Horses and Rabbits? Optimal Dynamic Capital Structure from Shareholder and Manager Perspectives*

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Abstract

This paper examines optimal capital structure choice using a dynamic capital structure model that is calibrated to reflect actual firm characteristics. We examine capital structure choices that maximize either the utility of managers or the value of a share. The model uses contingent-claims methods to value interest tax shields, allows for bankruptcy if firm value reaches a pre-specified boundary, and explicitly models the firm maintaining a long-run target debt/equity ratio by refinancing maturing debt at that ratio. Since the model's major forces favoring debt or equity are tax shields and bankruptcy costs, this model provides a way to calculate optimal capital structures in a realistic representation of the traditional 'tradeoff' model. The model's predicted optimal capital structures are much more consistent with actual capital structures than have been presumed by the literature dating at least to Miller (1977). One important reason for this difference is that the value of a firm's existing tax shields declines with incremental leverage as bankruptcy costs increase, a relation not commonly recognized by the existing literature.

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1. Introduction.

It is not much of an overstatement to say that since Modigliani and Miller's (1963) tax correction paper, the central issue in corporate finance research has been the question of why, despite the large tax advantage enjoyed by debt, actual firms have fairly low leverage ratios. This question has stimulated much of the early research on agency theory (Jensen/Meckling, 1976; Myers, 1977), some of the best-known work on information asymmetries (Myers and Majluf, 1984), three American Finance Association presidential addresses (Miller, 1977; Myers, 1984; and Leland, 1998), and continues to motivate well-regarded research (Graham, 2000). The consensus view underlying this vast literature is that bankruptcy costs alone are too small to offset the value of tax shields, and that other factors, such as agency costs, must be introduced into the cost-benefit analysis to explain actual capital structures. Miller (1977, p. 264) perhaps puts it best: "…the supposed trade-off between tax gains and bankruptcy costs looks like the recipe for the fabled horse-and-rabbit stew – one horse and one rabbit".

The underlying logic of this widespread view is that while tax shields are relatively large (about 9.7% of firm value according to Graham (2000)), bankruptcy costs are incurred infrequently and are relatively small when they are incurred. Yet, the tradeoff theory does not contain any predictions about the *level* of tax shields and bankruptcy costs; rather, it states that at the margin, adding an additional dollar of debt will not change firm value. An important consideration, largely neglected by the literature, is that tax shields themselves are options, and that adding additional debt decreases the value of *existing* tax shields.[1] Incremental debt increases the probability of bankruptcy, and tax shields lose value if firms go bankrupt.[2] In other words, in the tradeoff framework, a firm considering adding leverage should trade off

[1] Exceptions are Majd and Myers (1987), Ju (1998), and Parrino, Poteshman, and Weisbach (2002).
[2] The value of tax shields in bankruptcy is severely limited by the Tax Reform Act of 1986, which restricts the ability of firms to sell net operating losses. See Ginsburg and Levin (2001), Chapter 12, for details on how these restrictions work.

the value coming from the tax shields created by the new leverage against the increase in expected bankruptcy costs, *plus the decrease in the value of existing tax shields.*

This paper estimates optimal capital structure using a dynamic model of capital structure. We view the model as a representation of the traditional tradeoff model because the primary determinants of the optimal capital structure are tax shields and bankruptcy costs. However, the model incorporates a number of elements not typically seen in the capital structure literature. First, tax shields are valued using contingent-claims methods. Second, as in Black and Cox (1976), the firm defaults if the value of its assets reaches a pre-specified bankruptcy boundary. Third, the firm refinances its debt when it matures at a pre-specified target debt/equity ratio. The value of a share today reflects both tax shields and expected bankruptcy costs, both directly and through their impact on subsequent financing costs.

In the model, the manager of an unlevered firm undertakes a fairly priced debt/equity swap, in which he selects the fraction of equity to be exchanged for debt, with the objective of maximizing either the per-share value of the firm's equity or his own utility. The swap that maximizes the per-share value of equity is, from the shareholders' perspective, the optimal capital structure.

The model is calibrated to be representative of the typical public firm in the U.S. and U.S. capital markets. The value of the firm's assets is assumed to follow geometric Brownian motion with a drift of 5% and a standard deviation of 32%. Similarly, the stock and option holdings of the firm's senior manager are selected to reflect the typical levels for senior U.S. executives. Finally, the firm is assumed to issue 10-year, coupon-bearing debt, which is priced at par. The bankruptcy boundary and costs incurred in bankruptcy are selected so that the spread between the coupon rate and the Treasury rate and recovery rates in bankruptcy are consistent with values observed in practice.

We find that the optimal debt-equity swap leads to a market debt/total capital ratio of approximately 20%. In comparison, the median firm in the Compustat database in 1999 had a debt/total capital ratio of 21.9%. These two ratios are far closer to one another than are implied by the previous literature.

We next introduce agency costs into this framework by calculating the value of the swap that maximizes the utility of a potential manager. We assume the manager has a constant relative risk aversion utility function with a risk-aversion parameter of 2, owns 0.32% of the company's stock, has at-the-money options on 0.38% of the company's stock, and has non-firm wealth equal to the value of his shares. When the swap is chosen to maximize this utility function, the optimal leverage drops to approximately 15% of firm value.

We perform numerical comparative statics to evaluate the impact of sensitivity analysis on the major parameters on optimal capital structure. Not surprisingly, corporate tax rates, bankruptcy costs, and the ability of debtholders to force the firm into bankruptcy all impact optimal capital structure ratios.

We also calibrate the model to reflect 15 actual firms. For these 15 firms, the predicted stock-price maximizing leverage ratio is less than the firm's actual leverage ratio for 13 of them and the predicted utility-maximizing leverage is less than actual leverage for 14 of them. In general, the model is able to predict, within a reasonable degree of error, the leverage observed at firms that have relatively small to typical levels of debt, but substantially underestimates the level of debt observed at highly levered firms.

