quad \text{(A1)}$$

for all i. Note, however, that $s_i(Q_i,R_i)$ is an increasing function of R_i by the definition of "more reliable." Consequently, $S_i(Q_i, R_i) = \int_0^{Q_i} s_i(t, R_i)dt$ is an increasing function of R_i. This, in turn, implies that, for any given $\mathbf{Q}$, $S_T(\mathbf{Q})$ and, hence, $F_i(S_T(\mathbf{Q}))$ are increasing functions of R_i. In other words, both $s_i(Q_i,R_i)$ and $F_i(S_T(\mathbf{Q}))$ are increasing functions of R_i. Hence, from Karlin and Studden (1996),

$$E_i[s_i(Q_i, R_i)F_i(S_T(\mathbf{Q}))] \geq E_i[s_i(Q_i, R_i)]E_i[F_i(S_T(\mathbf{Q}))] = E_i[s_i(Q_i, R_i)]SL(\mathbf{Q}).$$

Substituting this into (A1) yields

$$M_i(\mathbf{Q}) \leq (p + \pi - v)E_i[s_i(Q_i, R_i)][\phi_i - SL(\mathbf{Q})]. \quad \square$$

Proof of Lemma 2. Assume that $Q_i^* = 0$. Then (8) and (11) imply that

$$0 \geq M_i(\mathbf{Q}^*)_{Q_i^*=0} = (p + \pi - v)E_i\left[s_i(0, R_i)\left(\phi_i - \Pr\left\{D \leq \sum_{k \neq i} S_k(Q_k^*, R_k)\right\}\right)\right], \quad \text{(A2)}$$

where, also from (11), the set of Q_k^*'s represent any N-1 dimensional vector $\{Q_1,\ldots,Q_{i-1}, Q_{i+1},\ldots,Q_N\}$ that simultaneously solves the N-1 equations $M_j(\{Q_1,\ldots,Q_{i-1},Q_{i+1},\ldots,Q_N\}) \leq 0$ for j = 1, …, N and j ≠ i, given that $Q_i = 0$. That is, given (8), $\{Q_k^*\}$ is any solution to the following system of equations:

$$E_j\left[s_j(Q_j,R_j)\left(\phi_j - \Pr\left\{D \le \sum_{k\ne i} S_k(Q_k,R_k)\right\}\right)\right] \le 0 \quad \text{for } j = 1,\ldots,N;\ j \ne i. \tag{A3}$$

Notice that this system of equations is independent of R_i. Therefore, any solution to this system is independent of R_i. That is, the set of $\{Q_k^*\}$ is independent of R_i, which implies that (A2) reduces to

$$0 \ge (p+\pi-v)E_i[s_i(0,R_i)]\left(\phi_i - \Pr\left\{D \le \sum_{k\ne i} S_k(Q_k^*,R_k)\right\}\right) = E_i[s_i(0,R_i)][\phi_i - SL(\mathbf{Q}^*)].$$

Therefore, $\phi_i \le SL(\mathbf{Q}^*)$. □

Proof of Lemma 3. Assume that $Q_i^* > 0$. Then, from (12), $M_i(\mathbf{Q}^*) = 0$. Thus, from (8) and the assumption that $s_i(Q_i,R_i) \le 1$ for all Q_i and realized values of R_i,

$$0 = (p+\pi-v)\{E_i[s_i(Q_i^*,R_i)]\phi_i - E_i[s_i(Q_i^*,R_i)F_i(S_T(\mathbf{Q}^*))]\} \ge (p+\pi-v)\{E_i[s_i(Q_i^*,R_i)]\phi_i - SL(\mathbf{Q}^*)\}.$$

Therefore, $E_i[s_i(Q_i^*,R_i)]\phi_i \le SL(\mathbf{Q}^*)$. □

Proof of Proposition 7. If each active supplier has an exogenous output function so that $K_i(R_i) = R_i$ for $i = 1, 2$, then (2) implies that $s_i(R_i) = 0$ for $R_i < Q_i$, and $s_i(R_i) = 1$ for $R_i \ge Q_i$. Moreover, it means that (10), from the illustration in Section 2, provides the optimality conditions for Q_i^*:

$$\phi_i = \Pr\{D \le Q_i^* + S_j(Q_j^*,R_j)\} = E_j[F(Q_i^* + S_j(Q_j^*,R_j))], \tag{A4}$$

where $i = 1, 2$; $j = 1, 2$; $i \ne j$. Taking the total derivative of (A4) with respect to ϕ_j (for $j = 1, 2$),

