The Complementary Impacts of e−Markets on Existing Supplier−Buyer Relationships in a Supply Chain

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Abstract

E−markets have been established in many industries as a sourcing option for buyers, yet in many situations they are used to complement long−term supplier−buyer relationships rather than replacing them. In this paper, we analyze the complementary role of e−markets when the buyer uses e−markets as an outside option in bargaining with the traditional supplier. The supplier may choose to make a relationship−specific investment to reduce the production cost. Two scenarios are considered: when the quality of e−market offering is the same as the traditional suppliers and when there is e−market quality uncertainty. We find that under a wide range of conditions, the e−market helps to stimulate the suppliers investment and improve the coordination of the existing supply chain. Therefore, the e−market can complement rather than substitute the traditional relationship−based supply chain. In addition, our result shows that while the buyer always benefits from the e−market option even if the search cost in the e−market is taken into account, the supplier is worse off. When there is quality uncertainty in the e−market offering, two effects of quality uncertainty on e−market adoption are identified, along with the implications for e−market providers who wish to improve e−market adoptions. While better quality on average will increase e−market adoption, it is surprising that increasing quality dispersion of e−market will also help.
E-markets have been established in many industries as a sourcing option for buyers, yet in many situations they are used to complement long-term supplier-buyer relationships rather than replacing them. In this paper, we analyze the complementary role of e-markets when the buyer uses e-markets as an outside option in bargaining with the traditional supplier. The supplier may choose to make a relationship-specific investment to reduce the production cost. Two scenarios are considered: when the quality of e-market offering is the same as the traditional supplier’s and when there is e-market quality uncertainty. We find that under a wide range of conditions, the e-market helps to stimulate the supplier’s investment and improve the coordination of the existing supply chain. Therefore, the e-market can complement rather than substitute the traditional relationship-based supply chain. In addition, our result shows that while the buyer always benefits from the e-market option even if the search cost in the e-market is taken into account, the supplier is worse off. When there is quality uncertainty in the e-market offering, two effects of quality uncertainty on e-market adoption are identified, along with the implications for e-market providers who wish to improve e-market adoptions. While better quality on average will increase e-market adoption, it is surprising that increasing quality dispersion of e-market will also help.

Keywords:
E-market, business-to-business e-commerce, supplier-buyer relationship, bargaining, outside option, quality uncertainty, supply chain

1. Introduction
The late 1990s and early 2000s have seen the rise and fall of e-marketplaces. In the late 1990s, e-markets were established in many industries. They were touted for their ability to help firms get better prices through dynamic pricing as well as lower transaction costs. Yet, along with the dot com shakeout, many e-markets failed dramatically, never reaching their expected potential (Digital Economy 2003, p97). The lack of activities was often blamed for
e-markets’ drastic failure. Many suppliers, especially those with long-term relationships with buyers, were reluctant to participate in e-markets out of the fear that their products would be commoditized and they would be forced to compete on price. On the buyer’s side, the decision of buying into a B2B e-market is often made by people in charge of procurement, who may feel their own role of negotiating deals with and managing relationship with suppliers being threatened by the e-market (Purchasing B2B, 2002). Moreover, e-markets introduce new suppliers, and buyers run a greater risk of acquiring defective or inferior goods by procuring components from suppliers they have not dealt with before (Banham 2000). As a result, e-markets, with the primary focus on offering lower prices, were often associated with high quality uncertainties, adding another disincentive for buyers to adopt e-markets.

Despite the decline, hundreds of e-marketplaces still survived (Laseter and Bodily 2004). For many buyers, e-markets remain as a sourcing option or a benchmarking tool. As time went by, buyers increasingly rely on a balanced mix of individual suppliers and e-markets to fulfill their procurement needs (Purchasing B2B 2002). For example, DowChemical participates in Omnexus, a plastics industry e-marketplace, and Elemica, a B2B exchange for chemical transactions to source their inputs, while running its own private extranet with long-term suppliers (Datz 2002). Grey et al. (2005) report that IBM and Hewlett-Packard use prices from spot markets such as Converge as a benchmarking tool during their negotiation with DRAM suppliers. In this case, buyers can get the best of both worlds—the stability of a long-term relationship as well as the price advantage from the e-market.

Yet the e-market’s role as a complement to buyers’ existing relationship has not been studied extensively. The existing studies on e-markets in the Operations Management and Information Systems literature mainly have dealt with new issues on e-markets, such as network structures (Tomak and Xia, 2002), offering selective subsidy to suppliers (Nickerson and Owan 2002), mitigation negative effects of the quality uncertainty of service products through contingency pricing (Bhargava and Sundareson 2003), the adoption of open platform procurement systems versus proprietary ones (Kauffman and Mohtadi 2004), information privacy (Zhu 2004), the application of options and futures (Wu and Kleidorfer 2005) and the sustainability and growth prospect of e-markets (Galbreth et al. 2005). Among the papers, a notable common theme is the impact of network effects on both sides of the market (Yoo et al. 2002, Nickerson and Owan 2002, Bhargava and Choudhary 2004, Kauffman and Mohtadi 2004, Galbreth et al. 2005). However, only a few papers examine e-market’s role when it co-exists with existing relationships. Lee and Whang (2002), motivated by Hewlett-
Packard’s exchange as a secondary market for resellers to trade their products online, study the dynamics between a manufacturer and its many resellers under the existence of such a secondary market. Kleindorfer and Wu (2002) investigate the integration of spot markets and contract relationships in capital-intensive industries. However, the question that many companies have with the existence of e-markets, “how does it change my relationship with existing suppliers/buyers?” remains unanswered.

In this paper, we depart from the traditional view of the “either-or” choice between online and offline sourcing, or the view of the two as substitutes, and concentrate on explaining the complementary impacts e-markets have on an existing offline buyer-supplier relationship. Specifically, we aim to study the dynamics between a supplier and a manufacturer in a supply chain context when the manufacturer can use e-markets as an outside option to negotiate with the existing supplier (which is also called "long-term supplier" in the paper). Examples of such e-markets include Converge, a global exchange for electronic components, and Alibaba, a global business-to-business e-market based in China where participating suppliers list their negotiable prices on the website.

We model a game of interaction between a supplier and a buyer (manufacturer) where the buyer has an e-market as a sourcing option and sells to the consumer market. The long-term supplier can make a relationship-specific investment to reduce the production cost.

By comparing the market equilibrium with and without the e-market, we find that when the e-market suppliers provide the same quality as the existing supplier, under certain conditions the e-market option for the buyer stimulates the existing supplier’s investment, reduces the purchase price and increases the sales volume, which benefits the buyer. The supply chain profit may increase due to the reduced double-marginalization effect. In addition, we characterize the two effects of e-market quality uncertainty on e-market adoption. From the e-market’s perspective, our results suggest that improving on the overall quality of its product offering and reputation would facilitate e-market adoption. Surprisingly, we find that increasing the quality dispersion may also help.

Placed in a supply chain context with end consumer market, our research also complements the understanding of the relationship between supply chain management and e-markets. The two seem to have opposite focus in the traditional view: supply chain management emphasizes on coordination, relationship and long-term strategic cooperation between vertical partners in the supply chain, while e-markets encourage horizontal competition and support transaction-based cooperation. Our result demonstrates that the e-market option
may help improve supply chain coordinations, showing the complementary role of e-markets in supply chain management.

The paper is organized as follows. In the next section we review relevant literature on e-market and its integration with long-term supplier-buyer relationships. The model is presented in Section 3, in which we analyze the impact of e-markets on supplier-buyer relationships in the traditional supply chain. In Section 4 we examine the effect of e-market quality uncertainty on e-market adoption, followed by the conclusion and discussions in Section 5. Proofs of all the results are in the Appendix.

2. Literature Review

Electronic marketplaces received extensive attention from researchers recently. This is evidenced by the recent special issue of Management Science on electronic markets, as well as the e-business special issue of Management Science with a significant proportion of papers discussing e-markets. However, as stated in the introduction by Anadalingam and Raghavan (2005), all but three papers study auctions. Overall, the papers in this special issue all deal with e-market as a separate way to buy or sell without considering the traditional, non-e-market or auction trading relationships. While they offer insights for e-market design and management, they do not explain if and how e-markets can sustain in the existence of the “old way” of trading-relationship-based contracts. When an e-market exists for the buyer who is in a one-to-one relationship with a long-term supplier, how does it change the negotiation result between the two partners? Will it lead to better supply chain coordination? How will the classic underinvestment result (Williamson 1985, Grout 1984) change? These are important questions that need to be answered for e-market to adopt its deserving role in e-business.

Related to the questions above is the research on the integration of spot markets and long-term contracts. Wu et al. (2002) study contracts between one supplier and one or multiple buyers in a capital-intensive industry where capacity is costly and has to be reserved well in advance of production. Subsequent papers by Kleindorfer and Wu (2003) and Wu and Kleindorfer (2005) have a similar set-up. Lee and Whang’s (2002) model studies the two-period trading between the manufacturer and the resellers. Their research is motivated by an electronic market in the hi-tech industry that featured short product life cycle and high uncertainty in demand. The resellers order from the manufacturer in the first period, and then trade inventories among themselves in a secondary market. They observe two
effects in the secondary market, a quantity effect on the sales of the manufacturer that is indeterminate, and an allocation effect that always improves supply chain performance. Moreover, the resellers are always better off given the secondary market.

In a more recent study, Tunca and Zenios (2006) examine the competition between the e-market procurement auctions and long-term relational contracts. The manufactures use a reverse auction for the procurement of low-quality parts and a relational contract for procuring high-quality parts. They identify conditions under which the two procurement channels coexist or one dominates the other. They find that high quality premium will discourage the existence of the e-market.

Grey et al. (2005) survey the e-market literature to answer the question "how can e-markets add value to supply chains in which long-term buyer-supplier relationships are important"? Their extensive survey identifies three benefits of e-markets: better resource allocation, better information aggregation and dissemination and better risk management. Better resource allocation refers to better matching of supply and demand. While theoretically elegant, the market approach is shadowed by long-term supplier-buyer relationship in the real world, thanks to fragmentation, imperfect competition, externalities or transaction costs in many industries. Therefore, the authors point out, e-market can serve a complementary role in industries—the majority of transactions are still conducted between suppliers and buyers in the traditional way while e-market serves as a channel for suppliers to offload excess inventory and for buyers to address periodic shortages. They also suggest that e-market prices can serve as a benchmark for contract price negotiations as well as for the supplier to evaluate investments in production capacity. Yet as a survey paper, it does not offer any analytical results.

In this paper, along the same dimension, we examine the role of e-markets in price negotiations and relationship-specific investments. Recognizing the importance of the long-term supplier-buyer relationship, we argue that e-market can be used not only as a channel to address supply or demand fluctuations but also an option for the buyer to utilize in negotiating with the supplier, therefore changing the supplier’s relationship-specific investments beforehand.

Our work differs from Wu’s stream of work with his co-authors in the role of e-markets. In their work, contracts between the supplier and the buyer are negotiated in advance while e-markets or spot markets serve as a backup when demand is realized. In our paper, we do not consider the capacity constraint. Instead, the supplier can choose to make relationship-
specific investments to lower the cost of production. Furthermore, the e-market is only used by the buyer as an outside option in negotiation with the supplier. The buyer still prefers to buy from the supplier, if the supplier can match the e-market price. Our work is also different from Tunca and Zenios (2006) in that we study the complementary role of e-market to the relational contract when the buyer uses the e-market as an outside option to pressure on the long-term supplier in the negotiation.

3. The Model

Consider a two-firm supply chain with a supplier and a buyer (manufacturer). With an existing relationship between the two firms, the buyer (referred to as "she") negotiates with the supplier (referred to as "he") on the wholesale price contract and makes purchases. The buyer sells products to the consumer market where she faces a downward-sloping demand. Assume the inverse demand function is \( r = a - bQ \), for \( 0 \leq r \leq a \), where \( r \) is the retail price and \( Q \) is the demand.\(^1\)

Besides the long-term supplier, there is an e-marketplace where buyer can search to find the price quotes from participating suppliers. We use the term "search cost" to encompass costs in finding supplier information, verifying supplier information, posting RFQs and qualifying suppliers, etc. She can then use the price quote as a benchmarking tool in the bargaining with the long-term supplier. The buyer first invests in searching to find a good price in the e-market\(^2\). The relationship between the buyer’s investment in searching and the e-market price he could get is not fully studied in the literature. To efficiently describe the relationship without too much distraction from the purpose of our paper, we make the following assumptions: the e-market price buyer could get is \( t = t_0 - x_b \), where \( t_0 \) is the e-market price if no searching is done, and \( x_b \) is the amount of price reduction achieved by her investment on searching. The cost of investment is given by \( I_b = \alpha_b x_b^2 \), where \( \alpha_b > 0 \) is a scaling factor. The more investment the buyer makes in searching, the better (lower) price she can enjoy. Moreover, this function models the decreasing returns to the investment on searching: getting a better price becomes increasingly difficult as she searches more.\(^3\)

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\(^1\)Also used in Tunca and Zenios (2006), the linear demand model is a standard assumption to capture consumers’ aggregate willingness-to-pay.

\(^2\)While we use the e-market in its singular form throughout the paper, the concept does not map to just one e-market—it corresponds to a collection of e-markets serving as a potential source from which the buyer can find the product in question.