Overall, the results in this paper suggest that the tradeoff model performs reasonably well in predicting capital structures for firms with typical levels of debt. Certainly, the "horse and rabbit stew" analogy seems inappropriate – actual capital structures appear to be of the same order of magnitude as those predicted by this model. If anything, our results suggest that the tradeoff model predicts leverage levels that are lower than those observed empirically, especially when it is adjusted for managerial preferences. As such, these results highlight the importance of theories such as Jensen (1986), in which debt provides advantages from an incentive perspective.

Our model implies that important factors determining capital structure are the underlying risk of the firm's assets, the ability of bondholders to force default for a given level of firm value, as well as the incremental costs conditional on default. Our ability to measure these variables is quite limited using current econometric methods; a better understanding of their relative importance can advance our understanding of capital structure choices, and potentially improve the financing choices of actual firms.

Our model also suggests several policy issues that have not been emphasized by the literature. First, we assume that firms cannot sell their tax shields when they enter bankruptcy. This assumption seems plausible given the restrictions on doing so imposed by the Tax Reform Act of 1986. However, our analysis suggests that easing this restriction could have important implications for capital structure and make debt much more attractive. Second, it suggests that the ability of bondholders to force default through covenants has an important effect on both the value of a firm's tax shields, and, consequently, its optimal leverage ratio. Reforms that increase the rights of bondholders to force default may have a meaningful impact on the *ex ante* financing decisions that firms make.

The rest of this paper is organized as follows: Section 2 describes the model in detail. Section 3 explains how we calibrate the model to reflect current market data. Section 4 discusses the implications of the calibrated model, while Section 5 contains a short conclusion.

2. A Dynamic Model of Capital Structure

The models that we use are based on Ju (1998). In these models, the firm issues debt with a maturity of T, which pays a continuous, constant (tax-deductible) coupon. The manager's wealth at time zero is divided between non-firm wealth and his stake in the firm, which consists of equity shares and standard European call options, which expire at time T_u. The manager cannot sell or hedge his shares or options. For simplicity, it is assumed that the manager's non-firm wealth grows at the risk-free rate, r, and is therefore uncorrelated with the value of the manager's stake in the firm. The manager's utility is given by a CRRA utility function defined over his entire wealth. The value process of the firm's assets (e.g., the value of the cash flows from operations) follows geometric Brownian motion.

The model is in continuous time with $0 < T_u < T$. At time zero the value of the firm's assets is $V(0)$. Before the swap, the firm's capital consists of N_{NS} shares of stock with a total market value of

$E_{NS}(0)=V(0)$.[3] The value of the firm's assets, $V(t)$, follows geometric Brownian motion described by:

$$\frac{dV(t)}{V(t)}=(\mu-\delta)dt+\sigma dZ(t) \tag{1}$$

where μ and $\sigma>0$ are constants and $dZ(t)$ is a standard Weiner process. The firm liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ is equal to a time varying dividend $div(t)dt$ paid to equity holders over the time interval dt:

$$\delta V(t)dt=div(t)dt. \tag{2}$$

The value of δ is specified exogenously as a model parameter.

We will consider a fair equity for debt swap at time zero that either (1) maximizes the value of a share of equity or (2) maximizes the manager's expected utility at time T_u. The swap is fair in the sense that the debt which is issued is done so at its correct market value. The debt has a face value of F_S and has a market value when it is issued at time zero of $D_S(0)$. The debt pays a coupon at a constant annualized rate C_S which is set so that the bond is priced at par, that is, so that $F_S=D_S(0)$. The firm deducts its coupon payments from its taxes at an effective rate τ, and the tax benefit of the debt at time zero has a value $TB_S(0)$. The bond has a protective covenant which specifies that if the firm value at anytime during the life of the bond $[0,T]$ decreases to an exponential boundary, the firm is forced into bankruptcy.[4] When this occurs, the stock becomes worthless and the debtholders recover $1-\alpha_{BC}$ of the value of the assets. The fraction of the value of the assets not recovered by the debtholders is assumed to

[3] The subscript NS refers to quantities before the swap and the subscript S refers to quantities after the swap is performed.

[4] Following Black and Cox (1976), we are implicitly assuming that this covenant acts somewhat like the actual covenants seen in bond indentures. The idea is that actual bond covenants are set up for the purpose of allowing bondholders to seize assets when they are in danger of being lost – this assumption models this process explicitly.

be consumed in the bankruptcy process. The bankruptcy boundary is an exponential curve that increases at a rate g and is equal to the face value of debt at time T. Consequently, the bankruptcy boundary is described by $F_S e^{g(t-T)}$. The bankruptcy costs for the firm are the present value of the expected losses in bankruptcy, and are denoted by $BC_S(0)$. After the swap the firm still liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ equals the sum of the after-tax coupon paid to bond holders [$(1-\tau)C_S dt$] and a time varying dividend $div(t)dt$ paid to equity holders over the time interval dt:

$$\delta V(t)dt = \left[div(t) + (1-\tau)C_S\right]dt. \tag{3}$$

In order to guarantee that the dividend rate is non-negative, we require that

$$\delta V(t) \geq (1-\tau)C_S. \tag{4}$$

We assume that the swap is fully transparent so that the post-swap values of the debt and equity exchanged are equal in magnitude and opposite in sign. That is,

$$D_S(0) = -\left(\frac{N_S - N_{NS}}{N_S}\right)E_S(0).$$

At time zero there is an infinite number of fair equity for debt swaps available to the firm. We will analyze two of these. The first type of swap that we consider maximizes the value of a share of equity. That is, it maximizes the quantity, $E_S(0)/N_S$. The second type of swap that we consider maximizes the manager's expected utility at time T_u. That is, it maximizes the expected value of the manager's CRRA utility function (which is defined over his total wealth) at time T_u.

