Returns Policies for a Pessimistic Retailer

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Abstract

Manufacturer buy−back policies are studied in the context of asymmetric demand information. A manufacturer offers a new product for sale to a retailer who makes a single stocking decision prior to the sales period. The two parties formulate different demand forecasts, either because they cannot share all relevant information or because the shared information lacks sufficient credibility. The manufacturer’s profit is, therefore, limited by the retailer’s forecast which is relatively pessimistic. Offering to buy−back unsold items can induce the retailer to order more while supporting higher wholesale prices. It can also signal credibility for the manufacturer’s optimistic forecast. Yet, the actual effect of buy−back depends on how the retailer regards the manufacturer’s information. This paper characterizes two extremes: the retailer either deems the manufacturer to be better informed or less informed about demand. Buy−back serves as a signaling mechanism on the one hand and as an inducement on the other.
Returns Policies for a Pessimistic Retailer

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Manufacturer buy-back policies are studied in the context of asymmetric demand information. A manufacturer offers a new product for sale to a retailer who makes a single stocking decision prior to the sales period. The two parties formulate different demand forecasts, either because they cannot share all relevant information or because the shared information lacks sufficient credibility. The manufacturer’s profit is, therefore, limited by the retailer’s forecast which is relatively pessimistic. Offering to buy-back unsold items can induce the retailer to order more while supporting higher wholesale prices. It can also signal credibility for the manufacturer’s optimistic forecast. Yet, the actual effect of buy-back depends on how the retailer regards the manufacturer’s information. This paper characterizes two extremes: the retailer either deems the manufacturer to be better informed or less informed about demand. Buy-back serves as a signaling mechanism on the one hand and as an inducement on the other.

1. Introduction

A retailer’s demand forecast for a new product is often lower than that of the manufacturer because of a natural skepticism for untried products. The manufacturer may, nevertheless, feel confident in its forecast because of prior market research. To overcome retailer skepticism, manufacturers commonly offer a return policy for unsold units. By taking away some of the retailer’s risk of over stocking, the manufacturer induces larger order quantities at higher wholesale prices than would otherwise be possible under a wholesale-price-only contract. Moreover, a returns policy can lend credibility to a manufacturer’s forecast. Yet, the extent to which a return policy can increase a manufacturer’s profit depends on the retailer’s forecast. And the retailer’s forecast may in turn depend on the manufacturer’s offer because it may give information about true demand.

This paper models the manufacturer profit when a buy-back contract is offered to a skeptical retailer. A manufacturer offers to sell a single product to a retailer. The manufacturer sets the wholesale price and a guaranteed buy-back price for units remaining unsold at the
end of a specified time period. The retailer participates in the contract if its expected profit exceeds a given value. The retailer then orders according to the newsvendor model where the sales price is exogenous.

In such a setting, the manufacturer’s wholesale and buy-back price might offer information about the manufacturer’s forecast. The amount of influence exerted on the retailer’s forecast depends on the retailer’s belief about the quality of the manufacturer’s information. If the retailer believes the manufacturer has inferior information about demand, then the retailer’s forecast remains unchanged after the manufacturer’s offer. If the retailer believes the manufacturer has superior information about demand, then the retailer will adopt the manufacturer’s forecast if it is reported credibly—that is, if there is no profit incentive for the manufacturer to inflate the forecast.

Returns policies occur in many industries where the manufacturer is more optimistic about demand. Bookstores choose among thousands of new titles each year from publishers who were optimistic enough to invest in an author’s work. The retailer ordering decision often comes prior to publication, so the decision is made without access to sample copies. Publishers usually offer buy-back policies to compete for retailer orders. Likewise, fashion apparel and cosmetics are sold by manufacturers who are optimistic about meeting new fashion trends. Retailers usually get a buy-back guarantee or a markdown money guarantee (Tsay 2001) because orders for the entire season must be made before sales data is available.

Newspapers and periodicals are often sold on consignment at convenience stores, newsstands, and grocery stores through some kind of vendor managed inventory agreement. Because demand is a function of news content, the publisher is better informed about potential demand for each new edition. Recognizing this, the retailer depends completely on the publisher forecast by allowing the publisher to choose the stocking quantity. Risk is removed from the retailer by a revenue sharing agreement because the retailer only pays the manufacturer for units sold. The only difference between buy-back and consignment is that funds change hands twice in buy-back and once in consignment. The decision rights over stocking quantity are not unique to consignment. This analysis considers buy-back and consignment as synonymous.

The model in this paper is developed in two scenarios distinguished by the influence that the manufacturer’s offer exerts on the retailer’s forecast. First, the manufacturer’s offer exerts no influence on the retailer’s forecast because the retailer assumes the manufacturer has inferior information. The manufacturer’s problem is to choose the profit maximizing
pair of wholesale and buy-back prices while satisfying the retailer’s requirement for expected profit. The analysis explores the effect of the retailer’s forecast on the optimal wholesale and buy-back price. It also explores the effect of the retailer’s requirement for expected profit as it interacts with the retailer’s forecast to reduce manufacturer profit.

In the second scenario, it is assumed that the retailer believes that the manufacturer has a superior forecast. In this context, there is a single order quantity that both the retailer and the manufacturer would agree maximizes the total channel profit if the retailer knew the manufacturer’s forecast. Knowing the retailer’s forecast can be influenced, the manufacturer has a profit incentive to inflate the retailer’s forecast as much as possible. The analysis explores conditions under which the manufacturer will offer prices that signal its true demand forecast and thus maximize total channel profit.