\(^3\)From now on, we use the term "search cost" to represent buyer’s investment in searching.
After the buyer acquires the e-market price, she brings it to the supplier and will use it as the outside option in the bargaining process. The supplier then decides whether to participate in the bargaining. If he decides to participate, he can first make some relationship-specific investment to lower the unit production cost, as a way to improve the margin and the profit. That is, the cost reductions are only effective for the transactions with this buyer. For example, improving the information system linked with this buyer is one of such investments. Assume the unit production cost is \( c = c_0 - x_s \), where \( c_0 \) is the unit cost if no relationship-specific investment is made, and \( x_s \) is unit cost reduction achieved by the supplier’s investment. The cost of investment is given by \( I_s = \alpha_s x^2_s \), where \( \alpha_s > 0 \) is a scaling factor. Therefore, the supplier can achieve lower production cost if he invests more, but it becomes increasingly difficult as the investment becomes larger. Similar assumptions have been made in the literature to model the relationship between unit production cost and R&D expenditures (Gupta and Loulou 1998).

After the cost-reduction investment is made, the bargaining process begins. The buyer uses the e-market price as the outside option to negotiate with the supplier over the transfer price (the wholesale price) \( p \). We assume that the buyer values the long-term relationship with the supplier, so that even though she looks for a lower price in the e-market, it is only used as an outside option to negotiate with the supplier. If the supplier can offer the same or lower price than the e-market, then she will still keep her business with the supplier. She goes to the e-markets only if the e-market price cannot be matched by the supplier. The reason that the buyer does not choose the e-market at the first step could be the large switching cost or the uncertainty about the e-market suppliers. The buyer usually has less information about the e-market suppliers, and therefore is less assured of the product quality they provide compared to the existing supplier. For example, IBM and HP use prices from spot markets as a benchmarking tool during their negotiation with DRAM suppliers (Grey et al. 2005).

Finally, the buyer sells the products to the consumer market, choosing the retail price \( r \) and ordering the corresponding quantity \( Q \) from the supplier who fulfills the order.

To evaluate the impact of e-markets on the existing supplier-buyer relationships, we first present two benchmarking models in Sections 3.1 and 3.2. The first model provides the centralized solution (first-best) for the supply chain when there is no e-market for the buyer. In the second model, the buyer and the supplier’s strategic investments and the equilibrium solution without the presence of e-market are derived. Then in 3.3 we present the main
model where the buyer uses the e-market as a bargaining tool in the procurement process with the long-term supplier. The sub-game perfect Nash equilibrium is characterized, along with the conditions under which the buyer will procure from the long-term supplier versus from the e-market, and the conditions under which the outside option is binding. Section 3.4 further analyzes the impact of e-market on the supplier-buyer relationships.

In Section 3, it is assumed that the e-market suppliers provide the same product quality as the long-term supplier does. This may apply to situation where the procurement products are commodities. 4

Section 4 further extends the model by incorporating quality uncertainties of e-market products, which is applicable to products whose quality may vary a lot.

3.1 First-best solution when there is no e-market

When no e-markets exist, the centralized optimization problem for supply chain is to choose the cost-reduction investment $I_s$ and the retail price $r$ to maximize the channel profit:

$$
\pi_c(I_s, r) = (r - c)Q - I_s = \left( r - \left( c_0 - \frac{I_s}{\alpha_s} \right) \right) \frac{a - r}{b} - I_s
$$

where the subscript $c$ refers to the channel. An equivalent problem is to choose the optimal unit cost reduction $x_s$ and the selling quantity $Q$ to maximize

$$
\pi_c(x_s, Q) = (r - c)Q - I_s = (a - bQ - (c_0 - x_s))Q - \alpha_s x_s^2
$$

subject to $0 \leq x_s \leq c_0, Q \geq 0$.

The following lemma gives the centralized solution for the supply chain.

**Lemma 1.** (First-best) Assuming $a \geq c_0$, i.e. the trade is feasible even if the supplier does not invest in cost reduction, and $\alpha_s > \frac{a}{4bc_0}$, the investment cost is high enough so that it is impossible for the supplier to reduce the unit production cost to zero, the centralized optimal solution is: 5

$$
I_s^c = \alpha_s \frac{(a - c_0)^2}{(4b\alpha_s - 1)^2}, x_s^c = \frac{a - c_0}{4b\alpha_s - 1}, r_s^c = \frac{2b\alpha_s(a + c_0) - a}{4b\alpha_s - 1}, Q_s^c = \frac{2\alpha_s(a - c_0)}{4b\alpha_s - 1} \quad (1)
$$

The centralized solution will be used to compare with the decentralized solution with and without the presence of e-markets.

4 Most of the products sold at Alibaba and Converge are commodity-like products.

5 The superscript $c$ denotes the centralized solution.
3.2 Equilibrium investment and contract without e-market

The analysis of this model aims to provide a benchmark to measure the impact of the e-market on the supplier-buyer relationships.

If there is no e-market for the buyer, the sequence of events is the following:

Stage 1. The supplier determines his investment $I_s$ to reduce production cost.

Stage 2. The buyer and the supplier bargain over the transfer price $p$.

Stage 3. The buyer orders quantity $Q$ from the supplier and sells to the consumer market at retail price $r$.

We will find the equilibrium investment and contract through backward induction.

At the last stage, after the transfer price $p$ is settled, the buyer chooses her optimal order quantity $Q$ and retail price $r$ to maximize her profit $\pi_b(Q) = (r - p)Q = (a - bQ - p)Q$, where the subscript $b$ represents the buyer. The optimal order quantity and retail price for the buyer are given by

$$Q(p) = \frac{a - p}{2b}, r(p) = \frac{a + p}{2} \quad (2)$$

Working backwards, at Stage 2 the supplier and the buyer bargain over the transfer price $p$, anticipating the retail price that the buyer is going to charge. We use the cooperative bargaining model to solve the transfer price. Let $u_s, u_b$ represent the supplier’s utility and the buyer’s utility, the Nash Bargaining Solution at Stage 2 is the solution of the following problem:

$$\max_{u_s, u_b} u_s \cdot u_b$$
subject to

$$(u_s, u_b) \in \Omega, u_s \geq 0, u_b \geq 0$$

where $\Omega = \{(u_s, u_b) : u_s = (p - c)Q(p), u_b = (r(p) - p)Q(p)\}$

The problem is equivalent to $\max_p \frac{(p-c)(a-p)^2}{8b^2}$, subject to $0 \leq x_s \leq c_0, Q \geq 0$.

Solving the optimization problem yields the Nash Bargaining Solution

$$p = \frac{a + 3c}{4} \quad (3)$$

Note that as the supplier invests more in cost reductions, the transfer price decreases along with the unit production cost. Therefore, the buyer also benefits from the cost reductions as evidenced by the increased margin $(r(p) - p) = \frac{a - p}{2}$ and selling quantities $Q(p) = \frac{a - p}{2b}$. Although supplier’s margin $(p - c) = \frac{a - c}{4}$ and selling quantities also increase with his investment, he does not enjoy the full benefit of cost reductions.
When the supplier needs to choose the amount of investment at Stage 1, he has to strike the balance between the benefit and the convex and increasing investment cost. Taking into account possible reactions at the subsequent stages, he determines the optimal investment $I_s$ (or equivalently, the unit cost reduction $x_s$) to maximize his profit $\pi_s = (p - c)Q(p) - I_s$.

From (2) and (3), the supplier’s problem is to choose the optimal unit cost reduction $x_s$ for profit maximization:

$$\max_{x_s} \pi_s(x_s) = \frac{3(a - c_0 + x_s)^2}{32b} - \alpha_s x_s^2$$

subject to

$$0 \leq x_s \leq c_0.$$ 

The solution is $x_s = \frac{3(a-c_0)}{32b\alpha_s - 3}$ under the previous assumption $a \geq c_0$ and $\alpha_s > \frac{a}{4bc_0}$.

Substitute the solution into equation (2) and (3), we obtain the equilibrium solution as described by the following proposition.

**Proposition 1.** *(Decentralized Solution without e-Markets)* When there is no e-market, the equilibrium investment, transfer price, retail price and selling quantities are:

$$I_d = \frac{9\alpha_s(a - c_0)^2}{(32b\alpha_s - 3)^2}, x_d = \frac{3(a - c_0)}{32b\alpha_s - 3}, p^d = \frac{8\beta\alpha_s(a + 3c_0) - 3a}{32b\alpha_s - 3} \quad (4)$$

$$r^d = \frac{4\beta\alpha_s(5a + 3c_0) - 3a}{32b\alpha_s - 3}, Q^d = \frac{12\alpha_s(a - c_0)}{32b\alpha_s - 3}$$

With bilateral bargaining between the supplier and the buyer, the supplier under-invests in cost reductions, which results in higher unit production cost, higher retail price and less channel profit compared to the first-best solution.

The proof of the second part follows by comparing the first best solution in (1) and the equilibrium solution in (4). Proposition 1 states that the supplier under-invests in the decentralized setting, which is because he can only internalize half of the benefit of the investment. Although the buyer would like larger investment by the supplier, she cannot give him any additional incentive for investment. As a consequence, the unit production cost is higher, the buyer charges higher retail price and sells less quantities than the first-best solution.
3.3 Equilibrium investments and contract with e-market as an outside option

With the existence of an e-market, in order to get the lowest price the buyer has to invest in time and effort to search, which is presented by $I_b$. The e-market price $t$ is then used as an outside option for the buyer in negotiating with the supplier over the transfer price $p$. If the supplier offers a lower price or matches the same price, the buyer will buy from the supplier. Otherwise, the buyer will exercise the e-market option.

The timeline is shown in Figure 1: First, the buyer determines her investment $I_b$ to search in the e-market and finds the lowest price $t$. After the supplier is informed of the e-market price, he decides whether to participate in the bargaining and determines his investment $I_s$ to reduce unit production cost to $c$. Then the bargaining process begins, with the buyer using the e-market as an outside option in the negotiation with the supplier over the transfer price $p$. The buyer orders quantity $Q$ from the supplier—either the existing one or the e-market one—and sells to the consumer market at retail price $r$.

Next, we solve through backward induction the unique subgame perfect Nash equilibrium and the conditions under which the e-market is effective in improving the existing supply chain performance are derived.

3.3.1 Supplier’s optimal investment

At the last stage, buyer’s optimal retail price and selling quantities are the same as in equation (2). At Stage 3, the buyer, with a price quotation $t$ from e-markets as an outside option, bargains with the supplier over the transfer price $p$. If no agreement is reached in the bargaining, the buyer’s utility is $(r(t) - t)Q(t)$, as she will procure from the e-market. The Nash bargaining solution is then given by the following lemma.
Lemma 2. (Transfer Price with e-Markets) With e-markets as outside option for the buyer, the Nash Bargaining Solution is

\[
p = \begin{cases} 
\frac{a+3c}{t} & \text{if } t \geq \frac{a+3c}{4} \\
\frac{a+3c}{4} & \text{if } t < \frac{a+3c}{4}
\end{cases}
\] (5)

The outside option renders the buyer the privilege to get the lower price among \(\frac{a+3c}{4}\) and \(t\): if the supplier invests a lot and proposes a price lower than e-market price \((\frac{a+3c}{4} < t)\), then the buyer takes the proposed price; otherwise, she uses the e-market price. Therefore, the e-market price is effective only when the supplier does not invest enough. It thus provides a stimulus for the supplier to invest in cost reductions.

At Stage 2, supplier has the right to decide whether to participate in the bargaining. Assume the supplier’s reservation utility is 0. If the e-market price is so low that it is difficult to gain profit whether he invests or not, he may not participate in the bargaining at all. We separate the supplier’s decision process of participation and investment into two steps: he first solves for the optimal investment if he participates in the bargaining, then decides whether to participate by comparing the optimal profit with the reservation utility.

Rewrite equation (5) as

\[
p(x_s) = \begin{cases} 
\frac{a+3c-3x_s}{4t} & \text{if } x_s \geq \frac{a+3c-4t}{3} \\
\frac{a+3c-4t}{3} & \text{if } x_s < \frac{a+3c-4t}{3}
\end{cases}
\]

The supplier solves \(\max_{x_s}(p(x_s) - c(x_s))Q(p(x_s)) - I_s(x_s) = (p(x_s) - c_0 + x_s)(\frac{a-p(x_s)}{2b}) - \alpha_s x_s^2\), which yields the following solution.

Proposition 2. (Supplier’s Optimal Investment) If the supplier decides to participate in the bargaining, his optimal cost-reduction investment is:

\[
x_s(x_b) = \begin{cases} 
\frac{3(a-c_0)}{32\alpha_s-3} & \text{if } x_b < x_B^1 \\
\frac{a+3c_0-4a_1+4x_B}{3} & \text{if } x_B^1 < x_b < x_B^2 \\
\frac{a-t_a+x_B}{4\alpha_s} & \text{if } x_b > x_B^2
\end{cases}
\] (6)

where \(x_B^1 = -(a - t_0) + \frac{24a_1(a-c_0)}{32\alpha_s-3}\), \(x_B^2 = -(a - t_0) + \frac{12a_1(a-c_0)}{16\alpha_s-3}\).

Figure 2 shows the supplier’s optimal investment as a function of the buyer’s investment in searching. When the buyer’s investment, \(x_b\), is small, the e-market has no effect on supplier’s investment as the e-market price is not low enough. As \(x_b\) exceeds a threshold, \(x_B^1\), the supplier feels threatened and his investment increases with the buyer’s investment in searching. Therefore, the buyer can stimulate the supplier’s investment by investing more
on searching for e-market price. A lower e-market original price $t_0$ has a similar effect in encouraging supplier’s cost reduction investment.

The following proposition provides the supplier’s optimal participation policy, which is a threshold policy.