$$0 = E_j\left[f(Q_i^* + S_j(Q_j^*,R_j))\left(\frac{dQ_i^*}{d\phi_j} + s_j(R_j)\frac{dQ_j^*}{d\phi_j}\right)\right] < E_j[f(Q_i^* + S_j(Q_j^*,R_j))]\left(\frac{dQ_i^*}{d\phi_j} + \frac{dQ_j^*}{d\phi_j}\right),$$

where the inequality follows because $dQ_j^*/d\phi_j > 0$ by Proposition 6 and because $s_j(R_j)$ is equal to either 0 or 1 for any given value of R_j. Therefore, for $j = 1, 2$,

$$0 < \left(\frac{dQ_i^*}{d\phi_j} + \frac{dQ_j^*}{d\phi_j} \right) = \frac{dQ_T^*}{d\phi_j}.$$

Next, for the purpose of this proof, let $G_i(u)$ be the c.d.f. of R_i. Then, given (A4),

$$\begin{aligned} SL(\mathbf{Q}^*) &\equiv E\left[F\left(S_i\left(Q_i^*, R_i\right) + S_j\left(Q_j^*, R_j\right)\right)\right] \\ &= \int_0^{Q_i^*} E_j\left[F\left(R_i + S_j\left(Q_j^*, R_j\right)\right)\right] g_i(u) du + \int_{Q_i^*}^{\infty} E_j\left[F\left(Q_i^* + S_j\left(Q_j^*, R_j\right)\right)\right] g_i(u) du \\ &= \int_0^{Q_i^*} E_j\left[F\left(R_i + S_j\left(Q_j^*, R_j\right)\right)\right] g_i(u) du + \int_{Q_i^*}^{\infty} \phi_i g_i(u) du, \end{aligned}$$

where i = 1, 2; j = 1, 2; $i \neq j$. Applying (A4) to the derivative with respect to ϕ_j (for j = 1, 2),

$$\begin{aligned} \frac{dSL(\mathbf{Q}^*)}{d\phi_j} &= \int_0^{Q_i^*} E_j\left[f\left(R_i + S_j\left(Q_j^*, R_j\right)\right) s_j\left(R_j\right) \frac{dQ_j^*}{d\phi_j} \right] g_i(u) du + \frac{dQ_i^*}{d\phi_j}\left[E_j\left[F\left(Q_i^* + S_j\left(Q_j^*, R_j\right)\right)\right] - \phi_i \right] g_i\left(Q_i^*\right) \\ &= \int_0^{Q_i^*} E_j\left[f\left(R_i + S_j\left(Q_j^*, R_j\right)\right) s_j\left(R_j\right) \frac{dQ_j^*}{d\phi_j} \right] g_i(u) du > 0, \end{aligned}$$

where the inequality follows because $dQ_j^* / d\phi_j > 0$ by Proposition 6. □

Proof of Proposition 8. To simplify notation in this proof, we denote $R_i(\rho_i,\varepsilon_i)$ simply as R_i, recognizing that the appropriate distribution function $G_i(u)$ is the distribution of ε_i. Likewise, we use z_i and z_i^* as shorthand for $z_i(Q_i,\rho_i)$ and $z_i\left(Q_i^*, \rho_i\right)$, respectively.

Part (a). For any given value of Q_1, the KKT optimality condition for Q_2 is given by (10):

$$\phi_2 = E_1\left[F\left(Q_2 + S_1\left(Q_1, R_1\right)\right)\right]. \qquad \text{(A5)}$$

The LHS of this equation is independent of Q_2, while the RHS is increasing in Q_2. Thus, given Q_1, (A5) has exactly one solution, namely $Q_2^*(Q_1)$, which is the optimal value of Q_2 as a function of Q_1. Taking the total derivative of (A5) with respect to Q_1,

$$0 = E_1\left[f\left(Q_2^*(Q_1) + S_1(Q_1, R_1)\right)\left(\frac{dQ_2^*(Q_1)}{dQ_1} + s_1(R_1) \right) \right]$$

$$\Rightarrow \quad \frac{dQ_2^*(Q_1)}{dQ_1} = -\frac{E_1\left[f\left(Q_2^*(Q_1) + S_1(Q_1, R_1)\right) s_1(R_1)\right]}{E_1\left[f\left(Q_2^*(Q_1) + S_1(Q_1, R_1)\right)\right]} \tag{A6}$$

Since $s_1(R_1) \geq 0$ for all realized values of R_1, (A6) implies that $dQ_2^*(Q_1)/dQ_1 \leq 0$. Moreover, since $s_1(R_1) \leq 1$ for all realized values of R_1, (A6) also implies that $1 + dQ_2^*(Q_1)/dQ_1 \geq 0$.