The next section explains the basic model, and section 3 reviews the related literature. The inferior information scenario is modeled in section 4 and the superior information scenario in section 5. All proofs are in the appendix.

2. The Model

A manufacturer (M) offers a single product for sale to a single retailer (R) in a specified market. The market demand is uncertain, and, for modeling simplicity, the sales price $p$ is exogenous. R represents demand as the random variable $X$ with cumulative distribution function $F_X(x)$ and density function $f_X(x)$. M represents demand as the random variable $Y$ with cumulative distribution function $F_Y(y)$ and density function $f_Y(y)$. It is assumed that $X$ and $Y$ have finite means, $F_X$ and $F_Y$ are invertible, $F_X(u) = F_Y(u) = 0$ for all $u \leq 0$, and $f_X(u) > 0$ and $f_Y(u) > 0$ for all $u > 0$. The results in this paper can also be derived for $X$ and $Y$ having supports with finite upper bounds.

M makes an offer to R that specifies a unit wholesale price $w$ and a unit buy-back price $b$. R accepts M’s offer if there exists an order quantity $q$ that gives R an expected profit of at least $u$. Let $\Pi_r(b, q, w)$ be R’s expected profit from $q$ units of product given $b$ and $w$:

$$\Pi_r(b, q, w) = E[(p - w - h)\min\{q, X\} + (b - w - h)\max\{q - X, 0\}]$$

$$= (p - w - h)q - (p - b)T_X(q)$$

where $h$ is the unit holding cost incurred by R, and $T_X(q) \equiv \int_0^q F_X(x)dx$ is the number of units that R expects to be left unsold. $\Pi_r(q)$ is a concave function with a maximum found...
by solving the first order optimality condition \( F_X(q) = (p - w - h)/(p - b) \). At optimality equation (1) simplifies to \( \Pi_r(q) = (p - b)I_X(q) \), where \( I_X(q) \equiv \int_0^q x f_X(x)dx \).

The unit production cost for M is \( c \) and the salvage value obtained by M for returned units is \( v \geq 0 \), where \( p - h \geq c \geq v \) and \( p \geq b \geq v \). Let \( \Pi_m(b, q, w) \) be M’s expected profit given \( q \), \( w \), and \( b \):

\[
\Pi_m(b, q, w) = \mathbb{E}[(w - c)q - (b - v)\max\{q - Y, 0\}] \\
= (w - c)q - (b - v)T_Y(q)
\]

where \( T_Y(q) \equiv \int_0^q F_Y(y)dy \) is the number of units that M expects to be returned. M wants to choose values of \( w \) and \( b \) that maximize \( \Pi_m(b, q, w) \) subject to R’s first order condition and R’s requirement for \( \Pi_r \). M’s optimization problem is labeled MP:

\[
\text{MP1}
\]

\[
\max_{b,q,w} \Pi_m(b, q, w) \\
\text{subject to: } (p - b)F_X(q) = p - w - h \\
(p - b)I_X(q) \geq u
\]

The first constraint in MP1 is R’s first-order optimality condition and the second is R’s participation constraint.

MP1 extends the model in Kandel (1996) by accounting for the difference between \( F_Y \) and \( F_X \) and by including a retailer participation constraint. The participation constraint is explicitly modeled in this paper because the amount that it diminishes M’s profit depends on \( F_X \).

A benchmark for M’s expected profit is the integrated channel profit computed using \( F_Y \). Let \( \Pi_{mc}(q) \) be M’s expectation for the integrated channel profit given a stocking quantity \( q \):

\[
\Pi_{mc}(q) = (p - c - h)q - (p - v)T_Y(q)
\]

The maximum of \( \Pi_{mc}(q) \) occurs at \( q_{mc}^* \), where \( q_{mc}^* \) is the solution to \( F_Y(q) = (p - c - h)/(p - v) \). If the cost of administering the contract between M and R is negligible, then \( \Pi_{mc}(q_{mc}^*) - u \) is the most profit M could expect from any contract with R. This is true as long as R is
not more optimistic, in some sense, than M. (Otherwise M could exploit R’s optimism by offering a contract that M expects to result in a loss for R).

Note that if channel profit is computed by substituting $T_X$ for $T_Y$ in equation (3), then the solutions to $F_X(q) = (p - c - h)/(p - v)$ would maximize the function. Let $q^*_c$ be that quantity; i.e., $q^*_c$ is the quantity that R would use to maximize channel profit.

Following is the list of key modeling assumptions:

1. $X$ and $Y$ are stochastically ordered according to three properties:
   
   (a) $F_X(q^m_c) \geq F_Y(q^m_c)$
   
   (b) $T_X(q) \geq T_Y(q), \; q \leq q^m_c$
   
   (c) $\frac{d}{dq} \left[ \frac{T_X(q) - T_Y(q)}{T_X(q)} \right] u > \frac{d}{dq} \Pi^m_m(q), \; q > q^m_c$

2. $F_X$ is known by M.

3. $p, h, v,$ and $b$ are common knowledge.

4. The net salvage value is the same for both R and M, namely $v$.

5. If demand exceeds $q$, then neither R nor M incurs additional cost.

Assumption 1 characterizes R’s relative pessimism about demand. Assumption 1 implies $X \geq_{st} Y$ in some special cases. For example, if $X$ and $Y$ are both exponential, then Assumption 1 implies that $E[X] \leq E[Y]$ which implies stochastic dominance. Likewise, if $X$ and $Y$ are both shifted uniform or normal (such that the probability of negative demand is negligible) with equal variances in both cases, then Assumption 1 implies $E[X] \leq E[Y]$ which implies stochastic dominance. However, $X \geq_{st} Y$ is not assumed in general because it would exclude meaningful probability distributions where the the variance of $X$ is greater than the variance of $Y$.