**Proposition 3.** (Supplier’s Participation Policy) The supplier participates in the bargaining if and only if the buyer’s investment is low enough, i.e. $x_b < X_b$, where $X_b$ is given by:

$$X_b = \begin{cases} \delta_1 & \text{if } b\alpha_s \leq \frac{3}{8b_0} \\ \delta_2 & \text{if } b\alpha_s > \frac{3}{8b_0} \end{cases}$$

where $\delta_1 = -(a - t_0) + \frac{3\delta(a - c_0)}{4\delta - 1}$, $\delta_2 = -(a - t_0) + \frac{8b_0(a - c_0)}{8b_0 - 1}$, $\delta = \sqrt{\frac{2b_0\alpha_s}{3}}$.

The threshold $X_b$ is decreasing in $\alpha_s$.

Proposition 3 states that if the buyer invests a lot in searching and obtains an e-market price that is too low for the supplier to gain any profit whether he invests in cost reduction or not, the supplier would rather not participate. The threshold decreases with the search parameter $\alpha_s$, which implies the supplier is less likely to participate as the cost reduction investment becomes more costly.

### 3.3.2 Buyer’s optimal investment in searching and the equilibrium

Now we characterize the buyer’s optimal investment in searching at the beginning of the game. Anticipating the supplier’s participation strategy and investment strategy, the buyer determines her optimal investment, described by the following proposition.

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7With our previous assumption $\alpha_s > \frac{a}{4b_0}$.
Proposition 4. *Buyer’s Optimal Search* The buyer’s optimal investment strategy in searching the e-market is a threshold policy:

\[ x_b^* = \begin{cases} 
0 & \text{if } \gamma < K_3 \text{ (i.e. } t_0 > a - (a - c_0)K_3) \\
\frac{a-t_0}{4ba_0 - 1} & \text{if } \gamma > K_3 \text{ (i.e. } t_0 < a - (a - c_0)K_3) 
\end{cases} \]

where \( \gamma = \frac{a-t_0}{a-c_0} \), \( K_3 = \frac{24ba_0}{32ba_0 - 3} \sqrt{1 - \frac{1}{4ba_0}} \).

Figure 3 shows the buyer’s optimal investment in searching. Conventional wisdom tells us that the higher the original e-market price is, the more the buyer has to search to get a low price, so buyer should invest more as \( t_0 \) goes higher. However, we see that the buyer’s optimal investment is actually decreasing in \( t_0 \) and jumps to zero at a certain threshold. This seemingly surprising result can be better understood after we examine the buyer’s incentive to invest. If the original e-market price is high, then getting a low enough price is hard and costly for the buyer, and if the price \( t \) she gets after searching is not low enough, the outside option will not be effective. Therefore she would rather not invest at all if \( t_0 \) is high, and just procures from the long-term supplier. That explains the jump to 0 at the threshold. If \( t_0 \) is low enough, getting a low e-market price is easier and the buyer has incentive to search to get the benefit: the supplier has to match the low price if he participates; even if the supplier does not participate, she could still buy from the e-market at this low price. In the same vein, as \( t_0 \) decreases further, the buyer will search more to capture the benefit of low price.

From Proposition 2, 3 and 4, we derive the subgame perfect Nash equilibrium of the game stated in the following theorem.
Theorem 1. The subgame perfect Nash equilibrium of the game is described by the following tables:

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Region in Fig.4</th>
<th>$x_b^*$</th>
<th>$x_s^*$</th>
<th>Outside option binding?</th>
<th>Procurement channel*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &lt; K_3$</td>
<td>$I$</td>
<td>0</td>
<td>$\frac{3(a-c_0)}{32b_0a_0-3}$</td>
<td>No</td>
<td>$S$</td>
</tr>
<tr>
<td>$K_3 &lt; \gamma &lt; K_5$</td>
<td>$II$</td>
<td>$\frac{a-t_0}{4b_0a_0-1}$</td>
<td>$c_0 - a + \frac{16b_0a_0(a-t_0)}{3(4b_0a_0-1)}$</td>
<td>Yes</td>
<td>$S$</td>
</tr>
<tr>
<td>$\gamma &gt; K_5$</td>
<td>$III$</td>
<td>$\frac{a-t_0}{4b_0a_0-1}$</td>
<td>$\frac{a_0(a-t_0)}{a_0(4b_0a_0-1)}$</td>
<td>Yes</td>
<td>$S$</td>
</tr>
</tbody>
</table>

* Whether procures from e-market (E) or long-term supplier (S)

If $b_0 a_0 > \frac{3}{8}$,

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Region in Fig.4</th>
<th>$x_b^*$</th>
<th>$x_s^*$</th>
<th>Outside option binding?</th>
<th>Procurement channel*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &lt; K_3$</td>
<td>$I$</td>
<td>0</td>
<td>$\frac{3(a-c_0)}{32b_0a_0-3}$</td>
<td>No</td>
<td>$S$</td>
</tr>
<tr>
<td>$K_3 &lt; \gamma &lt; K_5$</td>
<td>$II$</td>
<td>$\frac{a-t_0}{4b_0a_0-1}$</td>
<td>$c_0 - a + \frac{16b_0a_0(a-t_0)}{3(4b_0a_0-1)}$</td>
<td>Yes</td>
<td>$S$</td>
</tr>
<tr>
<td>$K_4 &lt; \gamma &lt; K_6$</td>
<td>$III$</td>
<td>$\frac{a-t_0}{4b_0a_0-1}$</td>
<td>$\frac{a_0(a-t_0)}{a_0(4b_0a_0-1)}$</td>
<td>Yes</td>
<td>$S$</td>
</tr>
<tr>
<td>$\gamma &gt; K_6$</td>
<td>$III$</td>
<td>$\frac{a-t_0}{4b_0a_0-1}$</td>
<td>$\frac{a_0(a-t_0)}{a_0(4b_0a_0-1)}$</td>
<td>Yes</td>
<td>$S$</td>
</tr>
</tbody>
</table>

where $K_4 = \frac{12b_0a_0}{6b_0a_0-3}(1 - \frac{1}{4b_0a_0})$, $K_5 = \frac{36}{43-1}(1 - \frac{1}{4b_0a_0})$, $K_6 = \frac{8b_0a_0}{8b_0a_0-1}(1 - \frac{1}{4b_0a_0})$.

The ratio $\gamma = \frac{a-t_0}{a-c_0}$ represents the degree of e-market threat for the supplier: it is low if original e-market price $t_0$ is high and/or initial cost $c_0$ is low, where the supplier would not worry about the e-markets; it is high if $t_0$ is low and/or $c_0$ is high, where the supplier would feel a lot of pressure from e-markets. We refer to $\gamma$ as the "pressure ratio".

Theorem 1 states that when the pressure ratio is sufficiently low, the e-market does not have any effect on the existing supplier-buyer relationship—buyer does not invest in searching while supplier would participate and invest in cost reductions, and the final transfer price is less than the e-market price. When the pressure ratio is sufficiently high, the buyer invests in searching, but matching the e-market price is too costly for the supplier that he would rather quit, so the buyer exercises the e-market option. In the region where the pressure ratio is moderate, buyer invests in searching, supplier participates and invests in cost reduction, and the outside option is binding.

Figure 4 illustrates Theorem 1. When the original e-market price $t_0$ is high enough or the initial cost $c_0$ is low enough (Region (I)), the buyer procures from the long-term supplier without investing in searching, as it is unrealistic to find a low e-market price. In contrast, when $t_0$ is low or $c_0$ is high (Region (III)), the buyer exercises the e-market option, as it is unprofitable for the supplier to match the e-market price. In Region (II) where $t_0$ and $c_0$ are
The observation that the buyer searches in the e-markets but finally procures from the long-term supplier has an interesting implication: if more and more buyers use the e-market as outside options, we may expect to see an increase in e-market price quotations without a significant increase in the transaction volume through e-markets. This may apply to the situation where the e-market is not mature and has not established the reputation so that the buyers would rather buy from the long-term suppliers and use the e-markets purely as a bargaining tool. More implications of this result are discussed in Section 5.

In Section 3.4, we will compare the equilibrium outcome with and without the e-market options to show the impact of e-markets on supplier-buyer relationships.

### 3.3.3 Comparative statics

Now we examine how the equilibrium outcome in Theorem 1 changes with market characteristics and investment cost parameters. The results are summarized in Table 1.

The four parameters have similar directional effects on the equilibrium transfer price, retail price, selling quantity and supplier and buyer’s profit. It is consistent with our intuition that their increases do not benefit and are even harmful for the supply chain efficiency.
effects of the parameters on supplier and buyer’s investments are worth discussing: As the search parameter $\alpha_b$ or the original e-market price $t_0$ increases, buyer invests less in searching the e-market. With less pressure to match the e-market price, the supplier invests less in cost reductions too. On the other hand, as the cost reduction parameter $\alpha_s$ increases, supplier invests less, but the buyer is *more willing to search* to pressure on the supplier: because if she does not, she will get a higher trading price in the bargaining. The initial production cost $c_0$ does not affect buyer’s investment, but it may change the supplier’s investment in two directions: if the pressure ratio $\gamma$ is low, his investment decreases with $c_0$; if the pressure ratio is moderate, his investment increases with $c_0$ to match the e-market price; if $\gamma$ is sufficiently high, his investment remains unchanged with $c_0$.

Table 1. The impact of parameter changes on the equilibrium outcome with e-market options

<table>
<thead>
<tr>
<th>Equilibrium outcome</th>
<th>$\alpha_s$</th>
<th>$\alpha_b$</th>
<th>$c_0$</th>
<th>$t_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_s^*$</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>$x_b^*$</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>$p^*$</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>$r^*$</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>$\pi_s^*$</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>$\pi_b^*$</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
</tr>
</tbody>
</table>

"-" denotes no change

To better understand Theorem 1, it helps to look at how the likelihood of each outcome in Theorem 1 changes with the parameters. As the search parameter $\alpha_b$ decreases or the cost reduction parameter $\alpha_s$ increases, the e-market is more likely to be effective in the bargaining and the buyer is more likely to exercise the e-market option. A decrease in the original e-market price $t_0$ or an increase in the initial production cost $c_0$ has a similar effect in making e-market more effective. These characterize the conditions under which the e-market plays a more active role in the existing supplier-buyer relationships.

### 3.4 The impact of e-market on the supplier-buyer relationships

To examine the impact of e-market on the existing supplier-buyer relationships, we contrast our findings from Section 3.3 to the no e-market case of 3.2. The comparison provides insights on the complementary role of e-market in the existing supply chain. We will examine how the e-market option affects the procurement contract with the long-term supplier, the supplier’s
investment in cost reductions, and the supply chain performance by contrasting all relevant
statistics from the two cases. The following proposition provides the results.

**Proposition 5.** *(Payoff Change)* When \( t_0 \) is sufficiently low or \( c_0 \) is sufficiently high (when \( \frac{a-t_0}{a-c_0} > K_3 \)), the supplier’s investment in cost reduction is larger under the competition from the e-market\(^8\), the total selling quantity is larger, the buyer and the consumers are better off, while the supplier is worse off than the benchmark no e-market case.

When \( t_0 \) is high or \( c_0 \) is low, the e-market option does not affect the equilibrium. When \( t_0 \) is sufficiently low or \( c_0 \) is sufficiently high (i.e. the pressure ratio is high), using e-market as an outside option stimulates supplier’s investment (as long as he participates), lowers the trading price and improves the buyer’s profitability whether she procures from the long-term supplier or the e-market. The long-term supplier is always worse off under the competition from the e-market, although he invests more in cost reductions to reduce the profit loss. The consumer surplus is higher due to the lower retail price and larger quantity sold. Consumers always benefit from the introduction of e-market competition.

Now we analyze the impact of e-market on supply chain profit, which is the sum of the buyer and the existing supplier’s profit. The effects of the e-market option on supply chain performance are two-sided: on one hand, the supplier’s underinvestment is moderated and the transfer price is lower, which reduces the double-marginalization and improves supply chain efficiency (if the supplier participates); on the other hand, there is the additional search cost to make the e-market option effective. If the benefits of reduced double-marginalization outbid the search cost, the supply chain profit increases. Otherwise it decreases. The exact conditions for which the channel profit increases or decreases are not in a clean form. To provide more insights, we first examine the extreme case where \( \alpha_s \rightarrow \infty \) and \( \alpha_b \rightarrow \infty \). The condition \( \alpha_s \rightarrow \infty \) applies to the case when the supplier does not have the ability or opportunity to invest in cost reductions because of, say, the budget or time constraint. The condition \( \alpha_b \rightarrow \infty \) refers to the perfectly competitive e-market where the e-market price is exogenously determined. We find that the supply chain profit always increases as long as \( \gamma \), the e-market pressure ratio, is large enough. Then we extend the result to the case with more general \( \alpha_s \) and \( \alpha_b \). The following proposition characterizes the conditions under which supply chain profit increases under the e-market option.

\(^8\)This only applies to the case when the supplier is willing to participate. If supplier does not participate, the investment is zero.
Proposition 6. (a) When the supplier does not have the option to invest in cost reductions and the e-market is perfectly competitive, i.e. $\alpha_s \to \infty$ and $\alpha_b \to \infty$, the supply chain profit is higher than the no e-market case if the pressure ratio is high enough ($\gamma > \frac{3}{4}$), and stays unchanged otherwise.

(b) For $\alpha_s > \frac{3}{8b}$ and large enough $\alpha_b$, there exists $\gamma$ such that the supply chain profit is higher than the benchmark no e-market case when $\gamma > \gamma$.

Part (a) of Proposition 6 states that when the e-market is perfectly competitive and supplier does not have the option to invest, the supply chain profit increases as long as the pressure ratio is large enough so that the outside option is binding ($\gamma > \frac{3}{4}$). As the supplier does not have the option to invest, the effect of e-market as the outside option is acted upon the transfer price—the e-market option gives the buyer a leverage for a better price, which indirectly reduces the double-marginalization effect and thus improves the supply chain efficiency. Since there is no search cost, the channel profit will always increase if the e-market outside option is binding.