At time zero the manager's stake in the firm consists of $N_{Man}\ (< N_{NS})$ shares and N_{Calls} European call options with strike price K that expire at time T_u. For purposes of computational tractability, we assume that the firm buys the manager's calls from a third party. Hence, if the manager exercises the calls at time T_u, he buys N_{Calls} shares from the third party at a price of $N_{Calls}K$ dollars. We assume that the manager cannot sell or hedge either his shares or his options. In addition, at time zero the manager has $NFW\left(0\right)$ dollars of non-firm wealth. For simplicity, this wealth is assumed to grow at the risk-free rate. When the swap is performed in order to maximize the manager's expected utility at time T_u, this utility is described by

$$U\left(Wealth_{T_u}\right) = \frac{\left(Wealth_{T_u}\right)^{1-\gamma} - 1}{1-\gamma} \tag{5}$$

where γ is a risk-aversion parameter and $Wealth_{T_u}$ is the manager's total wealth at time T_u.

The value of the debt, the bankruptcy costs, and the tax benefit of debt are computed from the probability density function for first hitting the exponential bankruptcy boundary. Let $f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)$ be the probability density for first hitting a boundary described by Ae^{gt} at a time t^*, where A is a constant, if the variable V initially has a value $V\left(0\right) > A$ and follows geometric Brownian motion with drift $r-\delta$ and volatility σ. In our model, A is the value of the bankruptcy boundary at time zero, so that A is equal to $F_S e^{-gT}$. An explicit expression for $f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)$ is provided in the Appendix. Next define:

$$G\left(T,V\left(0\right),A,g,r,\delta,\sigma\right) \equiv \int_0^T f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)dt^* \tag{6}$$

$$H\left(T,V\left(0\right),A,g,r,\delta,\sigma\right) \equiv \int_0^T e^{-rt} f\left(t;V\left(0\right),A,g,r,\delta,\sigma\right)dt \tag{7}$$

and

$$I\left(T,V\left(0\right),A,g,r,\delta,\sigma\right) \equiv \int_0^T e^{-(r-g)t^*} f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)dt^*. \tag{8}$$

Closed form solutions for these expressions are derived in the Appendix.

Following Leland and Toft (1996), the value of the debt at time zero is the sum of a contribution from the coupon, a contribution from the payment to debtholders if bankruptcy occurs, and the repayment of the face value at time T if bankruptcy does not occur:

$$
\begin{aligned}
D_S(0) = C_S \int_0^T e^{-rt^*} \left(1 - G\left(t^*, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right)\right) dt^* \\
+ \int_0^T e^{-rt^*} \left(1 - \alpha_{BC}\right) F_S e^{-g\left(T - t^*\right)} f\left(t^*, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right) dt^* \\
+ F_S \left(1 - G\left(T, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right)\right) e^{-rT}
\end{aligned} \tag{9}
$$

or

$$
\begin{aligned}
D_S(0) = \frac{C_S}{r}\left(1 - \left(1 - G\left(T, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right)\right) e^{-rT} - H\left(T, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right)\right) \\
+ \left(1 - \alpha_{BC}\right) F_S e^{-gT} I\left(T, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right) \\
+ F_S \left(1 - G\left(T, V(0), F_S e^{-gT}, g, r, \delta, \sigma\right)\right) e^{-rT}
\end{aligned} \tag{10}
$$

Another modeling decision involves the question of whether the firm should refinance the debt obtained in the swap when it matures. We consider two alternative models: The first is a "static" model, in which the firm does not refinance debt, and becomes an all-equity firm subsequent to the time the debt matures. The second is a "dynamic" model, in which new debt is reissued at the time of maturity. Since the dynamic framework seems *a priori* more appealing, and in fact Ju (1998) shows that the refinancing assumption can affect corporate financing decisions *ex ante*, we analyze the dynamic model. Nonetheless, it is convenient to present the solution of the dynamic model in terms of that for the static model that we develop now.

In the static model, when the firm is forced into bankruptcy at time t^*, the bankruptcy costs are $\alpha_{BC} V\left(t^*\right)$. Hence, at time zero the value of the bankruptcy costs are

$$
BC_S(0) = \int_0^T \alpha_{BC} F_S e^{g\left(t^* - T\right)} e^{-rt^*} f\left(t^*; V(0), F_S e^{-gT}, g, r, \delta, \sigma\right) dt^* \tag{11}
$$

or

$$BC_S\left(0\right)=\alpha_{BC}F_Se^{-gT}I\left(T,V\left(0\right),F_Se^{-gT},g,r,\delta,\sigma\right) \tag{12}$$

The tax benefits of debt accrue to the firm as long as it has not gone bankrupt. Consequently, the tax benefits of debt in the static model can be computed by

$$TB_S\left(0\right)=\int_0^T \tau C_S e^{-rt^*}\left(1-G\left(t^*,V\left(0\right),F_Se^{-gT},g,r,\delta,\sigma\right)\right)dt^* \tag{13}$$

or

$$TB_S\left(0\right)=\frac{\tau C_S}{r}\left(1-\left(1-G\left(T,V\left(0\right),F_Se^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT}-H\left(T,V\left(0\right),F_Se^{-gT},g,r,\delta,\sigma\right)\right) \tag{14}$$

The value of the equity is equal to the value of the assets plus the tax benefits of debt minus the bankruptcy costs minus the value of the debt:

$$E_S\left(0\right)=V\left(0\right)+TB_S\left(0\right)-BC_S\left(0\right)-D_S\left(0\right) \tag{15}$$

In order to compute the manager's time zero expectation of his utility at time T_u, let $V^K\left(T_u\right)$ be the value of the firm's assets at time T_u that makes a share of stock worth K at time T_u. Then the manager's time zero expectation of his utility at time T_u is the sum of three components. The first component is a function of the probability density for the value of the firm's assets being at various levels above $V^K\left(T_u\right)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The second component is a function of the probability density for the value of the firm's assets being at various levels below $V^K\left(T_u\right)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The third component is the utility derived from his non-firm wealth if the bankruptcy boundary is hit. Let $g\left(V\left(0\right),V\left(T\right),T,A,g,\mu,\delta,\sigma\right)$ be the density function for starting at a value $V\left(0\right)>A$ and being at $V\left(T\right)>Ae^{gT}$ at time $T>0$ without ever hitting the boundary Ae^{gt} in the interval $t\in\left[0,T\right]$ when the V process follows geometric Brownian motion with drift $\mu-\delta$ and volatility σ. An explicit expression for $g\left(V(0),V(T),T,A,g,\mu,\delta,\sigma\right)$ is presented in the Appendix.