Although Assumption 1 allows R to put greater probability density than M on high demand values, Assumption 1(c) creates a lower bound on $F_X(q) - F_Y(q)$ that prevents M from profiting more than $\Pi^m_m(q^m_c) - u$ by exploiting R’s optimism about high demand values. Thus the relative pessimism of the retailer is characterized by the cost parameters in the channel as well as $F_X$ and $F_Y$.

Let $\Pi^m_r(q)$ be M’s expectation for R’s profit:

$$\Pi^m_r(q) = (p - w - h)q - (p - b)T_Y(q)$$
A consequence of assumption Assumption 1 is that $\Pi^m_r(q) \geq \Pi_r(q)$ for $q \leq q^m_c$. Thus M expects R to have surplus, except under special conditions, unless M convinces R to adopt $F_Y$ as its forecast.

3. Literature Review

Buy-back contracts serve different functions in different settings. They can be a channel coordination mechanism; they can be a means for the more powerful channel member to capture channel profit (with or without regard to channel coordination); and they can signal high demand in the presence of information asymmetry.

Much of the literature on buy-back contracts evaluates them according to the degree to which they promote channel coordination (see Cachon (2002) for a comprehensive survey). Yet, channel coordination has no meaning in the case of asymmetric demand information unless both members of the channel agree on who is better informed. If each member of the channel thinks of itself as the better informed party, then consensus about the channel optimal quantity is unlikely. On the other hand, if a retailer and its supplier agree that the retailer is better informed, then the supplier can offer a channel coordinating buy-back contract without knowing R’s demand distribution (Pasternack 1995). Such a blind offer is not recommended if the supplier considers itself better informed because $q^r_c$ can be significantly lower than $q^m_c$.

Buy-back contracts are also used by manufacturers to extract surplus from the retailer with little or no improvement to channel profit as shown by Kandel (1996), Padmanabhan and Png (1997), and Granot and Yin (2003). Kandel (1996) does conclude, however, that buy-back can be mutually beneficial to the manufacturer and the retailer in the context of a lower retailer forecast. This paper extends Kandel (1996) by specifying the optimal buy-back and wholesale prices as a function of $F_X$ and analyzes the effects of R’s participation constraint on M’s profit.

In the supply chain contracting literature, papers that address asymmetric demand information can be classified into two categories, those that assume the better informed party initiates the contract and those that assume the least informed party initiates the contract. In all cases, however, it is implicitly assumed that both parties agree on who is better informed. Gilbert et al. (2000) model the least informed party as initiating the contract. They analyzes the use of a non-linear wholesale price mechanism for discriminating among
retailers with unknown demand distributions. Papers that model the better informed party as initiating the contract include Desai (2000), Lariviere and Padmanabhan (1997), Chu (1992), and Cachon and Lariviere (2001). Desai (2000), Lariviere and Padmanabhan (1997), and Chu (1992) model slotting allowances (an ex ante fee required by R to stock a manufacturers product) as a means for M to signal high demand and R to screen for high demand products. Cachon and Lariviere (2001) study quantity flexibility contracts as a signaling mechanism that allows the downstream member to credibly forecast demand to its supplier who then makes a production capacity decision.

This paper takes the perspective of the upstream channel member, M, who initiates the contract and who regards itself as better informed. In the case where the downstream member, R, considers itself better informed, M offers buy-back as an inducement to increase the order quantity. In the case where R agrees that M is better informed, M offers buy-back as a signal for high demand.

4. Manufacturer Forecast Deemed Inferior

In this section it is assumed that R believes $F_Y$ to be inferior to $F_X$, regardless of whether or not R knows $F_Y$. R’s belief pertains to M’s ability to understand the market in which R sells and not necessarily the credibility of M’s forecast if it is reported. M’s offer, therefore, exerts no influence on $F_X$.

The buy-back and wholesale prices that maximize $\Pi_m(b, q, w)$ for a given $q$ are denoted $b^*$ and $w^*$, respectively. They are specified in Theorem 1.

**Theorem 1** If $q$ is such that $(p - v)I_X(q) \geq u$, then

$$b^* = p - u/I_X(q)$$

and

$$w^* = p - h - uF_X(q)/I_X(q)$$

A wholesale-price-only contract is optimal (i.e., $b^* = v$) if and only if $u = \Pi_c(q_r^*)$.

Theorem 1 indicates that in the given modeling context, M would prefer buy-back over a wholesale-price-only contract except under the given condition. It also simplifies MP1 into a single variable optimization problem with one constraint. M’s optimal value for $q$ will be denoted $q^*$. In the extreme when $u = \Pi_c(q_r^*)$, M has no alternative but to withhold the
buy-back offer and set \( w^* = c \). Then \( q^* = q^c_r \), and \( R \) captures the entire channel profit, even though \( \Pi^m (q^c_r) \geq \Pi^c (q^c_r) \). If \( u = 0 \), then Proposition 1 implies that \( b^* = p \), \( q^* = q^m_c \), \( w^* = p - h \) and \( M \) captures the maximum channel profit. This is the same result stated in Kandel (1996) for the case when \( F_X (q) = F_Y (q) \), for all \( q \). In such a case, however, \( R \) has no particular incentive to order \( q^* \). \( M \) must, therefore, control the order quantity to achieve optimality. If \( M \) controls \( q \) for any \( u \), then \( R \)’s first-order condition is not relevant. \( M \)’s optimal contract would, therefore, be \( b^* = p \) and \( w^* = p - h - u/q \) for all \( q \), and \( M \) would achieve \( \Pi_m = \Pi^m_c - u \). \( M \) would essentially be renting shelf space from \( R \) at a price of \( u \) and selling directly to the market.