In the more general case, Part (b) states that when the cost-reduction investment is costly, the e-market option helps to stimulate supplier’s investment and improve supply chain profit if the pressure from the e-market is high enough. More specifically, if the e-market is almost perfectly competitive and the original e-market price is sufficiently low, using e-market in the bargaining process leads to a much lower transfer price and reduced double-marginalization without incurring significant search cost. Therefore, the benefits from improved supply chain efficiency outbid the buyer’s search cost, resulting in a better supply chain performance.

4. The Impact of Quality Uncertainty on e-Market Adoption

In the models above, we assume that e-market suppliers provide goods of the same quality as the long-term supplier does and it is known to the buyer. In reality, the buyer may not know the quality of the e-market offering. In this section, we extend our model to the setting where there is quality uncertainty of the products provided by the e-market suppliers.

Assume the product quality provided by the long-term supplier is $s_0$, which is a constant. The quality of goods provided by e-market supplier is $s = s_0 - \varepsilon$, where $\varepsilon$ is a random variable with support $[-\infty, s_0]$, mean $\mu$ and variance $\sigma^2$ which are all public information.
The buyer will not know the e-market quality realization if she procures from the long-term supplier. However, if she decides to procure from the e-market supplier, the quality uncertainty is realized after she commits to the purchase and before she sells to the consumer market. Therefore, the buyer can adjust the retail price according to the quality realization to maximize her profit. When the buyer sells the products to the end market, customers can detect the quality, either through consumer review or word of mouth. Therefore, the inverse demand function is assumed to be \( r = a - bQ + s \).

The timeline is based on that described in Figure 1 but expanded as follows: If the buyer decides to procure from the e-market, the quality uncertainty of e-market products is realized, then she determines the ordering quantity \( Q \) and sells to the consumer market at price \( r \).

Next we will find the equilibrium outcome and analyze the impact of e-market quality uncertainty on e-market adoption.

### 4.1 Supplier’s optimal investment when there is e-market quality uncertainty

At the last stage, if the buyer purchases from the e-market, the quality uncertainty is realized before she sells to the consumer market. With the consumer demand increasing in the quality delivered, the buyer’s optimal order quantity and retail price are given by \( Q(p, s) = \frac{a + s - p}{2b} \) and \( r(p, s) = \frac{a + s + p}{2} \), both of which are increasing in the realized e-market quality \( s \). If the buyer procures from the long-term supplier, the optimal order quantity and selling price are \( Q(p, s_0) \) and \( r(p, s_0) \).

In the bargaining process where the buyer uses e-market as an outside option, the quality uncertainty of e-market product turns the buyer’s utility of purchasing from e-market into a random variable. Assume the buyer is risk-neutral, then she uses the expected profit of purchasing from e-market \( E_s[(r(t, s) - t)Q(t, s)] \) as the utility of the outside option. The bargaining solution is then given by the following lemma.

**Lemma 3.** Under e-market quality uncertainty, the Nash Bargaining Solution is

\[
p = \begin{cases} 
\frac{a + s_0 + 3c}{4} & \text{if } t \geq a + s_0 - \mu - \sqrt{\frac{9}{16} (a + s_0 - c)^2 - \sigma^2} \\
\tilde{p} & \text{if } t < a + s_0 - \mu - \sqrt{\frac{9}{16} (a + s_0 - c)^2 - \sigma^2}
\end{cases}
\]

where \( \tilde{p} = a + s_0 - \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} \).
The bargaining price is decreasing in $\sigma^2$ and increasing in $\mu$. For the same expected e-market quality, the buyer could get a lower price in the bargaining as the variance of quality increases. While this may be surprising, the underlying rationale is that if the realized e-market quality is high, the buyer can charge a price premium which yields a profit gain that outweighs the profit loss when the realized e-market quality is low. Therefore, the buyer benefits from the variance of e-market quality and can get a lower bargaining price as the quality variance increases. If we fix the variance of quality, the bargaining price increases as the expected e-market quality decreases (i.e. as $\mu$ increases). Due to the information asymmetry— it is difficult for the buyer to assess e-market qualities—the e-market suppliers have an incentive to provide lower qualities to save production costs. Therefore, even if the buyer gets a low e-market price $t$, she may not be able to get a low price in the bargaining if the expected e-market quality is low.

In Stage 2, the supplier decides whether to participate in the bargaining. If he participates, the optimal investment is given by the following proposition.

**Proposition 7.** Under e-market quality uncertainty, if the supplier decides to participate in the bargaining, his optimal investment is:

$$x_s(x_b) = \begin{cases} \frac{3(a + s_0 - c_0)}{32b_0a_0 - 3} & \text{if } x_b < x_{B^1}\varepsilon \\ \frac{4}{3}g(x_b) - (a + s_0 - c_0) & \text{if } x_{B^1}\varepsilon < x_b < x_{B^2}\varepsilon \\ \frac{4}{3}g(x_b) & \text{if } x_b > x_{B^2}\varepsilon \end{cases}$$

where

$$x_{B^1}\varepsilon = \sqrt{\left[\frac{24b_0a_0}{32b_0a_0 - 3}(a + s_0 - c_0)\right]^2 - \sigma^2 - (a + s_0 - \mu - t_0)},$$

$$x_{B^2}\varepsilon = \sqrt{\left[\frac{12b_0a_0}{16b_0a_0 - 3}(a + s_0 - c_0)\right]^2 - \sigma^2 - (a + s_0 - \mu - t_0)}.$$
$g(x_b) = \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2}. \text{With the assumption that}$

$$\sigma^2 \leq \left[ \frac{24b\alpha_s}{32b\alpha_s - 3}(a + s_0 - c_0) \right]^2$$  \hspace{1cm} (A1)

Figure 6 shows how supplier’s optimal investment changes with $\mu$ and $\sigma^2$. The supplier’s investment is increasing in $\sigma^2$ and decreasing in $\mu$. As $\sigma^2$ increases, the supplier feels more pressure from the e-market competition and invests more to reduce the production cost. However, as $\mu$ increases, the expected e-market quality decreases, the supplier feels less pressure and thus invests less.

The supplier’s participation policy under e-market quality uncertainty is given by the following proposition, which mirrors the result in Proposition 3.

**Proposition 8.** Under e-market quality uncertainty, the supplier participates in the bargaining if and only if $x_b < X_{be}$, where $X_{be}$ is given by:

$$X_{be} = \begin{cases} 
\delta_{1e} & \text{if } b\alpha_s \leq \frac{3}{8\alpha_s - 1} \\
\delta_{2e} & \text{if } b\alpha_s > \frac{3}{8\alpha_s - 1}
\end{cases}$$

where $\delta_{1e} = \sqrt{\left[ \frac{3\delta}{4b - 1}(a + s_0 - c_0) \right]^2 - \sigma^2 - (a + s_0 - \mu - t_0)}$, $\delta_{2e} = \sqrt{\left[ \frac{8b\alpha_s}{8b\alpha_s - 1}(a + s_0 - c_0) \right]^2 - \sigma^2 - (a + s_0 - \mu - t_0)}$.

The threshold is decreasing in $\sigma^2$ and increasing in $\mu$, so the supplier is more likely to participate if the expected e-market quality is low and less likely to participate if the variance of quality is high, with the same reasoning for supplier’s optimal investment strategy.
4.2 Buyer’s optimal investment and equilibrium outcome when there is e-market quality uncertainty

The buyer’s optimal investment in searching at Stage 1 is given by the following proposition.

**Proposition 9.** *Under e-market quality uncertainty, the buyer’s optimal investment strategy is:*

\[
x_b^* = \begin{cases} 
0 & \text{if } \gamma_\varepsilon < \sqrt{1 - \frac{1}{4b_0}}, \\
\frac{a + s_0 - \mu - t_0}{4b_0 - 1} & \text{if } \gamma_\varepsilon > \sqrt{1 - \frac{1}{4b_0}},
\end{cases}
\]

where \( \gamma_\varepsilon = \frac{a + s_0 - \mu - t_0}{\sqrt{\frac{24b_0}{(a + s_0 - c_0)^2}}}. \)

Similar to the result in Proposition 4, the buyer will search and use the e-market price to bargain only if \( t_0 \) is low enough. However, as the expected e-market quality decreases (where \( \mu \) increases), the e-market becomes less valuable for the buyer and so she invests less in searching.

The following theorem characterizes the equilibrium outcome of the game under e-market quality uncertainty.

**Theorem 2.** *Under e-market quality uncertainty, the subgame perfect Nash equilibrium is described by the following tables:*

<table>
<thead>
<tr>
<th>Conditions</th>
<th>( x_b^* )</th>
<th>( x_s^* )</th>
<th>Outside option binding?</th>
<th>Procurement channel*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_\varepsilon \leq K_{3\varepsilon} )</td>
<td>0</td>
<td>( \frac{3(a + s_0 - c_0)}{32b_0 - 3} )</td>
<td>No</td>
<td>S</td>
</tr>
<tr>
<td>( \gamma_\varepsilon &gt; K_{3\varepsilon} )</td>
<td>( \beta_\varepsilon &lt; K_{5\varepsilon} )</td>
<td>( \frac{\beta_\varepsilon}{4}(\gamma_\varepsilon - 1)(a + s_0 - c_0) )</td>
<td>Yes</td>
<td>S</td>
</tr>
<tr>
<td>( K_{3\varepsilon} )</td>
<td>( \beta_\varepsilon &gt; K_{5\varepsilon} )</td>
<td>( x_{b_0}^* )</td>
<td>N/A</td>
<td>E</td>
</tr>
</tbody>
</table>

**Figure 7:** Buyer’s optimal investment with e-market quality uncertainty.
* Whether procures from e-market (E) or long-term supplier (S)

If \( b \alpha_s > \frac{3}{8} \),

<table>
<thead>
<tr>
<th>Conditions</th>
<th>( x_b^* )</th>
<th>( x_s^* )</th>
<th>Outside option binding?</th>
<th>Procurement channel*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_e &lt; K_{3e} )</td>
<td>0</td>
<td>( \frac{2(a + s_0 - c_0)}{3b\alpha_s - 3} )</td>
<td>No</td>
<td>S</td>
</tr>
<tr>
<td>( \gamma_e &gt; K_{3e} ) ( \beta_e &lt; K_{4e} )</td>
<td>( x_{bc}^* )</td>
<td>( \frac{4}{3}(\beta_e - 1)(a + s_0 - c_0) )</td>
<td>Yes</td>
<td>S</td>
</tr>
<tr>
<td>( K_{4e} &lt; \beta_e &lt; K_{6e} )</td>
<td>( x_{bc}^* )</td>
<td>( \frac{2}{4b\alpha_s}(a + s_0 - c_0) )</td>
<td>Yes</td>
<td>S</td>
</tr>
<tr>
<td>( \beta_e &gt; K_{6e} )</td>
<td>( x_{bc}^* )</td>
<td>N/A</td>
<td>N/A</td>
<td>E</td>
</tr>
</tbody>
</table>

where \( K_{3e} = \sqrt{1 - \frac{1}{4b\alpha_s}} \), \( K_{4e} = \frac{12b\alpha_s}{16b\alpha_s - 3} \), \( K_{5e} = \frac{34}{4b\alpha_s - 1} \), \( K_{6e} = \frac{8b\alpha_s}{8b\alpha_s - 1} \), \( x_{bc}^* = \frac{a + s_0 - \mu - t_0}{4b\alpha_s - 1} \), \( \beta_e = \frac{\left[ \frac{4b\alpha_s}{4b\alpha_s - 1} (a + s_0 - \mu - t_0) \right]^2 + \sigma^2}{a + s_0 - c_0} \).

The overall structure of the equilibrium results is the same as in Theorem 1. Here we will focus on the impact of e-market equality uncertainty on the equilibrium outcomes by contrasting the results in Theorem 2 to Theorem 1. The effect is shown in Table 2.

Table 2. The effect of quality uncertainty on the equilibrium outcomes

<table>
<thead>
<tr>
<th>( \sigma^2 )</th>
<th>( \mu )</th>
<th>( p^* )</th>
<th>( x_s^* )</th>
<th>( x_b^* )</th>
<th>( \pi_s^* )</th>
<th>( E\pi_b^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
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As the variance of e-market quality increases, the buyer searches more to gain the benefit of e-market, and the supplier invests more on cost reductions in face of the e-market competition. The buyer gains the benefit while supplier is worse off. This looks counterintuitive and we will give the explanations in the next section.

As the expected e-market quality decreases, e-market option becomes less powerful in the bargaining and the procurement price increases. Anticipating this, the buyer invests less in searching for e-market price, and so the supplier reduces his investment too. The long-term supplier benefits from the decreased e-market quality while the buyer suffers.

Further interpretations and managerial insights are provided in the next section.

4.3 The impact of e-market quality uncertainty on e-market adoption

There are two parts to the effect of quality uncertainty, the variability of the quality and the expected value of the quality. Before solving the model, our intuition was that if the buyer is uncertain about e-market quality, she is more likely to buy from the long-term supplier, and the e-market is less likely to be adopted. However, our result indicates that as the variability
of e-market quality increases, the buyer is more likely to adopt the e-market. More intuitive of the results is the impact of the expected value of e-market quality. As it decreases, the buyer is less likely to adopt the e-market.

Accordingly, we define the two conflicting forces at play: the "quality asymmetry effect", brought by $\sigma^2$, and the "information asymmetry effect", brought by $\mu$.