Then at time zero, the manager's expectation of his utility at time T_u after the swap is given by

$$\begin{aligned}
Utility_S(0) = & \int_{V^K(T_u)}^{\infty} U\left\{NFW(T_u) + \frac{N_{Man}+N_{Calls}}{N_S}\left[V(T_u)+TB_S(T_u)-BC_S(T_u)-D_S(T_u)\right]-N_{Calls}K\right\} \\
& \times g\left(V(0),V(T_u),T_u,F_S e^{-gT},g,\mu,\delta,\sigma\right)dV(T_u) \\
& + \int_{F_S e^{-g(T-T_u)}}^{V^K(T_u)} U\left\{NFW(T_u) + \frac{N_{Man}}{N_S}\left[V(T_u)+TB_S(T_u)-BC_S(T_u)-D_S(T_u)\right]\right\} \\
& \times g\left(V(0),V(T_u),T_u,F_S e^{-gT},g,\mu,\delta,\sigma\right)dV(T_u) \\
& + U\left(NFW(T_u)\right)\int_0^{T_u} f\left(t;V(0),F_S e^{-gT},g,\mu,\delta,\sigma\right)dt
\end{aligned} \tag{16}$$

where $V^K(T_u)$ satisfies the following equation:

$$K = \frac{V^K(T_u)+TB_S(T_u)-BC_S(T_u)-D_S(T_u)}{N_S}. \tag{17}$$

Note that all terms on the right hand side of equation (17) are a function of $V^K(T_u)$.

Next we extend the model to a more realistic dynamic setting. As in the static case, after the swap at time zero the firm has a bond outstanding with T years to maturity. Now, however, if the firm has not gone bankrupt at the end of T years, the firm issues a new T year bond at time T. The new bond has a coupon of $C_S V(T)/V(0)$. Similarly, as shown in the Appendix, all other securities will be scaled by a factor of $V(T)/V(0)$, because at time T the firm is identical to itself at time zero except that it is $V(T)/V(0)$ as large. The process of issuing a new T-year bond each time that a bond expires continues indefinitely until the firm goes bankrupt.

In this dynamic setting, the price of the debt is still given by equation (10). The firm value, however, will reflect the costs and benefits of the debt issued in the future until the firm goes bankrupt. In order to determine the total tax benefit and total bankruptcy cost of the current and potential future issues of debt, the following quantity will be useful:

$$\phi \equiv e^{-rT} E^{Q}\left[\mathbf{1}_{\{\text{Firm does not go bankrupt over } [0,\text{T}]\}} \frac{V(T)}{V(0)} \right] \tag{18}$$

The indicator function $\mathbf{1}_{\{\text{Firm does not go bankrupt over } [0,\text{T}]\}}$ is equal to one if the firm does not go bankrupt over the interval $[0,T]$ and zero otherwise. The expectation is taken over the risk-neutral Q measure. In the Appendix, we show that ϕ is given by the following expression:

$$\phi = e^{-\delta T}\left[N\left(d^1\right) - \left(\frac{F_S e^{-gT}}{V(0)} \right)^{2\left(1+\left(r-\delta-g-\sigma^2/2\right)/\sigma^2\right)} N\left(d^2\right) \right] \tag{19}$$

where

$$d^1 = \frac{-\log\left(F_S e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}} \tag{20}$$

and

$$d^2 = \frac{\log\left(F_S e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}}. \tag{21}$$

We also show in the Appendix that the total tax benefit of debt and the total bankruptcy costs are given by

$$TB_S^{Dynamic}(0) = \frac{TB_S(0)}{1-\phi} \tag{22}$$

and

$$BC_S^{Dynamic}(0) = \frac{BC_S(0)}{1-\phi} \tag{23}$$

Similar to equation (15), the value of the equity is equal to the value of the assets plus the tax benefits of debt minus the bankruptcy costs minus the value of the debt:

$$E_S^{Dynamic}(0) = V(0) + TB_S^{Dynamic}(0) - BC_S^{Dynamic}(0) - D_S(0). \tag{24}$$

Finally, the manager's utility after the swap in the dynamic model is given by

$$
\begin{aligned}
Utility_S^{Dynamic}(0) = & \int_{V^K(T_u)}^{\infty} U\left\{NFW(T_u) + \frac{N_{Man}+N_{Calls}}{N_S}\left[V(T_u)+TB_S^{Dynamic}(T_u)-BC_S^{Dynamic}(T_u)-D_S(T_u)\right]-N_{Calls}K\right\} \\
& \times g\left(V(0),V(T_u),T_u,F_S e^{-gT},g,\mu,\delta,\sigma\right)dV(T_u) \\
& + \int_{F_S e^{-g(T-T_u)}}^{V^K(T_u)} U\left\{NFW(T_u) + \frac{N_{Man}}{N_S}\left[V(T_u)+TB_S^{Dynamic}(T_u)-BC_S^{Dynamic}(T_u)-D_S(T_u)\right]\right\} \\
& \times g\left(V(0),V(T_u),T_u,F_S e^{-gT},g,\mu,\delta,\sigma\right)dV(T_u) \\
& + U\left(NFW(T_u)\right)\int_0^{T_u} f\left(t;V(0),F_S e^{-gT},g,\mu,\delta,\sigma\right)dt.
\end{aligned}
$$