In contrast to buy-back policies described in Pasternack (1985), the optimal solution in Proposition 1 does not guarantee channel coordination. Indeed, the notion of channel coordination is ambiguous in the context of this model because \( F_X \) differs from \( F_Y \). Since the objective of this analysis is to maximize manufacturer profit, let channel coordination be defined as the event that the order quantity maximizes \( M \)’s expectation for channel profit; i.e., channel coordination occurs when \( q^* = q^m_c \). \( M \) cannot realize the full benefit of channel coordination (i.e., \( \Pi_m (w^*, b^*) < \Pi^m_c (q^m_c) - u \)) unless the conditions in the following corollary exists.

**Corollary 1.1** If \( u > 0 \), then \( MP \) yields \( \Pi^m_c - u \) if and only if \( F_X (q^m_c) = F_Y (q^m_c) \) and \( T_X (q^m_c) = T_Y (q^m_c) \).

Under the conditions of Corollary 1.1, the optimal values for wholesale and buy-back price are

\[
\begin{align*}
w^* &= (p - h) - (p - c - h) u/\Pi^m_c \\
b^* &= p - (p - v) u/\Pi^m_c
\end{align*}
\]

The conditions of Corollary 1.1 would not occur in practice, except by pure mathematical coincidence, unless \( M \) and \( R \) shared a common CDF for demand. In general, therefore, it can be concluded that \( R \)’s pessimism precludes channel coordination from \( M \)’s perspective.

To maximize profit in spite of \( R \)’s pessimism Theorem 1 shows that it may be optimal for \( M \) to resort to \( b > w \). This creates, in effect, a net *ex post* payment for holding inventory that may exceed the inventory carrying cost \( h \). Such a payment may be undesirable to \( M \) as pointed out by Kandel (1996) because it introduces a moral hazard. For example, it may
tempt R to order excessively with the hope of forcing M to renegotiate under the threat of massive returns. Or it may tempt R to favor other products that compete for R’s time and shelf space. The next proposition gives the optimal policy, excluding net \textit{ex post} payments to R. It extends the second result in Kandel (1996). Let MP2 be the optimization problem MP1 with the additional constraint \( b \leq w \). Let \( \tilde{w} \), and \( \tilde{b} \) be the optimal wholesale and buy-back prices for any \( q \) that is feasible in MP2, and let \( \tilde{q} \) be the optimal order quantity in MP2. Let \( q_0 \) be the solution to \( b^* = w^* \) and let \( w_0 \) be equal to \( w^* \) when \( q = q_0 \).

\textbf{Corollary 1.2} MP2 has the following properties:

1. If \( q > q_0 \), then \( \tilde{w} = \tilde{b} = p - h/f_X(q) \) and \( \tilde{w} < w_0 \); otherwise, \( \tilde{w} = w^* \) and \( \tilde{b} = b^* \).

2. If \( q^* > q_0 \), then \( q_0 \leq \tilde{q} \leq F_X^{-1}(1 - h/(p - c)) \).

The restriction \( b \leq w \), therefore, reduces M’s profit by decreasing \( w \) and \( q \). It can also give R some expected surplus; i.e., the participation constraint is not necessarily binding at the optimal solution.

Going back to MP1, the participation constraint is binding at optimality. Yet, M expects R to receive a surplus. Besides being subtracted directly from M’s expected profit, \( u \) interacts with R’s forecast to further reduce M’s expected profit. This can be seen by writing out \( \Pi_m(b^*, q, w^*) \):

\[
\Pi_m(b^*, q, w^*) = \Pi_m^o(q) - \Pi_m^r(b^*, q, w^*)
= (p - c - h)q - (p - v)T_Y(q) - u - u \left[ \frac{T_X(q) - T_Y(q)}{I_X(q)} \right]
\]

Thus M expects R to have a surplus of \( u(T_X(q) - T_Y(q))/I_X(q) \). The following results describe how \( q^* \) and M’s expectation for R’s surplus change with \( u \).

\textbf{Theorem 2} If \( \Pi_m(b^*, q, w^*) \) is unimodal in \( q \) for all \( u \in [0, \Pi_r^c(q^c)] \), then \( u > 0 \) has the following effects:

(i) \( D_u[q^*] < 0 \)

(ii) \( D_u \left[ \frac{\Pi_m^r(b^*, q^*, w^*)}{u} \right] < 0 \)

The first effect of \( u \), that it decreases \( q^* \), follows the intuition that as M’s portion of channel profit diminishes M has less incentive to maximize channel profit. Thus \( q^* \) is contained in the
interval \([q_c^u, q_m^m]\) for all \(u \in [0, \Pi_c^r(q_c^u)]\). By way of contrast, the optimal wholesale-price-only contract results in \(q \leq q_c^r\) for all \(u \geq 0\) because \(w \geq c\) and \(b = v\) in the first-order condition for R’s optimal order quantity. The second effect of \(u\) is that M’s expectation for R’s surplus as a fraction of \(u\) decreases with \(u > 0\). If R requires next to no profit, then M expects that \(b^*\) and \(w^*\) increases R’s profit by a significant percentage. If R requires most or all of the channel profit, then M’s offer can be expected to profit R by little more than the required amount. However, the absolute amount of R’s surplus \(\Pi_m^r(b^*, q^*, w^*) - u\) is not monotone in \(u\). It increases with small values of \(u\) and decreases as \(u\) approaches \(\Pi_c^r(q_c^r)\).