The quality asymmetry effect refers to the effect of the variance of quality. As the consumers are willing to pay more for goods of better quality, the buyer can charge a price premium if the realized e-market quality is high, which leads to profit gains that more than offset the profit loss when the e-market quality turns out to be low. Therefore, the variance of quality increases the buyer’s expected utility if she procures from the e-market, and consequently decreases the bargaining price, forcing the supplier to invest more. As a result, buyer’s profit increases and she is more willing to search or adopt the e-market. The quality asymmetry effect increases with the variance $\sigma^2$.  

The information asymmetry effect, on the other hand, refers to the effect of asymmetric information in the e-market. Because it is difficult for the buyer to assess the e-market product qualities beforehand, and the contracts with e-market suppliers are usually short-term, the e-market suppliers have an incentive to provide lower quality to save the production cost. Therefore we would expect $\mu > 0$. Compared to the results in Section 3 where $\mu = 0$, the lower expected e-market quality reduces the buyer’s expected utility if purchasing from e-markets, and therefore increases the bargaining price and relieves the pressure for supplier’s investment. As a consequence, the buyer’s profit decreases and she is less likely to search or adopt the e-market. The larger the $\mu$ is, the larger the information asymmetry effect.

The final outcome is the combined effect and depends on the relative magnitude of the two effects. If the information asymmetry effect dominates the quality asymmetry effect, the buyer is less likely to adopt the e-market; otherwise, the e-market is more likely to be adopted.

Our results also have implications for e-market providers. When the original e-market price is low but not low enough (the middle case of Theorem 2), the buyer gets a price quote from the e-market, but ends up purchasing from the long-term supplier. In other words, e-market is used purely as a benchmarking tool in bargaining. How could the e-market

---

9The quality asymmetry effect still holds if we use a more general inverse demand $r = a - bQ + \varphi(s)$, where $\varphi(s)$ is convex and increasing in $s$, meaning that the premium the buyer could charge for high quality goods is more than the discount the buyer have to give for low quality goods.
providers possibly react to this non-revenue generating involvement? Our analysis points out two approaches for helping the e-market providers. First, increasing the average product quality from e-market suppliers could make the buyer more likely to adopt e-markets. It is important for the e-market to establish a quality image and good reputation so that the e-market suppliers are more respected for their often lower price. However, for small e-market providers who may not be able to improve on the average quality of their offerings, our result suggests that increasing the quality dispersion may help, and may even make the buyer more likely to adopt the e-market if she can adjust the price according to the realized quality.

5. Conclusions

In this section, we will draw conclusions first, and then provide implications of our results for industry practitioners, and discuss key assumptions and future research directions at the end.

In this paper, we study the impact of e-market on the long-term supplier-buyer relationship when it is used by the buyer as an outside option in the negotiation process. We place it in a supply chain context where the supplier can make relationship-specific investment to reduce the production cost. Comparisons between the market equilibrium with and without e-market option are drawn. Our results show that if the pressure ratio, a measure of the severity of the e-market threat for the supplier, is sufficiently low, the existing supplier-buyer relationship is not affected. On the other hand, if the pressure ratio is sufficiently high, the long-term supplier will not be able to compete with the low-price e-market, and therefore the procurement channel is switched to the e-market. When the pressure ratio of the e-market is moderate, however, it stimulates the long-term supplier to increase his relationship-specific investment and win the procurement contract. The supply chain profit increases if the benefit of improved channel coordination (reduced double-marginalization) overrides the cost of searching the e-market, which occurs if the cost-reduction parameter is large enough and the e-market is near perfectly competitive. The consumer welfare always increases in the presence of e-market options.

We further analyze the effects of e-market quality uncertainty on e-market adoption, which are two-sided. On the one hand, the information asymmetry gives the e-market suppliers incentive to provide goods of lower quality, which eventually hurts the buyer’s profit and e-market adoption; on the other hand, the buyer may benefit from the variability of the
e-market quality if she can adjust the consumer price accordingly. From the perspective of e-market suppliers, improving on the quality and reputation of e-market would facilitate its adoption. From the buyer’s point of view, the e-market quality uncertainty can be utilized if she could implement pricing in response of the realization of quality or the demand.

The results carry rich managerial implications for players in industries with the presence of the e-market. For the buyer, using e-market as the outside option can help her achieve better profitability, but if she wants to keep the long-term supplier for the reliability, she should use an e-market price that is not aggressively low. For the long-term supplier, his profit suffers with the threat of e-market competition. He has to agree on a lower price and may even lose the procurement contract. The underlying reason is that the supplier is passive and can only work with the buyer without any outside option of its own. Our results imply that the supplier will also have an incentive to participate in the e-market, maybe a different one, to gain some bargaining power in the negotiation, even if they do not actually have any transactions. This explains why more and more suppliers are indeed participating in e-markets, besides selling additional capacity. In turn, the growing e-market suppliers will attract more buyers to the e-market, which creates a positive feedback. Hence, we expect to see more participation in e-markets if more buyers use the e-market option in the bargaining.

For the e-market providers, however, our result shows that the large volume of price quotations may not imply large volume of transactions if more buyers use the e-market as outside options. This may apply to the situation where the e-market is not mature and has not established the reputation so that the buyers would rather buy from the reliable long-term suppliers and use the e-markets purely as a bargaining tool. Our results suggest that the e-market providers can boost the e-market sales/adoptions by improving on the quality and reputation of the e-market products. Indeed, such practice is common for many e-market providers. For example, Converge has developed quality control processes to ensure that the product sold through the exchange market meets the quality standards (http://www.converge.com/www-webapp/info/20_finance_logistics.jsp). As acknowledged by the CEO of Converge, Frank Cavallaro, the quality problem is critical in the success of Converge, and all its reputation is built on the quality. Another way is to make the e-market price less visible for non-serious buyers. In the late 90’s, many newly opened e-markets made the price list easy to access to attract buyers. Now they made the price less

visible by requiring buyer registration before releasing the product information (Converge) or by listing only the negotiable prices on the website (Alibaba). Our result helps to explain their behavior—to attract only the serious buyers.

One unique feature of our model is that we assume the system is “open”, in the sense that the long-term supplier and the e-market suppliers are different, and can sell to the buyer only through the designated channel. By contrast, existing models on the co-existence of contract-based relationship and e-markets (e.g. Tunca and Zenios 2006) study a “closed” system, where all players can participate in both channels. Our model provides an alternative observation of the industry, where firms in the e-market, if not trading surplus products, are often lesser known, smaller-sized ones who hope to reach potential buyers—whom they otherwise would have no way of reaching—through the e-market. 11 By contrast, the existing supplier, often bigger and more established than its e-market counterparts, does not want to join the e-market for fear of competing only on price. While buyers prefer to do business with the existing supplier for quality assurance, they still use the e-market as a benchmark for bargaining. We think this reflects the diversity of suppliers in many industries.

Although the model presented is a one-shot game, it actually applies to the long-term contracts, where the existing supplier is willing to make relationship-specific investments. We also studied the multi-period game where the long-term supplier’s relationship-specific investment is cumulative. We find that in the dynamic game, the long-term supplier will invest more and his chance of winning the contract over the e-market suppliers is larger than in the static game. (We omitted the analysis due to the length constraint.) The results provide insights that complement our findings in the static game.

Some technical assumptions are made to facilitate the analysis. First, we do not explicitly consider the switching cost. However, we do implicitly consider the effect of switching cost when we assume that the buyer prefers the long-term supplier over the e-market if she could reach the same profitability using the two options. With switching cost, e-market will be less likely to be searched or adopted. As the switching cost becomes sufficiently large, the e-market will have no impact on the traditional supply chain at all. Additionally, we model the cost of investment in the e-market option as a quadratic function for sake of analytical tractability. The same qualitative results are expected to hold under more general convex

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11 A good example is Alibaba.com, one of the most successful global B2B e-markets serving small and medium-sized enterprises (SME’s). Based in China, with its over two million members, Alibaba offers a gateway for SME’s in China and developing countries to reach buyers from around the world.
functions, but the analysis would be cumbersome and is left as a topic for future research.

Future research may focus on empirically testing the propositions derived from our theoretical model. Specifically, studies of those companies who use e-market as a bargaining tool may help explain whether the impact of e-market on IT investment (relationship-specific investment), procurement price and each party’s economic performances predicted in this paper holds in real-world environments.

Appendix

Proof of Lemma 1

The centralized decisions are solutions of the following problem:

$$\max_{0 \leq x_s \leq c_0, Q \geq 0} \pi_c(x_s, Q)$$

We solve the unconstrained problem first. From first order conditions

$$\frac{\partial \pi_c}{\partial Q} = 0, \frac{\partial \pi_c}{\partial x_s} = 0 \Rightarrow Q = \frac{a - c_0 + x_s}{2b}, x_s = \frac{Q}{2\alpha_s}$$

Second order conditions

$$\frac{\partial^2 \pi_c}{\partial Q^2} = -2b < 0, \frac{\partial^2 \pi_c}{\partial x_s^2} = -2\alpha_s < 0, H = 4b\alpha_s - 1 > 0$$

if we assume $b\alpha_s > \frac{1}{4}$.

The solutions

$$x_s = \frac{a - c_0}{4b\alpha_s - 1}, Q = \frac{2\alpha_s(a - c_0)}{4b\alpha_s - 1}$$

satisfies the constraints $0 \leq x_s < c_0, Q \geq 0$ if we assume $a \geq c_0$ and $\alpha_s > \frac{a}{4b\alpha_0}$. Then use $I_s = \alpha_s x_s^2$ and $r = a - bQ$ to get optimal investment and retail price. ◇

Proof of Lemma 2

The Nash Bargaining Solution is the solution of the following problem:

$$\max_{u_s, u_b} u_s \cdot u_b$$

subject to

$$(u_s, u_b) \in \Omega, u_s \geq 0, u_b \geq 0$$

where $\Omega = \{(u_s, u_b) : u_s = (p - c)Q(p), u_b = (r(p) - p)Q(p), u_b \geq (r(t) - t)Q(t)\}$

$(r(t) - t)Q(t)$ represents the buyer’s utility if no agreement is reached in the bargaining, as she will procure from e-market.

29
We first solve the optimization problem without the constraint \( u_b \geq (r(t) - t)Q(t) \), which is \( p = \frac{a+3c}{4} \). Then the constraint solution is
\[
p = \begin{cases} \frac{a+3c}{4} & \text{if } (r(\frac{a+3c}{4}) - \frac{a+3c}{4})Q(\frac{a+3c}{4}) \geq (r(t) - t)Q(t) \\ t & \text{if } (r(\frac{a+3c}{4}) - \frac{a+3c}{4})Q(\frac{a+3c}{4}) < (r(t) - t)Q(t) \end{cases}
\]

The conditions \( (r(\frac{a+3c}{4}) - \frac{a+3c}{4})Q(\frac{a+3c}{4}) \geq (r(t) - t)Q(t) \) are equivalent to \( t \geq (\frac{a+3c}{4}) \), so the result follows. \( \diamond \)

**Proof of Proposition 2**

At Stage 2, if the supplier chooses
\[
x_s \geq \frac{a+3c_0-4t_0+4x_b}{3}
\]
which implies \( t \geq \frac{a+3c}{4} \) at the next stage, then the transfer price is \( \frac{a+3c}{4} \). So the seller’s problem is:
\[
\max_{x_s} \frac{3(a - c_0 + x_s)^2}{32b} - \alpha_s x_s^2
\]
The objective function is maximized at:
\[
x_s^* = \max \left\{ \frac{3(a - c_0)}{32b\alpha_s - 3}, \frac{a + 3c_0 - 4t_0 + 4x_b}{3} \right\} \tag{9}
\]

And we define \( x_B^1 \) as the value of \( x_b \) where the two terms are equal:
\[
x_B^1 = -(a - t_0) + \frac{24b\alpha_s(a - c_0)}{32b\alpha_s - 3}
\]

Similarly, if the supplier chooses
\[
x_s < \frac{a+3c_0-4t_0+4x_b}{3}
\]
which implies \( t < \frac{a+3c}{4} \) at the next stage, then the transfer price is \( t \). The seller’s problem is:
\[
\max_{x_s} \frac{(t_0 - x_b - c_0 + x_s)(a - t_0 + x_b)}{2b} - \alpha_s x_s^2
\]
The objective function is maximized at \( \frac{a+t_0-x_b}{4b\alpha_s} \). Combining the pre-condition, we have
\[
x_s^* = \min \left\{ \frac{a - t_0 + x_b}{4b\alpha_s}, \frac{a + 3c_0 - 4t_0 + 4x_b}{3} \right\} \tag{10}
\]
And we define \( x_B^2 \) as the value of \( x_b \) where the two terms are equal:
\[
x_B^2 = -(a - t_0) + \frac{12b\alpha_s(a - c_0)}{16b\alpha_s - 3}
\]

We can show \( x_B^1 < x_B^2 \). So the value of \( x_b \) has three cases:

1. When \( x_b < x_B^1 < x_B^2 \), \( x_s^* = \frac{3(a - c_0)}{32b\alpha_s - 3} \).
2. When \( x_B^1 < x_b < x_B^2 \), \( x_s^* = \frac{a+3c_0-4t_0+4x_b}{3} \).
3.
3. When $x_B^1 < x_B^2 < x_b$, $x_s^* = \frac{a-t_0+x_b}{4b\alpha_s}$. $\diamond$

**Proof of Proposition 3**

From Proposition 2, supplier’s optimal investment, transfer price and profit are:

Case 1. If $x_b < x_B^1 < x_B^2$, then $x_s = \frac{3(a-c_0)}{32b\alpha_s-3}, p = \frac{a+3c}{4}, \pi_s = (p-c)\frac{a-t}{2b} - \alpha_s x_s^2 = \frac{3a(a-c_0)^2}{32b\alpha_s-3} > 0$.