Substituting $Q_2^*(Q_1)$ into (10) yields a KKT condition for Q_1 that is a function only of Q_1:

$$\phi_1 = E_2\left[F\left(Q_1 + S_2\left(Q_2^*(Q_1), R_2\right)\right)\right]. \tag{A7}$$

Notice,

$$\frac{dRHS}{dQ_1} = E_2\left[f\left(Q_1 + S_2\left(Q_2^*(Q_1), R_2\right)\right)\left(1 + s_2(R_2)\frac{dQ_2^*(Q_1)}{dQ_1} \right) \right]$$
$$\geq E_2\left[f\left(Q_1 + S_2\left(Q_2^*(Q_1), R_2\right)\right)\left(1 + \frac{dQ_2^*(Q_1)}{dQ_1} \right) \right] \geq 0,$$

where the first inequality follows because $s_2(R_2) \leq 1$ while $dQ_2^*(Q_1)/dQ_1 \leq 0$; and the second inequality follows because $1 + dQ_2^*(Q_1)/dQ_1 \geq 0$. Since the LHS of (A7) is independent of Q_1 while the RHS is increasing in Q_1, Q_1^* is uniquely determined by (A7). Thus, the KKT conditions are sufficient for determining optimality.

<u>Parts b and d</u>. From (A5) and (A7), Q_1^* and Q_2^* are defined as the solution to

$$0 = E_j\left[F\left(Q_i^* + S_j\left(Q_j^*, R_j\right)\right)\right] - \phi_i = \int_0^{z_j^*} F\left(Q_i^* + R_j\right) g_j(u)\,du + \int_{z_j^*}^{\infty} F\left(Q_T^*\right) g_j(u)\,du - \phi_i \tag{A8}$$

for i = 1, 2; j = 1, 2; and $i \neq j$. For the purpose of this proof, let $N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right)$ denote the RHS of this equation, where i = 1, 2; j = 1, 2; and $i \neq j$. Then, the following properties follow directly from (A8):

(b1) $N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right) = 0 = N_j\left(Q_j^*, Q_T^*, z_i^*, \rho_i\right)$;

(b2) $$\frac{\partial N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right)}{\partial Q_i^*} = \int_0^{z_j^*} f\left(Q_i^* + R_j\right) g_j(u) du > 0;$$

(b3) $$\frac{\partial N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right)}{\partial Q_T^*} = \int_{z_j^*}^{\infty} f\left(Q_T^*\right) g_j(u) du > 0;$$

(b4) $$\frac{\partial N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right)}{\partial z_j^*} = F\left(Q_i^* + Q_j^*\right) g_j\left(z_j^*\right) - F\left(Q_T^*\right) g_j\left(z_j^*\right) = 0;$$

(b5) $$\frac{\partial N_i\left(Q_i^*, Q_T^*, z_j^*, \rho_j\right)}{\partial \rho_j} = \int_0^{z_j^*} \frac{\partial R_j}{\partial \rho_j} f\left(Q_i^* + R_j\right) g_j(u) du > 0.$$

Given these properties, we next establish how Q_i^*, Q_j^*, and Q_T^* react to changes in ρ_i (for i = 1, 2) by starting with property (b1), taking total derivatives with respect to ρ_i, and then applying properties (b2) – (b5) to the resulting expressions. Accordingly, from properties (b1) and (b4),

$$0 = \frac{\partial N_i}{\partial Q_i^*}\frac{dQ_i^*}{d\rho_i} + \frac{\partial N_i}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i} + \frac{\partial N_i}{\partial z_j^*}\frac{dz_j^*}{d\rho_i} = \frac{\partial N_i}{\partial Q_i^*}\frac{dQ_i^*}{d\rho_i} + \frac{\partial N_i}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i} \tag{A9}$$

$$\Rightarrow \quad 0 = \left(\frac{\partial N_i}{\partial Q_i^*} + \frac{\partial N_i}{\partial Q_T^*}\right)\frac{dQ_i^*}{d\rho_i} + \frac{\partial N_i}{\partial Q_T^*}\frac{dQ_j^*}{d\rho_i}, \tag{A10}$$

where (A10) follows because $Q_T^* = Q_i^* + Q_j^*$. Similarly, from properties (b1), (b4), and (b5),

$$0 = \frac{\partial N_j}{\partial Q_j^*}\frac{dQ_j^*}{d\rho_i} + \frac{\partial N_j}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i} + \frac{\partial N_j}{\partial z_i^*}\frac{dz_i^*}{d\rho_i} + \frac{\partial N_j}{\partial \rho_i} > \frac{\partial N_j}{\partial Q_j^*}\frac{dQ_j^*}{d\rho_i} + \frac{\partial N_j}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i}. \tag{A11}$$

