Chapter B - Finance

B-1:

11.9 years

\[ \$10,000 \times (1.06)^t = \$20,000 \]
\[ (1.06)^t = 2 \]
\[ t \ln(1.06) = \ln 2 \]
\[ t = \frac{\ln 2}{\ln 1.06} \]
\[ t = 11.89566105 \]

B-2:

\$12,621.45

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
X & 6\% & 6\% & 6\% & 6\% & 6\% & 6\% & 6\% & -5,000 & -6,000 & -7,000 \\
\end{array}
\]

\[ X = \frac{\$5,000}{(1.06)^2} + \frac{\$6,000}{(1.06)^4} + \frac{\$7,000}{(1.06)^7} \]

\[ X = \$3,735.29 + \$4,229.76 + \$4,655.40 \]

\[ X = \$12,621.45 \]

Bank balances are thus

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\$12,621.45 & \$13,378.74 & \$14,181.46 & \$15,032.35 & \$15,934.29 & \$11,890.35 & \$6,603.77 & \$0.00 \\
\$5,000.00 & \$6,000.00 & \$7,000.00 & \$8,000.00 & \$9,000.00 & \$10,000.00 & \$11,000.00 & \$12,000.00 \\
\end{array}
\]
B-3:

<table>
<thead>
<tr>
<th></th>
<th>May 1, 2012</th>
<th>May 1, 2013</th>
<th>May 1, 2014</th>
<th>May 1, 2015</th>
<th>May 1, 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>$100</td>
<td>$200</td>
<td>$300</td>
<td>$1,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. Present Value at 3% = $1,448.64

Use [CF] to enter the cash flows. Remember that since you are calculating the NPV as of May 1, 1998 the initial cash flow is 0.

B. Future Value at 3% = $1,630.45

Since there is no future value (that I could find) take the net present value forward 4 years by multiplying by \((1.03)^4\).

C. Future Value:

The business calculator is built around the assumption that the discount or interest rate used stays constant. When the rate changes from period to period then we need to calculate present and future values payment by payment.

<table>
<thead>
<tr>
<th></th>
<th>May 1, 2012</th>
<th>May 1, 2013</th>
<th>May 1, 2014</th>
<th>May 1, 2015</th>
<th>May 1, 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
<td>4%</td>
<td>5%</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>$100</td>
<td>$200</td>
<td>$300</td>
<td>$1,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Future Value:

\[
\begin{align*}
\text{Future Value:} & = \$1000.00 \\
\$300 \times (1.06) & = \$318.00 \\
\$200 \times (1.05)^2 \times (1.06) & = \$222.60 \\
\$100 \times (1.04)^3 \times (1.05) \times (1.06) & = \$115.75 \\
\text{Total} & = \$1,656.35 \\
\end{align*}
\]

Present Value:

\[
\begin{align*}
\text{Present Value:} & = \frac{\$100}{(1.03)} \\
\$97.09 & = \frac{\$200}{(1.03)^2} \\
\$186.71 & = \frac{\$300}{(1.03)^3} \\
\$266.72 & = \frac{\$1000}{(1.03)^4} \\
\$838.75 & = \frac{\$1,656.35}{(1.03)^4} \\
\text{Total} & = \$1,389.27 \\
\end{align*}
\]
Answers to Questions & Problems

B-4:

A. $4,416.32

You can do this one two different ways:

\[
\begin{align*}
&\text{\$1000} \times (1.04) = \text{\$1,040.00} \\
&\text{\$1000} \times (1.04) \times (1.04) = \text{\$1,081.60} \\
&\text{\$1000} \times (1.04) \times (1.04) \times (1.04) = \text{\$1,124.86} \\
&\text{\$1000} \times (1.04) \times (1.04) \times (1.04) \times (1.04) = \text{\$1,169.86} \\
&\text{\$4,416.32}
\end{align*}
\]

or

\[
\begin{align*}
&\text{\$1000} \times (1.04) = \text{\$1,040.00} \\
&\text{\$2,040.00} \times (1.04) = \text{\$2,121.60} \\
&\text{\$3,121.60} \times (1.04) = \text{\$3,246.46} \\
&\text{\$4,246.46} \times (1.04) = \text{\$4,416.32}
\end{align*}
\]