Case 2. If $x_B^1 < x_b < x_B^2$, then $x_s = \frac{a+3c_0-4t_0+4x_b}{36b}\pi_s = (t-c)\frac{a-t}{2b} - \alpha_s x_s^2 = \frac{(a-t)^2}{6b} - \frac{\alpha_s(a+3c_0-4t)^2}{9}, \pi_s > 0$ iff $x_b < -(a-t_0) + \frac{3\delta(a-c_0)}{4\delta - 1} = \delta_1$, where $\delta = \sqrt{\frac{2b\alpha_s}{3}}$.

Case 3. If $x_B^1 < x_B^2 < x_b$, then $x_s = \frac{a-t_0+x_b}{4b\alpha_s}, p = t, \pi_s = (t-c)\frac{a-t}{2b} - \alpha_s x_s^2 = \frac{(t-c)(a-t)}{2b} + \frac{(a-t)^2}{16b\alpha_s}. \pi_s > 0$ iff $x_b < -(a-t_0) + \frac{8b\alpha_s(a-c_0)}{8b\alpha_s-1} = \delta_2$.

Since $\delta_1 < x_B^1 \Leftrightarrow b\alpha_s < \frac{3}{32}, \delta_1 < x_B^2 \Leftrightarrow b\alpha_s < \frac{3}{32}, \delta_2 < x_B^2 \Leftrightarrow b\alpha_s < \frac{3}{8}$, we have the following result:

If $b\alpha_s < \frac{3}{32}$, then $\delta_1 < x_B^1, \delta_2 < x_B^2$, participate iff $x_b < x_B^1$. But based on our previous assumption $\alpha_s > \frac{a}{4b\alpha_c}$, this case will not occur.

If $\frac{3}{32} < b\alpha_s < \frac{3}{8}$, then $x_B^1 < \delta_1 < x_B^2, \delta_2 < x_B^2$, participate iff $x_b < \delta_1, \delta_1$ is decreasing in $\alpha_s$.

If $b\alpha_s > \frac{3}{8}$, then $\delta_1 > x_B^2, \delta_2 > x_B^2$, participate iff $x_b < \delta_2, \delta_2$ is decreasing in $\alpha_s$. $\diamond$

**Proof of Proposition 4**

Case 1. If $x_b < x_B^1$, then from proof of Proposition 3, supplier will participate. From Proposition 2, $x_s = \frac{3(a-c_0)}{32b\alpha_s-3}, p = \frac{a+3c}{4}a-t_0$, so buyer’s profit is $\pi_b(x_b) = (r-p)\frac{a-t}{2b} - I_b = \frac{(a-t)^2}{4b} - \alpha_b x_b^2 = \frac{9(a-c_0)^2}{64b} - \alpha_b x_b^2 - \frac{144b\alpha_s(a-c_0)^2}{(32b\alpha_s-3)^2}, \alpha_b x_b^2$, which is concave decreasing in $x_b$, and from FOC we have $x_b^* = 0, \pi_b(x_b^*) = \frac{144\alpha_s(a-c_0)^2}{(32b\alpha_s-3)^2}$.

Case 2. If $x_B^1 < x_b < x_B^2$, then if supplier participates, $x_s = \frac{a+3c_0-4t_0+4x_b}{36b}, p = t$, we have $\pi_b(x_b) = (r-t)\frac{a-t}{2b} - I_b = \frac{(a-t)^2}{4b} - \alpha_b x_b^2 = \frac{(a-t_0+x_b)^2}{4b} - \alpha_b x_b^2 = \frac{24b\alpha_s(a-c_0)}{32b\alpha_s-3}, \pi_b(x_b^*) = \frac{\alpha_s(a-t_0)^2}{4b\alpha_s-1}$. If the supplier does not participate, the buyer will buy from e-market at price $t$, which yields the same profit for her, if we assume the e-market is reliable and provides the same quality as the long-term supplier does.

Case 3. If $x_b > x_B^2$, then $x_s = \frac{a-t_0+x_b}{4b\alpha_s}, p = t$. Buyer’s profit function is the same as Case 2.

Therefore, the optimal investment depends on the relative position of 0, $x_B^1$ and $\frac{a-t_0}{4b\alpha_s-1}$. $x_B^1 > 0$ iff $\frac{a-t_0}{a-c_0} < \frac{24b\alpha_s}{32b\alpha_s-3} = K_2$. $x_B^1 < \frac{a-t_0}{4b\alpha_s-1} \text{ iff } \frac{a-t_0}{a-c_0} > \frac{24b\alpha_s}{32b\alpha_s-3}(1-\frac{1}{4b\alpha_s}) = K_1$. Since $K_1 < K_2$, we have three cases:
If \( \frac{a-t_0}{a-c_0} < K_1 \), then \( x^*_B > \frac{a-t_0}{4b_0-1} \) and \( x^*_B > 0 \). Then \( x^*_b = 0, \pi_b(x^*_b) = \frac{144a_0^2(a-c_0)^2}{(32b_0-3)^2} \).

If \( K_1 < \frac{a-t_0}{a-c_0} < K_2 \), then \( 0 < x^*_B < \frac{a-t_0}{4b_0-1} \). Compare the profit at 0 and \( \frac{a-t_0}{4b_0-1} \), we have

\[
x^*_b = \begin{cases} 
0 & \text{if } \frac{a-t_0}{a-c_0} < \frac{24b_0}{32b_0-3} \sqrt{1 - \frac{1}{4b_0}} \triangleq K_3 \\
\frac{a-t_0}{4b_0-1} & \text{if } \frac{a-t_0}{a-c_0} > K_3
\end{cases}
\]

If \( \frac{a-t_0}{a-c_0} > K_2 \), then \( x^*_B < 0 < \frac{a-t_0}{4b_0-1} \). Then \( x^*_b = \frac{a-t_0}{4b_0-1}, \pi_b(x^*_b) = \frac{a_0(a-t_0)^2}{4b_0-1} \).

Since \( ba_0 > \frac{1}{4}, 1 - \frac{1}{4b_0} \in (0, 1) \), we have \( K_1 < K_3 < K_2 \). Therefore, the threshold is \( K_3 \), and the optimal policy is that the buyer does not invest if the ratio \( \frac{a-t_0}{a-c_0} \) is below \( K_3 \) but does invest to get a price reduction of \( \frac{a-t_0}{4b_0-1} \) if the ratio exceeds \( K_3 \).

**Proof of Theorem 1**

From Proposition 4, buyer will not invest if \( \frac{a-t_0}{a-c_0} < K_3 \). Since \( K_3 < K_2 \), we have \( \frac{a-t_0}{a-c_0} < K_2 \) which implies \( x^*_B > 0 \), i.e. \( x^*_b = 0 < x^*_B \). From Proposition 3 and Proposition 2, supplier will participate and \( x^*_s = \frac{3(a-c_0)}{32b_0-3}, p^* = \frac{a+3c}{4} < t^* = t_0 \), the outside option is not binding.

If \( \frac{a-t_0}{a-c_0} > K_3 \), supplier’s participation and investment depend on parameters. Since \( K_3 > K_1 \), we have \( \frac{a-t_0}{a-c_0} > K_1 \), i.e. \( \frac{a-t_0}{4b_0-1} > x^*_B \). There are two cases:

1) \( ba_0 < \frac{3}{8} \), then supplier participates if \( \frac{a-t_0}{4b_0-1} < \delta_1 \), i.e. \( \frac{a-t_0}{a-c_0} < \frac{3\delta_1}{4b_0} \triangleq K_5 \).

And since \( \delta_1 < x^*_B \) when \( ba_0 < \frac{3}{8} \), we have \( x^*_s = \frac{a+3c-4t_0+4x^*_b}{3} = c_0 - a + \frac{16b_0a_0(a-t_0)}{3(4b_0-1)} \). The outside option is binding since \( x^*_b > x^*_B \). Supplier will not participate otherwise.

2) \( ba_0 > \frac{3}{8} \), then supplier participates if \( \frac{a-t_0}{4b_0-1} < \delta_2 \), i.e. \( \frac{a-t_0}{a-c_0} < \frac{8b_0}{8a_0} \). \( (1 - \frac{1}{4b_0}) \triangleq K_6 \).

There are three sub-cases:

1) If \( \frac{a-t_0}{a-c_0} < \frac{12b_0}{16b_0-3}(1 - \frac{1}{4b_0}) \triangleq K_4 \), then \( x^*_B < \frac{a-t_0}{4b_0-1} < x^*_B \), we have \( x^*_s = \frac{a+3c-4t_0+4x^*_b}{3} = c_0 - a + \frac{16b_0a_0(a-t_0)}{3(4b_0-1)} \). The outside option is binding.

2) If \( \frac{12b_0}{16b_0-3}(1 - \frac{1}{4b_0}) < \frac{a-t_0}{a-c_0} < K_6 \), then \( x^*_B < \frac{a-t_0}{4b_0-1} < \delta_2 \), we have \( x^*_s = \frac{a-t_0+x^*_b}{4b_0} = \frac{a_0(a-t_0)}{a_0(4b_0-1)} \). The outside option is binding.

3) If \( \frac{a-t_0}{a-c_0} < K_6 \), supplier will not participate.

Reorganize the results we have the theorem.

**Proof of Proposition 5**

When \( \gamma < K_3 \), the equilibrium result is the same as when there is no e-market.

When \( \gamma > K_3 \):

Trading price: Without e-market: \( p^d = \frac{a+3c}{4} = \frac{a+3c_0-3x^*_B}{4} = c_0 + \frac{8b_0a_0-3}{32b_0-3} (a-c_0) \). With e-market: \( p^* = t^* = t_0 - x^*_b = t_0 - \frac{a-t_0}{4b_0-1} \). \( p^* < p^d \) if \( \gamma > \frac{24b_0}{32b_0-3}(1 - \frac{1}{4b_0}) \), which is true since \( \gamma > K_3 \). So trading price is lower.
Retail price: \( r^* = \frac{a + p_d^*}{2} < \frac{a + p_d}{2} = r_d \). Retail price is lower.

Selling quantity: \( Q^* = \frac{a - p_d^*}{2b} < \frac{a - p_d}{2b} = Q_d \). Selling quantity is larger.

Consumer surplus: Without e-market: 
\[
CS_d = \int_0^{Q_d} (a - bQ - r_d) dQ = \frac{72b_0^2(a-c_0)^2}{(4b_0 - 3)^2}.
\]

With e-market, 
\[
CS^* = \int_0^{Q^*} (a - bQ - r^*) dQ = \frac{2b_0^2(a-t_o)^2}{(4b_0 - 1)^2}.
\]

\[ CS^* > CS_d \text{ iff } \gamma > \frac{2b_0}{32b_0 - 3}(1 - \frac{1}{4b_0}), \] which is true since \( \gamma > K_3 \). So consumer surplus increases.

Buyer’s profit: Without e-market, 
\[
\pi_b^d = (r^* - p^d)Q^* - \alpha_s x^* = \frac{\alpha_s(a-t_o)}{4b_0 - 3}. \] \( \pi_b^* \) iff \( \gamma > K_3 \). So e-market increases buyer’s profit.

Supplier’s investment and profit:

Without e-market, 
\[
\pi_s^d = (p^d - c_0 + x^d)Q^* - \alpha_s x^d = \frac{3\alpha_s(a-c_0)^2}{32b_0 - 3}. \]

\[ \text{With e-market option:} \]

Case 1. \( b_0a_0 < \frac{3}{8} \) and \( K_3 < \gamma < K_5 \) or \( b_0a_0 > \frac{3}{8} \) and \( K_3 < \gamma < K_4 \), \( x^* = c_0 - a + \frac{16b_0(a-t_o)}{3(4b_0 - 1)} > x^d \) iff \( \gamma > \frac{24b_0}{32b_0 - 3}(1 - \frac{1}{8b_0}), \) which is true since \( \gamma > K_3 \). \( \pi_s^* = (p^* - c_0 + x^*)Q^* - \alpha_s x^* \) \[ \pi_s^* = \frac{8b_0^2(a-t_o)^2}{3(4b_0 - 1)^2} - \alpha_s(a - c_0 - a + \frac{16b_0(a-t_o)}{3(4b_0 - 1)})^2. \]

\[ \pi_s^d - \pi_s^* = \frac{8b_0}{32b_0 - 3}(2\alpha_s(a - c_0) - \alpha_s(32b_0 - 3)(a-t_o))^2 > 0 \text{ iff } \gamma \neq \frac{24b_0}{32b_0 - 3}(1 - \frac{1}{8b_0}), \] which is true since \( \gamma > K_3 \).

Case 2. \( b_0a_0 > \frac{3}{8} \) and \( K_4 < \gamma < K_5 \), \( x^* = \frac{\alpha_s(a-t_o)}{a_0(4b_0 - 1)} > x^d \) iff \( \gamma > \frac{12b_0}{32b_0 - 3}(1 - \frac{1}{8b_0}), \) which is true since \( \gamma > K_3 \). \( \pi_s^* = (p^* - c_0 + x^*)Q^* - \alpha_s x^* \) \[ \pi_s^d - \pi_s^* = \frac{(8b_0 - 1)b_0^2}{a_0(4b_0 - 1)^2}(a - t_o)^2 + \frac{2a_0}{4b_0 - 1}(a - t_o)(a - c_0). \]

\[ \pi_s^d - \pi_s^* = \frac{8b_0^2}{32b_0 - 3}(a - t_o)^2 - \frac{2a_0}{4b_0 - 1}(a - t_o)(a - c_0) + \frac{3\alpha_s(a-c_0)^2}{32b_0 - 3} > 0 \text{ iff } \gamma > \frac{1}{2}K_6(1 + \sqrt{\frac{8b_0}{32b_0 - 3}}) \text{ or } \gamma < \frac{1}{2}K_6(1 - \sqrt{\frac{8b_0}{32b_0 - 3}}). \] Therefore supplier is worse off because \( \frac{1}{2}K_6(1 - \sqrt{\frac{8b_0}{32b_0 - 3}}) < K_4 \).