5. Manufacturer Forecast Deemed Superior

In some situations R may believe that M has better information about demand and would be willing to use M’s forecast if it were known. Yet, M may not be able to credibly report its forecast under a wholesale-price-only contract because it has an incentive to inflate the order quantity. A manufacturer forecasting higher demand than R must, therefore, offer a contract that a manufacturer with lower demand would not mimic.

This section presents a model of a signaling game between M and R wherein M signals its belief about demand through the wholesale and buy-back prices. Conditions are identified under which a separating equilibrium exists wherein \(F_X\) becomes equal to \(F_Y\). (See Fudenberg and Tirole (1991) for more on signaling games).

In this model, M is characterized as being one of two types: high demand and low demand. M’s type is specified by the parameter \(\theta \in \{H, L\}\) and M’s CDF will now be denoted \(F_Y(y|\theta)\). The expected quantity unsold for an order of size \(q\) is \(T_Y(q|\theta)\). The only distinction between a type H manufacturer and a type L manufacturer is that the latter expects a larger number of unsold items from any given order quantity; more precisely, \(T_Y(q|H) \leq T_Y(q|L)\) for all \(q\).

\(F_X\) is based on R’s belief about \(\theta\). Assuming R knows the functional form of \(F_Y\), let \(\lambda\) be the probability that \(\theta = H\). In the first stage of the game, M offers a unit wholesale price \(w\) and a unit buy-back price \(b\) for any unsold item. R then updates its belief about \(\theta\) using Bayes rule and, in the second stage of the game, orders the quantity that maximizes its expected profit. The posterior distribution for \(\theta\) is \(\mu(\theta|w, b)\). \(F_X\) becomes \(F_X(a|w, b) = \mu(H|w, b)F_Y(a|H) + (1 - \mu(H|w, b))F_Y(a|L)\) for all \(a\). Thus, \(F_X\) has an inverse and \(T_X(q|w, b) \leq T_X(q|w', b')\) for all \(q\) whenever \(\mu(H|w, b) \geq \mu(H|w', b')\). If \(\theta\) is known
to R, then R would agree with M about the order quantity that is channel optimal, which will be denoted $q^\theta_c$.

A consequence of the formula for $F_X$ is that Assumption 1 will not always hold. If $\mu(H|w, b) > 0$ when a type L manufacturer offers $(w, b)$, then $T_X(q|w, b) \leq T_Y(q)$ for all $q$. Then a type L would have an incentive to mislead R into believing it is of type H because R would expect to make at least $u$ while $\Pi_m^L < u$.

In a separating equilibrium, a type $\theta$ manufacturer signals its type by offering $(w, b)$ such that $\mu(\theta|w, b) = 1$. Let

$$w^\theta \equiv p - h - (p - c - h) \frac{u}{\Pi^\theta_c(q^\theta_c)}$$

$$b^\theta \equiv p - (p - v) \frac{u}{\Pi^\theta_c(q^\theta_c)}$$

where $\Pi^\theta_c(q^\theta_c)$ is the maximum expected channel profit given $\theta$. The conditions under which $(w^\theta, b^\theta)$ are equilibrium prices are given in the following theorem.

**Theorem 3** The strategy wherein a type $\theta$ manufacturer offers $(w^\theta, b^\theta)$ and R orders $q^\theta_c$ is a separating equilibrium, if and only if

$$\frac{u}{\Pi^H_c(q^H_c)} < \frac{\Pi^L_c(q^L_c) - \Pi^L_c(q^H_c)}{\Pi^H_c(q^H_c) - \Pi^L_c(q^H_c)}$$

(6)

The separating equilibrium, if it exists, is unique.

Theorem 3 shows that R cannot enjoy the benefit of a separating equilibrium when it requires too much profit. In other words, a type M manufacturer cannot distinguish itself unless R requires less than some amount that is less than total channel profit. Moreover, the existence of a separating equilibrium depends implicitly on the differences between $F_Y(\cdot|L)$ and $F_Y(\cdot|H)$ through the profit functions $\Pi^L_c$ and $\Pi^H_c$. The question thus arises of how precise the buy-back mechanism is at distinguishing between possible manufacturer CDF’s when $F_Y$ is unknown to R. The answer depends on $u$ and $c_c$. When $u$ is small $F_Y(\cdot|L)$ can be closer to $F_Y(\cdot|H)$ in some sense than when $u$ is large. The next section uses a uniform distribution model for $F_Y$, because of its tractibility, to explore the effect of $u$ and $c_c$ on the separating equilibrium.