3. Calibrating the Model

In choosing the amount of debt that will be swapped for outstanding equity, the manager is assumed to select the face value of 10-year debt (i.e., T = 10 years) such that he maximizes his expected utility one year in the future (i.e., $T_u = 1$). The total value of the firm's assets before the swap, $V(0)$, is normalized to \$100. Since the firm has no leverage before the swap, the value of the firm's assets equals the value of the equity, which is divided among a total of 100 shares. We assume that the manager of the firm owns a 0.32 share of stock and a 1-year exchange traded European call option on an additional 0.38 shares.[5] The strike price for the call option is set equal to the time zero value of a share of equity of the firm before the swap, \$1. For the base-case, the manager's non-firm wealth is assumed to equal the time-zero value of the shares that the manager owns without the project, \$0.32. Consistent with the literature, we assume the manager's risk aversion parameter γ equals 2 (see pp. 258-260 of Ljungqvist and Sargent (2000) for a discussion of the interpretation of this value and other values of γ used in the sensitivity analysis).

[5] The manager's stock and option holdings represent the median values for managers at 1,405 firms for which sufficient data to estimate these figures are available for 1999 on the ExecuComp database.

Given these assumptions, calibration of the model requires estimates of (1) the risk-free rate, *r*, (2) the effective tax rate, τ, (3) the drift parameter for the total value of the firm, μ, (4) the volatility of the total value of the firm, σ, (5) the level of dividends, DivRate, paid by the firm, (6) the debtholder bankruptcy recovery rate, $\left(1-\alpha_{BC}\right)$, and (7) the bankruptcy boundary's exponential growth rate, *g*. We estimate these parameters using data from the end of January 2001.

As our estimate of the risk-free rate, we use the rate on 10-year Treasury bonds as of January 30, 2001 as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. This rate equals 5.22 percent.

We estimate the tax rate used to calculate the tax shields from the debt using data on estimated marginal tax rates (before interest expense) provided by John Graham, who constructed these estimates using the approach described in Graham (1996). In particular, for the base case, we assume that the tax rate equals the median marginal tax rate of 34 percent for the 5,519 firms for which 1999 estimates are available.

We set the drift parameter of the firm, μ, equal to 5 percent. This value is consistent with an expected long-term inflation rate of 2.5 percent and 2.5 percent real growth. The 2.5 percent long-term inflation rate is consistent with five-year estimates published by WEFA (formerly Wharton Econometric Forecasting Associates) for the Consumer Price Index in its *US Outlook* report for December 2000.

To estimate the volatility of the total value of the firm's assets, σ, we examine the sample of 1,043 firms for which the necessary data are available on Compustat for the entire 1980 to 1999 period. The median value of the annual standard deviation of the percentage change in firm value for the 1,043 firms, 0.2852, provides a lower bound for our estimate of σ.[6] This value is a lower bound because there is a survivorship bias in the sample. We use 0.32 as our estimate of the value of σ for the universe of

[6] This estimate is only an approximation, as it does not incorporate bankruptcy costs, which are not observable. It is relatively insensitive to the sample and period. Estimates of σ range from 0.2513 to 0.3333 for different time periods (ten and 20 years) and samples (firms for which all data are available for the full 20 year period and for which data are only available for ten years).

firms. Because we are interested in examining how the volatility of the firm's assets affects the financing decision, we examine the impact of σ values ranging from 0.15 to 0.50 on the swap decision in the sensitivity analyses below.

We set the dividend rate, DivRate, equal to 1.5 percent in the base case. Because this rate is stated as a percentage of the unlevered value of the firm, we use a number that is on the lower end of the 1.50 to 2.0 percent dividend yield paid by public firms at the beginning of 2001.

The debtholder bankruptcy recovery rate and the exponential growth rate for the bankruptcy boundary are selected to yield an expected recovery rate of 45 percent and a spread over the 10-year Treasury bond rate for the firm's debt equal to 1.90 percent when the firm has a debt to total capital ratio of 21.9 percent (the median value for all firms in the Compustat database in 1999). The 45 percent recovery rate is broadly consistent with recovery rates published by Hamilton, Gupton, and Berhault (2001). For the 1981 to 2000 period, Hamilton, Gupton, and Berhault estimate the mean default recovery rates for senior secured bonds, senior unsecured bonds, and subordinated bonds of all ratings to equal 53.9 percent, 47.4 percent, and 32.3 percent, respectively. The 1.90 percent spread over the Treasury bond rate equals the spread for 10-year A-rated corporate debt as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. The bankruptcy recovery and bankruptcy boundary growth rates for our base case equal 0.5194 (α_{BC} = 0.4806) and 5.19 percent, respectively.

Panel A of Table I summarizes our parameter choices. These choices are used to derive the set of parameters that are presented in Panel B of Table I.

4. Optimal Capital Structure in this Model

In this model, each potential capital structure leads to a different value of tax shields and bankruptcy costs, and ultimately different price distributions for the firm's securities. To determine the 'optimal' capital structure, we begin with an all equity firm, and consider the range of potential debt/equity swaps. Potential swaps are assumed to be fair in the sense that the new debt is issued at its fair market value. We also assume that the swap is fully transparent so that the post-swap values of the debt and equity that are exchanged in the swap are equal in magnitude and opposite in sign. That is,

$$D_S(0) = -\left(\frac{N_S - N_{NS}}{N_S}\right) E_S(0).$$

The optimal capital structure from a particular viewpoint is defined as the swap that maximizes the objective function in question. So, from the shareholders' perspective, the optimal capital structure is defined by the swap that maximizes the post-swap value of each share of equity, and from the managers' viewpoint, the optimal capital structure maximizes the manager's expected (post-swap) utility at time T_u.