So supplier’s investment is always higher while supplier’s profit is always lower.

\[ \diamond \]

**Proof of Proposition 6**

To see Part (a), use Thm 1 and take the limit of \( \alpha_s \to \infty, \alpha_b \to \infty. \)

When \( \gamma < \frac{3}{4} \), the equilibrium is the same as when there is no e-market, so the supply chain profit remains the same. When \( \frac{3}{4} < \gamma < 1 \), we have \( \pi_b^* = \frac{(a-t_o)^2}{4b_0}, \pi_s^* = \frac{(t_0-c_0)(a-t_o)}{2b_0} \), supply chain profit \( \pi_c^* = \frac{(a-t_o)^2}{4b_0} + \frac{(a-c_0)(a-t_o)}{2b_0} \). In the equilibrium when there is no e-market, we have \( \pi_b^d = \frac{9(a-c_0)^2}{64b_0}, \pi_s^d = \frac{3(a-c_0)^2}{32b_0}, \) channel profit \( \pi_c^d = \frac{15(a-c_0)^2}{64b_0}. \) We have \( \pi_c^* > \pi_c^d \) iff \( \frac{3}{4} < \gamma < \frac{5}{4} \) which is satisfied. When \( \gamma > 1, \pi_c^* = 0, \pi_c^* = \pi_b^* > \pi_c^d \) iff \( \gamma > \frac{\sqrt{15}}{4} \) which is satisfied using \( \gamma > 1. \) The result follows.

For Part (b), in the benchmark no e-market case, we have \( \pi_b^d = \frac{144b_0^2(a-c_0)^2}{(32b_0 - 3)^2}, \pi_s^d = \frac{3\alpha_s(a-c_0)^2}{32b_0 - 3}, \pi_c^d = \pi_b^d + \pi_s^d. \)
For the case with e-market option, we first analyze the case where the e-market is perfectly competitive, i.e. $\alpha_b \to \infty$.

Use Thm 1 and take the limit of $\alpha_b \to \infty$. When $\gamma > \frac{24b_0\alpha_s}{32b_0\alpha_s-3}$. $\pi^*_b = \frac{(a-t_0)^2}{4}$. Now consider the case where $b_0\alpha_s > \frac{3}{8}$. Case 1: $\frac{24b_0\alpha_s}{32b_0\alpha_s-3} < \gamma < \frac{12b_0\alpha_s}{16b_0\alpha_s-3}$. $\pi^*_s = \frac{(a-t_0)^2}{66} + \alpha_s(c_0 - a + \frac{4(a-t_0)}{3})^2$. We have $\pi^*_c > \pi^*_c$ iff $\frac{24b_0\alpha_s}{32b_0\alpha_s-3} < \gamma < \frac{12b_0\alpha_s}{16b_0\alpha_s-3}$, which is satisfied by the condition of case 1. Case 2: $\frac{12b_0\alpha_s}{16b_0\alpha_s-3} < \gamma < \frac{8b_0\alpha_s}{8b_0\alpha_s-1}$. $\pi^*_s = \frac{(a-t_0)^2}{16b_0\alpha_s-1} + \frac{1}{2b_0}(a-t_0)(a-c_0)$. We have $\pi^*_c > \pi^*_c$ iff $\frac{8b_0\alpha_s}{8b_0\alpha_s-1} > \gamma > \frac{4b_0\alpha_s}{32b_0\alpha_s-3}(1+\Delta)$, where $\Delta = \frac{2b_0\alpha_s(16b_0\alpha_s+37)}{(32b_0\alpha_s-3)}$. Since $\frac{8b_0\alpha_s}{8b_0\alpha_s-1} > \frac{4b_0\alpha_s}{32b_0\alpha_s-3}(1+\Delta)$, the upper limit is always satisfied in case 2. Case 3: $\gamma > \frac{8b_0\alpha_s}{8b_0\alpha_s-1}$. $\pi^*_s = 0$. We have $\pi^*_c > \pi^*_c$ iff $\gamma > \frac{12b_0\alpha_s}{16b_0\alpha_s-3}$, which is satisfied by $\gamma > \frac{8b_0\alpha_s}{8b_0\alpha_s-1}$. Therefore, the supply chain profit is larger than no e-market case if $b_0\alpha_s > \frac{3}{8}$ and $\gamma > \frac{12b_0\alpha_s}{16b_0\alpha_s-3}$ when the e-market is perfectly competitive.

For general $\alpha_s$ and $\alpha_b$, consider the case where $\gamma > K_3$. The buyer’s profit under the e-market option $\pi^*_b = \frac{\alpha_s(a-t_0)^2}{4b_0\alpha_s-1}$ is decreasing in $\alpha_b$. Supplier’s profit does not change much when $\alpha_b$ is large enough, since $\frac{d\pi^*_s}{d\alpha_b} \bigg|_{\alpha_b \to \infty} = 0$. Therefore, from the result where $\alpha_b \to \infty$, we can infer that there exist $\gamma$ such that supply chain profit is larger than the no e-market case if $b_0\alpha_s > \frac{3}{8}$ and $\gamma > \gamma$ when the e-market is almost perfectly competitive ($\alpha_b$ is large enough).

This completes the proof. ◦

**Proof of Lemma 3**

The bargaining solution is the solution of

$$\max_{u_s,u_b} u_s \cdot u_b$$

subject to

$$(u_s, u_b) \in \Omega, u_s \geq 0, u_b \geq 0$$

where $\Omega = \{(u_s, u_b) : u_s = (p-c)Q(p, s_0), u_b = (r(p, s_0) - p)Q(p, s_0), u_b \geq E_s[(r(t, s) - t)Q(t, s)]\}$.

We first solve the optimization problem without the constraint $u_b \geq E_s[(r(t, s) - t)Q(t, s)]$, which is $p = \frac{a + s_0 + 3c}{4} \equiv \bar{p}$. Then the constraint solution is

$$p = \begin{cases} \bar{p} & \text{if } u_b(\bar{p}, s_0) \geq E_s u_b(t, s) \\ \bar{p} & \text{if } u_b(\bar{p}, s_0) < E_s u_b(t, s) \end{cases}$$

where function $u_b(p, s) = (r(p, s) - t)Q(p, s)$ and $\bar{p}$ satisfies $u_b(\bar{p}, s_0) = E_s u_b(t, s)$.

We have $u_b(\bar{p}, s_0) = (r(\bar{p}, s) - t)Q(\bar{p}, s) = \frac{(a + s_0 - \bar{p})^2}{4b} = \frac{9(a + s_0 - c)^2}{6b}$, $u_b(t, s) = \frac{(a + s - t)^2}{4b}$. 

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Using \( s = s_0 - \varepsilon \), we have \( E u_b(t, s) = E_{s \leftarrow \varepsilon}(a + s_0 - s - t)^2 = E_{s \leftarrow \varepsilon}(a + s_0 - s - t)^2 = \frac{(a + s_0 - \mu - t)^2 + \sigma^2}{4b} \), where the last equality uses \( E[\varepsilon^2] = E^2[\varepsilon] + \sigma^2 \).

Therefore, \( u_b(\tilde{p}, s_0) \geq E u_b(t, s) \) if and only if \( \frac{9(a + s_0 - c)^2}{16} - \sigma^2 \geq (a + s_0 - \mu - t)^2 \).

Assume \( \sigma^2 < \frac{9(a + s_0 - c)^2}{16} \) (which we will show later), then \( u_b(\tilde{p}, s_0) \geq E u_b(t, s) \) if and only if \( t \geq a + s_0 - \mu - \sqrt{\frac{9}{16}(a + s_0 - c)^2 - \sigma^2} \).

If \( t \geq a + s_0 - \mu - \sqrt{\frac{9}{16}(a + s_0 - c)^2 - \sigma^2} \), then

\[
p = \begin{cases} 
\tilde{p} & \text{if } t \geq a + s_0 - \mu - \sqrt{\frac{9}{16}(a + s_0 - c)^2 - \sigma^2} \\
\tilde{p} & \text{if } t < a + s_0 - \mu - \sqrt{\frac{9}{16}(a + s_0 - c)^2 - \sigma^2}
\end{cases}
\]

\( \tilde{p} \) satisfies \( u_b(\tilde{p}, s_0) \geq E u_b(t, s) \), i.e., \( \frac{(a + s_0 - \mu - t)^2 + \sigma^2}{4b} = \frac{(a + s_0 - \mu - t)^2 + \sigma^2}{4b} \). We have \( \tilde{p} = a + s_0 - \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} \).

Now we show that \( \sigma^2 < \frac{9(a + s_0 - c)^2}{16} \):

\[
\sigma^2 = E[\varepsilon^2] - E^2[\varepsilon] \leq E[\varepsilon \cdot s_0] - E^2[\varepsilon] = s_0 \mu - E^2[\varepsilon] = \mu(s_0 - \mu)
\]

\[
\leq \frac{s_0^2}{4} < \frac{9s_0^2}{16} < \frac{9(a + s_0 - c)^2}{16}.
\]

Thus we have the bargaining solution stated in the lemma. ⊗

**Proof of Proposition 7**

First, rewrite the bargaining solution (equation (7)) as

\[
p(x_s) = \begin{cases} 
\frac{a + s_0 + 3c_0 - 3x_s}{4} & \text{if } x_s \geq \frac{4}{3} \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} - (a + s_0 - c_0) \\
\tilde{p} & \text{if } x_s < \frac{4}{3} \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} - (a + s_0 - c_0)
\end{cases}
\]

(11)

The supplier solves

\[
\max_{x_s}(p(x_s) - c(x_s))Q(p(x_s), s_0) - I_s(x_s) = (p(x_s) - c_0 + x_s)(a + s_0 - p(x_s)) - \alpha_s x_s^2
\]

If he chooses \( x_s \geq \frac{4}{3} \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} - (a + s_0 - c_0) \), then \( p(x_s) = \frac{a + s_0 + 3c_0 - 3x_s}{4} \), seller solves

\[
\max_{x_s} \frac{3(a + s_0 - c_0 + x_s)^2}{32b} - \alpha_s x_s^2
\]

The objective function is maximized at FOC \( x_s = \frac{3(a + s_0 - c_0)}{32b(\alpha_s)^{-2}} \).

If he chooses \( x_s < \frac{4}{3} \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} - (a + s_0 - c_0) \), then \( p(x_s) = \tilde{p} \), seller

\[
\max_{x_s} (\tilde{p} - c_0 + x_s)(a + s_0 - \tilde{p}) - \alpha_s x_s^2
\]

The objective function is maximized at FOC \( \frac{a + s_0 - \tilde{p}}{4b \alpha_s} = \frac{\sqrt{(a + s_0 - \mu - t)^2 + \sigma^2}}{4b \alpha_s} \).
Define \( g(x_b) = \sqrt{(a + s_0 - \mu - t)^2 + \sigma^2} \). Then the threshold is \( \frac{4}{3} g(x_b) - (a + s_0 - c_0) \). Since
\[
\frac{3(a + s_0 - c_0)}{32b_0a_0 - 3} > \frac{4}{3} g(x_b) - (a + s_0 - c_0) \iff g(x_b) < \frac{24b_0a_0}{32b_0a_0 - 3}(a + s_0 - c_0), \quad \frac{4}{3} g(x_b) - (a + s_0 - c_0) > \frac{g(x_b)}{4b_0a_0} \iff g(x_b) > \frac{12b_0a_0}{16b_0a_0 - 3}(a + s_0 - c_0),
\]
we have three cases:

1. If \( g(x_b) < \frac{24b_0a_0}{32b_0a_0 - 3}(a + s_0 - c_0) \), (i.e. \( x_b < x_{1B}^1 \)) then \( x_s^* = \frac{3(a + s_0 - c_0)}{32b_0a_0 - 3} \). (Relaxing the assumption would generate too many cases which do not generate more insights.)

2. If \( \frac{24b_0a_0}{32b_0a_0 - 3}(a + s_0 - c_0) < g(x_b) < \frac{12b_0a_0}{16b_0a_0 - 3}(a + s_0 - c_0) \), (i.e. \( x_{1B}^1 < x_b < x_{2B}^2 \)) then \( x_s^* = \frac{4}{3} g(x_b) - (a + s_0 - c_0) \). (Assumption A1 and the fact that \( \frac{24b_0a_0}{32b_0a_0 - 3} < \frac{12b_0a_0}{16b_0a_0 - 3} \) will guarantee the feasibility.)

3. If \( g(x_b) > \frac{12b_0a_0}{16b_0a_0 - 3}(a + s_0 - c_0) \), (i.e. \( x_b > x_{2B}^2 \)) then \( x_s^* = \frac{g(x_b)}{4b_0a_0} \). Therefore we have the response function stated in the lemma.

**Proof of Proposition 8**

From Proposition 7, supplier's optimal investment, transfer price and profit are:

Case 1. If \( x_b < x_{1B}^1 < x_{2B}^2 \), then \( x_s = \frac{3(a + s_0 - c_0)}{32b_0a_0 - 3}, p = \frac{a + s_0 + \tilde{p}}{2b} - \alpha_s x_s^2 = \frac{3a_s(a + s_0 - c_0)^2}{32b_0a_0 - 3} > 0 \).