### 5.1 Uniform Distribution Example

This section illustrates how $u$ and $c_c$ can effect the separating equilibrium for distinguishing between $F_Y(\cdot|H)$ and $F_Y(\cdot|L)$. Let $F_Y(\cdot|L)$ be uniform on the interval $[\alpha, \beta]$, and let $F_Y(\cdot|H)$
be uniform on the interval $[\gamma, \delta]$, where $\gamma > \alpha$. Then

$$\Pi_c^L = 0.5(p-v)(q_c^{L2} - \alpha^2)/(\beta - \alpha)$$

$$\Pi_c^H = 0.5(p-v)(q_c^{H2} - \gamma^2)/(\delta - \gamma)$$

and

$$\Pi_c^L(q_c^H) = \begin{cases} (p-v)((\alpha + \beta)/2 - (1-c_c)q_c^H) & \beta < q_c^H \\ (p-c-h)q_c^H - 0.5(p-v)(q_c^H - \alpha)^2/(\beta - \alpha) & q_c^H \leq \beta \end{cases}$$

where $q_c^L = c_c(\beta - \alpha) + \alpha$, $q_c^H = c_c(\delta - \gamma) + \gamma$, and $c_c = (p-c-h)/(p-v)$.

Let $p = 2$, $c = 1$, $v = 1/2$, $h = 1/5$, $[\alpha, \beta] = [1000,3000]$, and $[\delta, \gamma] = [3000,5000]$. Then $\Pi_c^H = 2826.67$ and $\Pi_c^L = 1226.67$. If $u < 1134.90$, then the separating equilibrium exists and it consists of $(w^H, b^H) = (1.47880, 1.39776)$. Therefore, a retailer who demands more than 40% of $\Pi_c^H$ would not benefit from a credible manufacturer forecast. For example, if $u = 1200$ and $(w,b) = (w^L,b^L)$, then the type L manufacturer expected profit is about 26.67. But if $(w,b) = (w^H,b^H)$ and $\mu(H|w^H,b^H) = 1$, then the type L manufacturer expects a profit of about 91.78. So a type L manufacturer would have an incentive to offer the same prices as a type H manufacturer.

Some general conclusions can be made: First, it can be shown that $b^H > b^L$ and $w^H > w^L$ for all $\alpha < \gamma$ and $\beta < \delta + (\gamma - \alpha)(1-c_c)/c_c$. Second, consider the existence of a separating equilibrium as $F_Y(\cdot|L)$ becomes closer, in some sense, to $F_Y(\cdot|H)$. It can be shown that the threshold given by (6) decreases with $\alpha < \gamma$ whenever $\beta < q_c^H$ or whenever both $\beta \geq q_c^H$ and $\beta - \alpha \leq \delta - \gamma$. The value of $u$ must, therefore, decrease as the location of $F_Y(\cdot|L)$ (as measured by $\alpha$) approaches the location of $F_Y(\cdot|H)$ for a separating equilibrium to exist.

This brings up the question of how sensitive the equilibrium can be to differences in $F_Y(\cdot|L)$ and $F_Y(\cdot|H)$ for $u$ fixed. In the case when $\beta - \alpha = \delta - \gamma$, the threshold in (6) is

$$\frac{u}{\Pi_c^H} \leq \begin{cases} \frac{(\delta - \gamma)(1-c_c)^2 - 2(\gamma - \alpha)(1-c_c)}{(\delta - \gamma)(1-c_c)^2 - 2(\gamma - \alpha)} & \beta \leq q_c^H \\ \frac{\gamma - \alpha}{\gamma - \alpha + 2c_c(\delta - \gamma)} & \beta \geq q_c^H \end{cases}$$

The threshold is decreasing in $\alpha$ over the interval $\alpha \in [0, \gamma]$ taking on values between 0 and the upper bound

$$\frac{\beta(1-c_c)^2 - 2\gamma(1-c_c)}{\beta(1-c_c)^2 - 2\gamma}$$
which occurs when \( \alpha = 0 \). If \( c_c = 0 \), then the upper bound on the threshold is 1. And if \( c_c = 1 \), then the upper bound is 0. A low profit margin, therefore, enables the type H manufacturer to distinguish itself, even when \( R \)'s portion of channel profit is high. A high profit margin makes it difficult for the type H manufacturer to distinguish itself unless \( R \)'s portion is low.

5.2 Ex Ante Profit Sharing Agreement

When \( R \) regards \( M \)'s information as superior, using expected profit as a participation requirement leaves \( R \) vulnerable to be fooled into expecting a higher profit than \( M \) expects. That is because \( R \)'s belief about demand depends on \( M \)'s offer. An alternative participation constraint is for \( R \) to require a fixed portion \( \rho \) of the channel profit. Replacing the participation constraint in MP1 with a profit sharing requirement translates to \( M \) having no alternative to offering \( w^* = p - h - (p - c - h)\rho \) and \( b^* = p - (p - v)\rho \). Let \( \hat{F}_Y \) denote \( M \)'s reported predictive distribution. Under a profit sharing agreement, \( M \) has no incentive to report any value except \( \hat{F}_Y(a) = F_Y(a|\theta) \) for \( a = q^\theta_c \). Indeed \( M \) could credibly tell \( R \) to order \( q^\theta_c \). To do otherwise would reduce channel profit, \( M \)'s portion of which is fixed.

Relating the profit-sharing contract to the signaling game, \((w^*, b^*)\) is equivalent to \((w^H, b^H)\) when \( u = \rho \Pi^H_c \) and it is equivalent to \((w^L, b^L)\) when \( u = \rho \Pi^L_c \). The difference is that an ex ante agreement on \( \rho \) makes \( \Pi_c \) a function \( F_Y \), which may be high or low. Requiring \( \Pi_c > u \) makes \( \Pi_c \) fixed, but it can be a false expectation unless a separating equilibrium exists.