Given properties (b2) and (b3), (A9) implies that

$$sign\left(dQ_i^*/d\rho_i\right) = -sign\left(dQ_T^*/d\rho_i\right), \tag{A12}$$

(A10) implies that

$$sign\left(dQ_i^*/d\rho_i\right) = -sign\left(dQ_j^*/d\rho_i\right), \tag{A13}$$

and (A11) implies that either Q_j^* is decreasing in ρ_i, Q_T^* is decreasing in ρ_i, or both Q_j^* and Q_T^* are decreasing in ρ_i. However, (A12) and (A13) together imply that $dQ_j^*/d\rho_i$ and $dQ_T^*/d\rho_i$ have the same sign. Therefore, both Q_j^* and Q_T^* are decreasing in ρ_i. Consequently, given (A12) and (A13), Q_i^* is increasing in ρ_i.

<u>Part c</u>. By definition,

$$\begin{aligned} SL\left(\mathbf{Q}^*\right) &\equiv E\left[F\left(S_1\left(Q_1^*, R_1\right) + S_2\left(Q_1^*, R_2\right)\right)\right] \\ &= \int_0^{z_1^*}\left[\int_0^{z_2^*} F\left(R_1 + R_2\right) g_2\left(u_2\right) du_2\right] g_1\left(u_1\right) du_1 + \left[1 - G_2\left(z_2^*\right)\right]\left[\int_0^{z_1^*} F\left(R_1 + Q_2^*\right) g_1\left(u_1\right) du_1\right] \\ &\quad + \left[1 - G_1\left(z_1^*\right)\right]\left[\int_0^{z_2^*} F\left(Q_1^* + R_2\right) g_2\left(u_2\right) du_2 + \left[1 - G_2\left(z_2^*\right)\right] F\left(Q_T^*\right)\right]. \end{aligned}$$

Therefore, applying (A8),

$$\begin{aligned} SL\left(\mathbf{Q}^*\right) = \int_0^{z_1^*}\int_0^{z_2^*} F\left(R_1 + R_2\right) g_1\left(u_1\right) g_2\left(u_2\right) du_1 du_2 - \left[1 - G_1\left(z_1^*\right)\right]\left[1 - G_2\left(z_2^*\right)\right] F\left(Q_T^*\right) \\ + \left[1 - G_2\left(z_2^*\right)\right]\phi_2 + \left[1 - G_1\left(z_1^*\right)\right]\phi_1 . \end{aligned} \tag{A14}$$

Let $SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)$ denote the RHS of (A14). Then, for i = 1, 2; j = 1, 2; and i ≠ j, the following properties follow directly from (A14):

(c1) $$\frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial Q_T^*} = -\left[1 - G_1\left(z_1^*\right)\right]\left[1 - G_2\left(z_2^*\right)\right] f\left(Q_T^*\right) < 0 ;$$

(c2) $$\frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial z_i^*} = g_i\left(z_i^*\right)\left[\int_0^{z_j^*} F\left(Q_i^* + R_j\right) g_j\left(u_j\right) du_j + \left[1 - G_j\left(z_j^*\right)\right] F\left(Q_T^*\right) - \phi_i\right] = 0 ;$$

(c3) $$\frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial \rho_i} = \int_0^{z_i^*}\int_0^{z_j^*} \frac{\partial R_i}{\partial \rho_i} f\left(R_i + R_j\right) g_i\left(u_i\right) g_j\left(u_j\right) du_i du_j > 0 .$$

Thus, for i = 1, 2; j = 1, 2; and i ≠ j,

$$\frac{dSL\left(\mathbf{Q}^*\right)}{d\rho_i} = \frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i} + 0 \cdot \left(\frac{dz_i^*}{d\rho_i} + \frac{dz_j^*}{d\rho_i}\right) + \frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial \rho_i}$$

$$> \frac{\partial SL\left(Q_T^*, z_1^*, z_2^*, \rho_1, \rho_2\right)}{\partial Q_T^*}\frac{dQ_T^*}{d\rho_i} > 0 ,$$

where the final inequality follows because $dQ_T^* / d\rho_i < 0$ from the proof of part d. □

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