4.1. Optimal Capital Structures for a Representative Firm

4.1.1. The Shareholders' Perspective

Table II presents calculations of shareholders' optimal capital structure, assuming the model is calibrated as discussed above. Each column represents a different level of asset volatility; all other parameters are the 'base-case' ones discussed above. The optimal capital structure is shown in Row 1. It is clearly very sensitive to asset volatility, equaling 42.2% when asset volatility is 12% and 9.9% when asset volatility is 52%.

This relation is consistent with casual empiricism, as well as with more formal studies suggesting that riskier firms do in fact use less leverage (see for example Titman and Wessels, 1988; or Rajan and Zingales, 1995). Yet, to evaluate the model's quantitative prediction requires a precise estimate of asset volatility. As stated above, we use 0.32 as our point estimate for the volatility of the firm's asset value. Our estimates of volatility are likely to be somewhat crude, given that they rely on book values of debt, and are net of the unobservable level of bankruptcy costs, which presumably vary over time. Yet, despite their crudeness, the model does surprisingly well at predicting capital structures. For the median firm, the model predicts a debt/total capital ratio of 19.7%. Using the same sample of 1,043 Compustat firms, the actual debt/total capital ratio is 21.9%. This comparison suggests that asset risk is an important factor in financing decisions. Improved estimates of asset risk are likely to lead to better capital structure decisions in practice.

These findings are counter to the conventional wisdom that the tradeoff approach to capital structure implies substantially more leverage than is observed in the data. To understand why our model leads to different findings from the usual intuition, one should examine Rows 10 and 11 of Table II, showing the values of bankruptcy costs and tax shields predicted by our model. For each level of volatility, the value of tax shields is substantially higher than the value of bankruptcy costs. These differences exist despite the fact that the leverage levels shown in Table II maximize share value. The reason for these differences is that adding leverage beyond that shown in Table II would decrease the value of tax shields sufficiently to lower share values. The true marginal cost of additional leverage includes not just bankruptcy costs, but a decline in the value of existing tax shields. This potential decline in the value of tax shields is one reason the predicted values from our model are lower than those produced by the common intuition about the tradeoff theories of capital structure (Miller, 1977).

4.1.2. The Manager's Perspective

Much of the capital structure literature has concerned the implications of agency problems for financing decisions. We next evaluate our model in an agency framework by replacing the assumption that capital structure is chosen to maximize the per share value of equity with the assumption that it is chosen to maximize the manager's utility function.[7]

We first assume that the manager maximizes a constant relative risk aversion (CRRA) utility function with a risk aversion parameter of 2 and has 50 percent of his non-option wealth invested in

[7] Much of the attention on agency problems and capital structure has historically focused on conflicts between stockholder and bondholders [Jensen and Meckling, 1976 and Myers, 1977]. While researchers have spent much time on the impact of these conflicts elsewhere (Mello and Parsons (1992), Leland (1998), Parrino and Weisbach, 1999; Parrino, Poteshman, and Weisbach, 2002), we ignore these conflicts here. To incorporate stockholder/bondholder conflicts would require specifying a distribution of potential projects, integrating over this distribution given a particular manager's preferences, and then seeing how the value of the firm's securities varies with the investment opportunity set. Such an exercise is beyond the scope of this paper, although our opinion is that it would be an excellent direction for future research.

shares of the firm, with the remainder invested in risk-free assets. As noted in Section 3, we assume that this stake in the firm equals 0.32% of the firm's equity and at the money call options to purchase 0.38% more of the firm's equity. We calculate the optimal capital structure from the manager's perspective by choosing the swap that maximizes the value of this utility function rather than the value of a share of common stock.

The results from this managerial model are presented in Table III. Since the manager is assumed to be risk-averse and the risk of the firm's equity increases with leverage, it is not surprising that the manager prefers less leverage than the shareholders. Comparing Tables II and III, for each level of risk, the optimal leverage from manager's perspective appears to be about 4 percentage points lower than from the shareholders' perspective.

4.2. Sensitivity of Optimal Capital Structure to Model Parameters

4.2.1. Tax Rates

Since the major factor leading to a preference for debt is its tax-deductibility, we expect the model's results to be especially sensitive to tax rates. We compute optimal capital structures (from the shareholder's perspective) as a function of corporate tax rates in Table IV.

Table IV indicates, unsurprisingly, that optimal leverage ratios are negatively related to the firm's tax rate. However, this relation appears to be nonlinear and is not as strong as one might expect. With a corporate tax rate of just one percent, the optimal leverage ratio equals 4.5%. This ratio rises 7.3 percentage points to 11.8% when the tax rate rises to 12%. In contrast, at higher tax rates the same 11 percentage point increase in tax rates (from 23% to 34%) leads to only a 3.6 percentage point increase in leverage, from 16.1% to 19.7%. Only when tax rates are twice as high as the current median rate estimated by Graham (1996) does the estimated debt to total capital ratio exceed 30%.

4.2.2. Bankruptcy Boundary

An important element of our model is that the firm is assumed to default if it hits a pre-specified bankruptcy boundary. The idea underlying this assumption is that most publicly traded debt contains covenants enabling debtholders to force default when the value of the firm is sufficiently low. In our model, the parameter g represents the steepness of this boundary, so that a lower g increases the likelihood that the firm defaults given poor performance. Intuitively, g can be thought of as a negative function of the strength of the bond's covenants. It is not clear conceptually how we expect this variable to be related to the shareholders' optimal leverage: Stronger bondholder rights make debt more attractive allowing debt to be issued at lower interest rates. Whether these lower interest rates are sufficient to compensate shareholders for the increased bankruptcy probabilities is not obvious.