In practice profit sharing agreements are common and this may well be due to a more practical reason for avoiding signaling games other than the pure vulnerability they create. The difficulty to \( R \) of predicting the functional form of \( F_Y \) and the computational burden to \( M \) of computing \((w^\theta, b^\theta)\) is overwhelming in the context of numerous transactions. For example, newspaper consignment agreements are usually based on a revenue split between the retailer and publisher fixed over the long term at an industry standard of about 20/80. The impracticality of engaging in a daily signalling game prior to delivery is obvious.

6. Conclusion

This paper specifies the optimal buy-back and wholesale price for a manufacturer to offer to a single retailer who is pessimistic about demand and who may or may not consider
the manufacturer to be better informed about demand. In contrast, previous literature on asymmetric demand information assumes that both parties agree on who is better informed.

When the manufacturer is deemed to be less informed, the manufacturer expects the retailer’s profit to be greater than the retailer expects it to be. Therefore, the degree to which the manufacturer can overcome retailer pessimism depends on the retailer’s participation requirement for expected profit. As the retailer’s required profit increases, the manufacturer’s optimal (induced) order quantity decreases and the manufacturer’s expectation for retailer surplus also decreases. Channel coordination is not achieved from the perspective of either party except in the extreme cases where one party has the power to capture the entire channel profit.

When the manufacturer is deemed to be better informed, the retailer relies on the wholesale and buy-back prices to predict demand. In the case of two manufacturer types—one expecting high demand and the other expecting low demand—a separating equilibrium exists if and only if the retailer’s required portion of channel profit is sufficiently low. As the two types of manufacturer demand distributions come closer together, a lower portion of channel profits for the retailer is required to distinguish between them in equilibrium. The profit margin of sales price to manufacturing cost attenuates the incentive for a low demand manufacturer to mimic a high demand manufacturer. That is, the lower the profit margin the more definitive the buy-back contract can be at distinguishing between high and low demand manufacturers.

A key assumption in the formulation of the manufacturer’s optimization problem is that the retailer’s probability distribution for demand is known. This, of course, does not represent reality, especially since retailers in practice do not take the time to fully specify probability functions representing their own beliefs about demand. The fact that the manufacturer’s optimal prices can change a lot when the retailer’s probability function changes a little points to the need for further exploration on this topic. An interesting question would be to identify conditions under which a menu of wholesale and buy-back prices could adequately discriminate among retailers who differ by probability function and, to some degree, overcome retailer pessimism.
Appendix: Proofs of Theorems and Corollaries

Proof of Theorem 1

This proof follows much the same logic as the proof of result 2 in Kandel (1996). Substituting w from R first order condition into Π:

\[ Π_M(b, q, w) = (1 - F_X(q))pq + F_X(q)bq - (c + h)q - T_Y(q)(b - v) \]

Then for all q

\[ \frac{d}{db}Π_M = qF_X(q) - T_Y(q) > qF_X(q) - T_X(q) > 0 \]

by Assumption 1(b). Thus, at optimality, the participation constraint is binding because R’s maximum expected profit is decreasing in b. The restriction that \( u \leq (p - v) \int_0^1 xf_X(x)dx \) guarantees the existence of a feasible b value. \( w^* \) follows from substituting \( b^* \) into R’s first-order condition.

If \( u = Π_r(q^*_c) \), then \( b = v, q^*_c, \) and \( w = c \) are the only values that satisfy the constraints. Conversely, suppose that \( b^* = v \). If \( w^* > c \), then R first-order condition implies that \( q^* < q^*_c \). But this contradicts the fact that \( Π_m(v, q^*_r, w^*) \geq Π_m(v, q^*_c, w^*) \). Therefore, \( b^* = v \) implies that \( w^* = c \) which implies that \( q^* = q^*_c \).

Proof of Corollary 1.1

If \( F_X(q^*_m) = F_Y(q^*_m) \) and \( T_X(q^*_m) = T_Y(q^*_m) \), then it is easily shown that \( q^*_m \) is feasible in MP by substituting \( q^*_m \) for q in (4) and (5).

Conversely, if MP yields \( Π^m_e - u \), then \( q^* = q^*_m, q^*_r = q^*_m \) and

\[ Π^m_e(q^*_m) - u = Π^m_e(q^*_m) - Π^m_r(q^*_m) \]

\[ \leq Π^m_e(q^*_m) - Π_r(q^*_m), \text{ [by Assumption 1]} \]

\[ \leq Π^m_e(q^*_m) - u \]

which implies that \( Π^m_e(q^*_m) = Π_r(q^*_m) \). Therefore, \( T_X(q^*_m) = T_Y(q^*_m) \). Substituting \( u = Π^m_r(q^*_m) \) into the participation constraint, then, leads easily to the conclusion that \( F_X(q^*_m) = F_Y(q^*_m) \).
Proof of Corollary 1.2

If \( q \leq q_0 \), then \( b^* \leq w^* \) because \( db^*/dq > dw^*/dq \). Therefore, \( \tilde{w} = w^* \) and \( \tilde{b} = b^* \). If \( q > q_0 \), then \( \tilde{b} = \tilde{w} \) because \( d\Pi_m(b, q, w)/db > 0 \) for all \( v \leq b \leq p \) and \( c \leq w \leq p - h \) by the R’s first-order optimality condition as shown in the proof of Proposition 1. Therefore, substituting \( w = b \) in R’s first-order condition yields the given formula for \( \tilde{b} \) and \( \tilde{w} \), \( q > q_0 \). The fact that \( \tilde{w} \) is strictly decreasing in \( q \) and \( w^* \) is increasing in \( q \) implies that \( \tilde{w} < w^*(q_0) \), \( q > q_0 \).