Case 2. If \( x_{1B}^1 < x_b < x_{2B}^2 \), then \( x_s = \frac{4}{3} g(x_b) - (a + s_0 - c_0), p = \tilde{\tilde{p}} = a + s_0 - g(x_b). \pi_s = (\tilde{\tilde{p}} - c) \frac{a + s_0 - \tilde{p}}{2b} - \alpha_s x_s^2 = \frac{g(x_b)}{6b} - \alpha_s \left[ \frac{4}{3} g(x_b) - (a + s_0 - c_0) \right]^2. \pi_s > 0 \iff x_b < \sqrt{\left[ \frac{a + s_0 - c_0}{3} \right]^2 - \sigma^2 - (a + s_0 - \mu - t_0) = \sqrt{\left[ \frac{3\tilde{p}}{4b} - (a + s_0 - c_0) \right]^2 - \sigma^2 - (a + s_0 - \mu - t_0) \triangleq \delta_{1e}}}, \)

where \( \delta = \sqrt{\frac{2b_0a_0}{3}} \). (Assumption A1 and the fact that \( \frac{24b_0a_0}{32b_0a_0 - 3} < \frac{3\delta}{4b} \) will guarantee the feasibility.)

Case 3. If \( x_{1B}^1 < x_{2B}^2 < x_b \), then \( x_s = g(x_b) \frac{2b}{4b_0a_0}, p = \tilde{\tilde{p}}, \pi_s = \left( \tilde{\tilde{p}} - c \right) \frac{a + s_0 - \tilde{p}}{2b} - \alpha_s x_s^2 = g(x_b) \frac{2b}{4b_0a_0} \left[ a + s_0 - c_0 - \frac{8b_0a_0 - 1}{8b_0a_0} g(x_b) \right]. \pi_s > 0 \iff x_b < \sqrt{\left[ \frac{8b_0a_0 - 1}{8b_0a_0 - 3} (a + s_0 - c_0) \right]^2 - \sigma^2 - (a + s_0 - \mu - t_0) \triangleq \delta_{2e}} \). (Assumption A1 and the fact that \( \frac{24b_0a_0}{32b_0a_0 - 3} < \frac{8b_0a_0 - 1}{8b_0a_0 - 3} \) will guarantee the feasibility.)

Since \( \delta_{1e} > x_{1B}^1 \iff b_0a_0 > \frac{3}{32}, \) which is true; \( \delta_{1e} < x_{2B}^2 \iff b_0a_0 < \frac{3}{8}, \delta_{2e} < x_{2B}^2 \iff b_0a_0 \) we have the following result:

If \( b_0a_0 < \frac{3}{8} \), then \( x_{1B}^1 < \delta_{1e} < x_{2B}^2, \delta_{2e} < x_{2B}^2 \), participate iff \( x_b < \delta_{1e}, \delta_{1e} \) is decreasing in \( \alpha_s \).

If \( b_0a_0 > \frac{3}{8} \), then \( \delta_{1e} > x_{2B}^2, \delta_{2e} > x_{2B}^2 \), participate iff \( x_b < \delta_{2e}, \delta_{2e} \) is decreasing in \( \alpha_s \).

**Proof of Proposition 9**
Case 1. If $x_b < x_B$<sup>1</sup>, then from proof of Proposition 8, supplier will participate, because

$$x_B < 3 \frac{a+s_0-c_0}{2b_\alpha s_0 - 3}, \ p = \frac{a+s_0+3c}{4},$$

so buyer's profit is $\pi_b(x_b) = (r - p) \frac{(a+s_0-p)^2}{2b} - I_b = \frac{(a+s_0-p)^2}{2b} - \alpha_b x_b^2 = \frac{9(a+s_0-c)^2}{6b} - \alpha_b x_b^2 = \frac{144b_\alpha^2(a+s_0-c)^2}{(32b_\alpha s_0-3)^2} - \alpha_b x_b^2$, which is concave decreasing in $x_b$, and from FOC we have $x_b = 0, \pi_b(x_b^*) = \frac{144b_\alpha^2(a+s_0-c)^2}{(32b_\alpha s_0-3)^2}$.

Case 2. If $x_B > x_b < x_B^2$, then if supplier participates, $x_s = \frac{4}{3} g(x_b) - (a + s_0 - c_0), \ p = \tilde{p}$, we have $\pi_b(x_b) = (r - \tilde{p}) \frac{(a+s_0-\tilde{p})^2}{2b} - I_b = \frac{(a+s_0-\tilde{p})^2}{2b} - \alpha_b x_b^2 = \frac{2^2(x_b)}{4b} - \alpha_b x_b^2 = \frac{(a+s_0-c_0)^2}{2b} - \alpha_b x_b^2$, which is concave in $x_b$. From FOC we have $x_b = \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$, $

\pi_b(x_b^*) = \frac{a_\alpha^2(a+s_0-c_0)^2 + \sigma^2}{4b_\alpha s_0 - 1}$.

Case 3. If $x_b > x_B^2$, then $x_s = \frac{g(x_b)}{4b_\alpha s_0}, \ p = \tilde{p}$. Buyer’s profit function is the same as Case 2.

Therefore, the optimal investment depends on the relative position of $0, x_B^2$ and $\frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$.

$x_B^2 > 0$ iff $\frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$.

$x_B^2 > x_B > x_B^1 = 0$ iff $\frac{a+s_0-c_0}{2b_\alpha s_0 - 3} < \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$.

If $\gamma_\varepsilon < 1 - \frac{1}{4b_\alpha s_0}$, then $x_B^1 < \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$, compare the profit at 0 and $\frac{a+s_0-c_0}{2b_\alpha s_0 - 3}$, we have

$$x_b^* = \begin{cases} 
0 & \text{if } \gamma_\varepsilon < 1 - \frac{1}{4b_\alpha s_0} \\
\frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1} & \text{if } \gamma_\varepsilon > 1 - \frac{1}{4b_\alpha s_0} 
\end{cases}$$

If $\gamma_\varepsilon > 1$, then $x_B^1 > 0 < \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$. Then $x_b^* = \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$, $\pi_b(x_b^*) = \frac{a_\alpha^2(a+s_0-c_0)^2 + \sigma^2}{4b_\alpha s_0 - 1}$.

Since $b_\alpha > 1, 1 - \frac{1}{4b_\alpha s_0} \in (0, 1)$, we have $1 - \frac{1}{4b_\alpha s_0} < \sqrt{1 - \frac{1}{4b_\alpha s_0}} < 1$. Therefore, the threshold is $\sqrt{1 - \frac{1}{4b_\alpha s_0}}$, and the optimal policy is that the buyer does not invest if the ratio $\gamma_\varepsilon$ is below $\sqrt{1 - \frac{1}{4b_\alpha s_0}}$, but does invest to get a price reduction of $\frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}$ if the ratio exceeds $\sqrt{1 - \frac{1}{4b_\alpha s_0}}$.

**Proof of Theorem 2**

From Proposition 9, buyer will not invest if $\gamma_\varepsilon < 1 - \frac{1}{4b_\alpha s_0}$. Since $\gamma_\varepsilon < 1 - \frac{1}{4b_\alpha s_0} < 1$, we have $x_B^1 > 0$, i.e. $x_b^* = 0 < x_B^1$. From Proposition 8 and Proposition 7, supplier will participate and $x_b^* = \frac{3(a+s_0-c_0)}{2b_\alpha s_0 - 3}, p = \frac{a+s_0+3c}{4} < \tilde{p}$, the outside option is not binding.

If $\gamma_\varepsilon > 1 - \frac{1}{4b_\alpha s_0}$, supplier’s participation and investment depend on parameters. Since $\gamma_\varepsilon > \sqrt{1 - \frac{1}{4b_\alpha s_0}} > 1 - \frac{1}{4b_\alpha s_0}$, we have $x_B^1 < \frac{a+s_0-\mu-\tau}{4b_\alpha s_0 - 1}(\Delta x_B)$.

There are two cases:
1. $b\alpha_s < \frac{3}{8}$, then supplier participates iff $x^*_b < \delta_1$, i.e. $\sqrt{\left(\frac{4b\alpha_s}{8b\alpha_s - 1}\right)(a + s_0 - \mu - t_0)} + \frac{\sigma^2}{\alpha_s} \leq \beta_\varepsilon$. And since $\delta_1 < x^*_b < \beta_\varepsilon$ when $b\alpha_s < \frac{3}{8}$, we have $x^*_b = \frac{4}{3}g(x^*_b) - (a + s_0 - c_0) = \frac{4}{3}\sqrt{\left(\frac{4b\alpha_s}{8b\alpha_s - 1}\right)(a + s_0 - \mu - t_0)} + \frac{\sigma^2}{\alpha_s} - (a + s_0 - c_0) = \left(\frac{4}{3}\beta_\varepsilon - 1\right)(a + s_0 - c_0)$. The outside option is binding since $x^*_b > x^*_b$. Supplier will not participate otherwise.

2. $b\alpha_s > \frac{3}{8}$, then supplier participates iff $x^*_b < \delta_2$, i.e. $\beta_\varepsilon < \frac{b\alpha_s}{8b\alpha_s - 1}$. There are three sub-cases:
   
   1) If $\beta_\varepsilon < \frac{12b\alpha_s}{16b\alpha_s - 3}$, (i.e. $x^*_b < x^*_b$), then $x^*_b = \frac{4}{3}g(x^*_b) - (a + s_0 - c_0) = \left(\frac{2}{3}\beta_\varepsilon - 1\right)(a + s_0 - c_0)$. The outside option is binding.
   
   2) If $\frac{12b\alpha_s}{16b\alpha_s - 3} < \beta_\varepsilon < \frac{8b\alpha_s}{8b\alpha_s - 1}$, then $x^*_b < \delta_2$, we have $x^*_b = \frac{g(x^*_b)}{4b\alpha_s} = \beta_\varepsilon (a + s_0 - c_0)$. The outside option is binding.
   
   3) If $\beta_\varepsilon > \frac{8b\alpha_s}{8b\alpha_s - 1}$, supplier will not participate.

Reorganize the results we have the theorem.

**Proof of the results in Table 2**

The equilibrium transfer price and each party’s profit under e-market quality uncertainty are given by:

If $b\alpha_s < \frac{3}{8}$,

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$p^*$</th>
<th>$E\pi_b^*$</th>
<th>$\pi_b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_\varepsilon &lt; K_3\varepsilon$</td>
<td>$c_0 + \frac{8b\alpha_s - 3}{32b\alpha_s - 3}(a + s_0 - c_0)$</td>
<td>$\frac{144\alpha_2^2(a + s_0 - c_0)^2}{32b\alpha_s - 3} + \frac{\sigma^2}{4b}$</td>
<td>$\frac{3\alpha_s(a + s_0 - c_0)^2}{32b\alpha_s - 3}$</td>
</tr>
<tr>
<td>$\gamma_\varepsilon &gt; \beta_\varepsilon &lt; K_{5\varepsilon}$</td>
<td>$a + s_0 - \beta_\varepsilon(a + s_0 - c_0)$</td>
<td>$\frac{\alpha_3(a + s_0 - \mu - t_0)}{4b\alpha_s - 1} + \frac{\sigma^2}{4b}$</td>
<td>$\Pi^1_s$</td>
</tr>
<tr>
<td>$K_{3\varepsilon}$</td>
<td>$N/A$</td>
<td>$\frac{\alpha_3(a + s_0 - \mu - t_0)}{4b\alpha_s - 1} + \frac{\sigma^2}{4b}$</td>
<td>$N/A$</td>
</tr>
</tbody>
</table>

If $b\alpha_s > \frac{3}{8}$,

<table>
<thead>
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</tr>
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<td>$\gamma_\varepsilon &lt; K_3\varepsilon$</td>
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</tr>
<tr>
<td>$\gamma_\varepsilon &gt; \beta_\varepsilon &lt; K_{4\varepsilon}$</td>
<td>$a + s_0 - \beta_\varepsilon(a + s_0 - c_0)$</td>
<td>$\frac{\alpha_3(a + s_0 - \mu - t_0)}{4b\alpha_s - 1} + \frac{\sigma^2}{4b}$</td>
<td>$\Pi^1_s$</td>
</tr>
<tr>
<td>$K_{3\varepsilon}$</td>
<td>$N/A$</td>
<td>$\frac{\alpha_3(a + s_0 - \mu - t_0)}{4b\alpha_s - 1} + \frac{\sigma^2}{4b}$</td>
<td>$N/A$</td>
</tr>
</tbody>
</table>

Where $\Pi^1_s = \left[\frac{b^2}{16} - \alpha_s(\frac{2}{3}\beta_\varepsilon - 1)^2\right](a + s_0 - c_0)^2$, $\Pi^2_s = \frac{\beta_\varepsilon - \beta_\varepsilon^2(1 - \frac{8b\alpha_s}{8b\alpha_s - 1})}{2b}(a + s_0 - c_0)^2$.

It is straightforward to show how the transfer price, supplier and buyer’s investments and buyer’s profit change with $\sigma^2$ and $\mu$.

To see how the supplier’s profit changes with $\sigma^2$ and $\mu$:

$\frac{\partial \Pi^1_s}{\partial \sigma^2} < 0$ iff $\beta_\varepsilon > \frac{24b\alpha_s}{32b\alpha_s - 3}$, which is satisfied using $\gamma_\varepsilon > K_{3\varepsilon}$. Similarly we can show $\frac{\partial \Pi^1_s}{\partial \mu} > 0$.

$\frac{\partial \Pi^2_s}{\partial \sigma^2} < 0$ iff $\beta_\varepsilon > \frac{4b\alpha_s}{8b\alpha_s - 1}$, which is satisfied using $\beta_\varepsilon > \frac{24b\alpha_s}{32b\alpha_s - 3}$. Similarly we can show $\frac{\partial \Pi^2_s}{\partial \mu} > 0$. 

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Therefore, supplier’s profit is decreasing in $\sigma^2$ and increasing in $\mu$. ◀

References