Table V presents estimates of the optimal capital structure as a function of g. The results in this Table indicate that optimal leverage is a positive function of g. As the rights of debtholders to force default increase, firms find it optimal to use less leverage. Thus, it appears that the direct effect of a lower g through increased bankruptcy probabilities is more than sufficient to offset the indirect effect of lower interest rates.

4.2.3. Costs Conditional on Reaching Bankruptcy

The bankruptcy cost parameter in our model, α_{BC}, represents the proportional value lost to bankruptcy costs conditional on hitting the default boundary. We examine the sensitivity of optimal capital structure to this parameter in Table VI.

Not surprisingly, leverage is negatively related to bankruptcy costs. With α_{BC} equal to 10%, the optimal leverage ratio is 31.3%, compared to 19.4% with α_{BC} of 50%. However, the relation is relatively weak. While one might expect that as α_{BC} approaches zero, the firm will become extremely highly leveraged, the results in Table VI suggest that when α_{BC} declines to 10%, the leverage ratio "only" increases to 31.3%. These results from Tables V and VI suggest that the threshold at which the debtholders can force the firm into bankruptcy is likely to be as important as the magnitude of the value that is consumed in the bankruptcy process. Perhaps this finding should not be surprising since the relevant measure of bankruptcy costs in the capital structure decision is the expected bankruptcy costs at the time the financing decision is made. Yet, in most textbook discussions of the effect of bankruptcy on capital structure, much is discussed about bankruptcy costs while the rights of bondholders to force bankruptcy are not usually emphasized.

4.3. Model Estimates for Individual Firms

In addition to estimating the model using parameters for a typical firm, we examine its ability to predict the capital structures observed in a sample of 15 actual firms, five firms from each of three industries – wholesale distribution, beer and wine manufacturing, and paper and allied products. To calibrate the model for individual firms and yet retain comparability across them, the nature of the debt contract is assumed to be similar across all firms. That is, we assume that each firm is financed with 10-year debt, and that the bankruptcy boundary and bankruptcy costs are the same for all firms. We allow the volatility of the firm's assets and the CEO's stock and option holdings to vary across firms. The volatility for each firm is estimated, using the model, by computing the volatility that yields the observed spread between each firm's actual current cost of debt and the yield on Treasury Bonds. These values range from 12.9% to 56.4% with a median value of 27.3%. The stock and option holdings for the individual CEO's are from the 2000 proxy statements filed by the sample firms with the SEC.

Table VII reports the estimated asset volatility, actual leverage, and estimated leverage, both value maximizing and utility maximizing, for each of the 15 sample firms. The striking feature of these data is that, while the model appears to do a good job of predicting leverage for firms with relatively little to typical levels of debt, such as Stewart & Stevenson, Ravenswood, Kimberly-Clark, P H Glatfelter, and Wausau-Mosinee, it substantially underestimates leverage for firms with large amounts of debt. The fact that the model tends to underestimate rather than overestimate leverage is counter to the usual intuition that tax shields are far too large to be offset by bankruptcy costs. Our model emphasizes that bankruptcy costs are not the only factor limiting leverage in the tradeoff framework; rather, the fact that tax shields lose value in bankruptcy limits the leverage ratio that maximizes shareholder value.

5. Conclusions

This paper considers a model of optimal capital structure in which the major forces affecting firms' financing decisions are corporate taxes and bankruptcy costs. As such, this model incorporates the effects that have been discussed at great length in the corporate finance literature since Modigliani and Miller (1963). The model contains a number of features designed to capture key elements of the capital structure decision as realistically as possible, including contingent-claims valuation of tax shields, a bankruptcy boundary on firm value below which firms default, and a target capital structure at which the firm refinances its debt at maturity. We calculate closed-form solutions for the important variables in this model, calibrate it using recent market data, and solve for the optimal capital structures from both the shareholders' and manager's perspectives.

In contrast to most of the literature since at least Miller (1977), we find that the tradeoff model does *not* predict that firms are underlevered. For a hypothetical firm constructed to be typical of large, publicly-traded companies, the model predicts a leverage ratio close to actual sample median – the predicted debt to total capital ratio is 19.7% compared to a sample median of 21.9%. When we calibrate the model to reflect actual firms, the model performs less well. However, the model's failure goes in the opposite direction from what it usually presumed. In contrast to the usual intuition, the model suggests that a number of firms appear to be overlevered, at least when only taxes and bankruptcy costs are considered.

These findings occur in a model in which the average tax benefits are substantially larger than the average expected bankruptcy costs. The explanation for the difference between our results and the usual intuition probably lies in the fact that we explicitly value a firm's tax shields. These shields lose their value when a firm defaults, so any decision that increases the probability of default, lowers the value of a firm's tax shields. Therefore, when a firm adds additional leverage, the additional debt creates new tax shields; however, in doing so it increases bankruptcy probabilities and decreases the value of the tax shields from the existing debt. This last effect has, to our knowledge, been ignored by the existing literature.

We also perform comparative statics on the model's underlying parameters, to determine their impact on capital structure choice. One parameter that appears to be particularly important is g, the slope of the bankruptcy boundary, which we interpret as measuring the strength of a firm's bond covenants. Our model assumes that this parameter is set exogenously; in a more realistic model of capital structure the strength of these covenants should be thought of as an important decision variable in a firm's financing decisions.

By focusing on the tradeoff between taxes and capital structure, we do not mean to downplay the importance of other factors. Clearly, the literature has identified agency and information issues as key issues that must be considered in financing decisions. An interesting recent paper applying methods similar to ours that incorporates some of these factors is Titman and Tsyplacov (2001). In addition, we do not address the surprising lack of evidence on adjustments that managers make to capital structure in order to keep capital structure ratios equal to some long-run target (Welch 2002). Rather, our message is that the simple tradeoff framework actually does much better at predicting average leverage levels than has typically been supposed, and should not be dismissed lightly as at least a first-pass way of understanding a firm's financing choices.