Substituting \( \tilde{b} \) into the participation constraint simplifies to \( hI_X(q)/F_X(q) \geq u \), which is binding when \( q = q_0 \) by definition. If \( q^* > q_0 \), then \( q_0 \) bounds \( \tilde{q} \) from below because \( hI_X(q)/F_X(q) \) decreases with \( q \). \( \tilde{w} \geq c \) simplifies to the upper bound \( F_X^{-1}(1 - h/(p - c)) \) for \( q \).

Proof of Theorem 2

If \( \Pi_m(b^*, q, w^*) \) is unimodal in \( q \), then \( q^* \) is the solution to the first order condition:

\[
0 = \frac{d}{dq} \Pi_m(b^*, q, w^*) = p - c - h - (p - v)F_Y(q) - u \frac{d}{dq} \left[ \frac{T_X(q) - T_Y(q)}{I_X(q)} \right] \tag{A1}
\]

(i) Let \( q_1 < q^* \) be the solution to (A1) when \( u = u_1 \) for any \( u_1 \in (0, \Pi^*_c(q^*_c)) \). Then for any \( u \in (u_1, \Pi^*_c(q^*_c)) \),

\[
0 = p - c - h - (p - v)F_Y(q_1) - u \frac{d}{dq} \left[ \frac{T_X(q_1) - T_Y(q_1)}{I_X(q_1)} \right] > p - c - h - (p - v)F_Y(q_1) - u \frac{d}{dq} \left[ \frac{T_X(q_1) - T_Y(q_1)}{I_X(q_1)} \right]
\]

because \( p - c - h - (p - v)F_Y(q_1) = d\Pi^*_c(q_1)/dq > 0 \). Therefore, \( q^* < q_1 \) for all \( u \in [u_1, \Pi^*_c(q^*_c)] \). Similarly, \( q_1 < q^* \leq q^*_c \) for all \( u \in [0 < u_1] \).

(ii) It can be shown that

\[
\frac{d}{du} \left[ \frac{\Pi^*_m(q^*)}{u} \right] = \frac{d}{dq} \left[ \frac{T_X(q^*) - T_Y(q^*)}{I_X(q^*)} \right] = \frac{d}{dq} \left[ \frac{T_X(q^*) - T_Y(q^*)}{I_X(q^*)} \right] \cdot \frac{d}{du}[q^*] \tag{A2}
\]

by the chain rule. The first term of the product on the RHS of (A2) must be positive to satisfy the condition in (A1). Therefore, (A2) is negative since \( dq^*/du < 0 \).
Proof of Theorem 3

Let $\theta = H$ and suppose $\mu(H|w^H, b^H) = 1$. Then $(w^H, b^H)$ is a unique solution to MP1 by $F_Y$ invertible. If a type L manufacturer offered $(w^H, b^H)$ its expected profit would be $(w^H - c)q^H_e - (b^H - v)T_Y(q^H_e|L)$. Therefore, a necessary condition for $(w^H, b^H)$ to form a separating equilibrium is $(w^H - c)q^H_e - (b^H - v)T_Y(q^H_e|L) < \Pi^L_c - u$, which, by substitution, simplifies to the right-hand inequality in (6).

Conversely, suppose condition (6) is satisfied, and consider the following strategies and beliefs: The type H manufacturer offers $(w^H, b^H)$ from which R infers that M is type H and therefore orders $q^H_e$; the type L manufacturer offers $(w^L, b^L)$ from which R infers that M is type L and therefore orders $q^L_e$. If the type H manufacturer offered any other contract consisting of $w' \neq w^H$ or $b' \neq b^H$ and if $\mu(H|w', b') = 1$, then $\Pi^H_m(b', q, w') < \Pi^H_c(q^H_e) - u$ because $(w^H, b^H)$ uniquely (by the assumption that $F_Y$ has an inverse) solves MP1. If $\mu(H|w', b') < 1$, then $T_Y(q|H) \leq T_X(q|w', b')$ for all $q$ and so $\Pi^H_r(b', q, w') \geq \Pi^L_r(b', q, w')$.

Therefore,

$$
\Pi^H_m(b', q, w') = \Pi^H_c(q) - \Pi^H_r(b', q, w')
$$

$$
< \Pi^H_c(q^H_e) - \Pi^H_r(b', q, w')
$$

$$
\leq \Pi^H_c(q^H_e) - \Pi^L_r(b', q, w')
$$

Moreover, a type L manufacturer would not offer $(w^H, b^H)$ under condition (6) because $\mu(H|w^H, b^H) = 1$ would be unfavorable. Thus, $(w^H, b^H, q^H_e)$ is a unique perfect Bayesian equilibrium for a type H manufacturer.

If the type L manufacturer offered $w'' \neq w^H$ or $b'' \neq b^H$, then $\mu(H|w'', b'') = 0$ by the fact that $(w^H, b^H, q^H_e)$ is a unique equilibrium point for a type H manufacturer. Thus, $F_X(a|w^H, b^H) = F_Y(a|L)$ for all $a$. Thus, $(w^H, b^H)$ is a unique solution to MP1 when $\theta = L$, and $(w^L, b^L, q^L_e)$ forms a perfect Bayesian equilibrium for a type L manufacturer.

References


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