June 1 | Freshman 5% | Sophomore 6% | Junior 7% | Senior 8% | Graduation
---|---|---|---|---|
$1000 | $1000 | $1000 | $1000 |

B. $4,746.72

This is exactly the same problem but with different interest rates

\[
\begin{align*}
&\text{\$1000} \times (1.05) = \text{\$1,050.00} \\
&\text{\$2,050.00} \times (1.06) = \text{\$2,173.00} \\
&\text{\$3,173.00} \times (1.07) = \text{\$3,395.11} \\
&\text{\$4,395.11} \times (1.08) = \text{\$4,746.72}
\end{align*}
\]

or

\[
\begin{align*}
&\text{\$1000} \times (1.05) = \text{\$1,050.00} \\
&\text{\$2,050.00} \times (1.06) = \text{\$2,173.00} \\
&\text{\$3,173.00} \times (1.07) = \text{\$3,395.11} \\
&\text{\$4,395.11} \times (1.08) = \text{\$4,746.72}
\end{align*}
\]
B-5:

<table>
<thead>
<tr>
<th>June 1</th>
<th>June 1</th>
<th>June 1</th>
<th>June 1</th>
<th>Graduation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman 4%</td>
<td>Sophomore 4%</td>
<td>Junior 4%</td>
<td>Senior 4%</td>
<td></td>
</tr>
<tr>
<td>$1000</td>
<td>$2000</td>
<td>$3000</td>
<td>$4000</td>
<td></td>
</tr>
</tbody>
</table>

A. Aunt Mabel must set aside $8,896.86

\[
\begin{align*}
$961.54 &= \frac{$1000}{(1.04)} \\
$1,849.11 &= \frac{$2000}{(1.04)^2} \\
$2,666.99 &= \frac{$3000}{(1.04)^3} \\
$3,419.22 &= \frac{$4000}{(1.04)^4} \\
\end{align*}
\]

$8,896.86

To double check that this is right put $8,896.86 in the bank

\[
\begin{align*}
$961.54 &= \frac{$9,252.72}{(1.04)} \\
$1,849.11 &= \frac{$8,582.83}{(1.04)^2} \\
$2,666.99 &= \frac{$6,846.15}{(1.04)^3} \\
$3,419.22 &= \frac{$4,000.00}{(1.04)^4} \\
\end{align*}
\]

B. If the interest rates increase to 12% in the last year it affects only the last payment. After Aunt Mabel pays you your $3000 after your junior year she has $3,846.15 \[4000/(1.04)\] left in the bank. At the end of the year this has increased to $4,307.69 \[3,846.15(1.12)\], so she after she gives you your $4000 for graduation she has $307.69 left over.

C. If Aunt Mabel had known this she would have set aside $8,652.63; $244.23 less than originally planned.

\[
\begin{align*}
$961.54 &= \frac{$1000}{(1.04)} \\
$1,849.11 &= \frac{$2000}{(1.04)^2} \\
$2,666.99 &= \frac{$3000}{(1.04)^3} \\
$3,174.99 &= \frac{$4000}{(1.04)^4} \\
$3,174.99 &= \frac{$4000}{(1.04)(1.04)^2} \\
\end{align*}
\]

$8,652.63

You can get the same answer by calculating the net present value of the $307.69 Aunt Mabel had left after the interest rate rose.
B-6:

$37,123.52

$50,000.00 = X \left(1 + \frac{0.06}{4}\right)^{5 \text{ years } \times \frac{4}{\text{year}}}

$50,000.00 = X \left(1.015\right)^{20}

$37,123.52 = X

B-7:

$37,068.61

$50,000.00 = X \left(1 + \frac{0.05}{12}\right)^{5 \text{ years } \times \frac{12}{\text{year}}}

$50,000.00 = X \left(1.005\right)^{60}

$37,068.61 = X