We also want to emphasize the usefulness of the approach of taking models seriously and calibrating them using market data. This quantitative approach has been usefully applied in other branches of economics, notably macroeconomics. Its main appeal is that it allows for quantitative comparisons between alternative theories. Given the multitude of theories in corporate finance together with the general lack of exogenous variation across firms facing any researcher attempting to do traditional empirical work, it seems likely that subsequent advances are likely to come from taking some of these models seriously and applying numerical methods to them.

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Table I
Model Parameters

Panel A: Chosen Parameters

Variable	Calibrated Value	Variable Description
T_u	1	Time at which manager evaluates utility and options mature
T	10	Time at which debt matures
r	0.0522	Annualized risk-free rate
$V(0)$	\$100	Value of assets before swap
μ	0.05	Drift of value of firm assets
σ	0.32	Volatility of value of firm assets
N_{NS}	100	Total shares outstanding before swap
γ	2	Manager's risk aversion parameter
N_{Man}	0.32	Number of shares owned by manager
N_{Calls}	0.38	Number of exchange traded European calls owned by manager
K	\$1	Strike price of calls
$NFW(0)$	\$0.32	Manager's non-firm wealth in dollars at time zero
α_{BC}	0.4806	1 - Debtholder bankruptcy recovery rate
g	0.0519	Bankruptcy boundary exponential growth rate
τ	0.34	Effective tax rate for debt tax shield
DivRate	0.015	Dividend payout rate to equity holders as a percentage of the unlevered value of the firm.

Table I (continued)

Panel B: Derived Variables

Variable	Variable Description
F_S	Face value of debt after swap
C_S	Constant annualized coupon rate paid on debt after swap. This is set to price the debt at par.
$D_S(0)$	Initial total value of debt after swap
N_S	Total shares outstanding after swap
$E_S(0)$	Initial total value of equity after swap
$BC_S(0)$	Initial total value of bankruptcy costs after swap
$TB_S(0)$	Initial total value of tax benefits of debt after swap
$NFW(T_u)$	Value of manager's non-firm wealth at time T_u
$Utility(0)$	Expected future value of manager's utility before swap
$Utility_S(0)$	Expected future value of manager's utility after swap
ϕ	Discounted risk-neutral expected value of the quantity $V(T)/V(0)$
$E_S^{Dynamic}(0)$	Initial total value of equity before swap
$BC_S^{Dynamic}(0)$	Initial total value of bankruptcy costs after swap
$TB_S^{Dynamic}(0)$	Initial total value of tax benefits of debt after swap
δ	After tax cash payout rate to both debtholders and equity holders as a percentage of the unlevered value of the firm.
$V^K(T_u)$	Value of assets that makes a share of stock worth *K* dollars at time T_U.

Table II

Model Output for Firms with Different Firm Asset Volatilities Where Objective is to Maximize Share Value

		Volatility of Firm Asset Value								
Row	**Variable**	**0.1200**	**0.1700**	**0.2200**	**0.2700**	**0.3200**	**0.3700**	**0.4200**	**0.4700**	**0.5200**
1)	**Debt/Total Capital After Swap**	42.2%	34.3%	28.2%	23.5%	19.7%	16.6%	14.0%	11.8%	9.9%
	Equity:									
2)	Value of Equity Before Swap	$100	$100	$100	$100	$100	$100	$100	$100	$100
3)	Number of Shares Before Swap	100	100	100	100	100	100	100	100	100
4)	Value of Equity After Swap	$73.05	$79.76	$84.14	$87.09	$89.17	$90.72	$91.93	$92.94	$93.83
5)	Number of Shares After Swap	57.79	65.71	71.77	76.52	80.32	83.42	86.01	88.19	90.08
6)	Change in Share Price	$0.2642	$0.2139	$0.1724	$0.1382	$0.1102	$0.0874	$0.0689	$0.0538	$0.0417
	Debt:									
7)	Face Value of Debt After Swap	$53.37	$41.63	$33.10	$26.73	$21.85	$18.02	$14.96	$12.44	$10.34
8)	Value of Debt After Swap	$53.37	$41.63	$33.10	$26.73	$21.85	$18.02	$14.96	$12.44	$10.34
9)	Coupon After Swap	$2.8584	$2.2749	$1.8553	$1.5448	$1.3078	$1.1210	$0.9685	$0.8397	$0.7273
10)	**Bankruptcy Costs After Swap**	$1.7881	$2.5841	$3.3156	$3.9439	$4.4442	$4.8036	$5.0183	$5.0904	$5.0263
11)	**Tax Benefit After Swap**	$28.2047	$23.9757	$20.5515	$17.7619	$15.4646	$13.5431	$11.9038	$10.4738	$9.1970
12)	**Change in Utility**	0.2640	0.2017	0.1518	0.1121	0.0806	0.0557	0.0361	0.0210	0.0095
13)	**Firm Value At Which E=K After Swap**	$87.5399	$87.9569	$89.0410	$90.3892	$91.7863	$93.1196	$94.3339	$95.4067	$96.3334

Table III

Model Output for Firms with Different Firm Asset Volatilities Where Objective is to Maximize the Manager's Utility

Values are for a manager with risk aversion parameter of 2, 0.32 shares, 0.38 options.

		Volatility of Firm Asset Value							
Row	**Variable**	**0.1200**	**0.1700**	**0.2200**	**0.2700**	**0.3200**	**0.3700**	**0.4200**	**0.4700**
1)	**Debt/Total Capital After Swap**	38.9%	30.9%	24.7%	19.6%	15.5%	12.1%	9.2%	6.9%
	Equity:								
2)	Value of Equity Before Swap	$100	$100	$100	$100	$100	$100	$100	$100
3)	Number of Shares Before Swap	100	100	100	100	100	100	100	100
4)	Value of Equity After Swap	$77.06	$83.69	$88.08	$91.18	$93.45	$95.16	$96.47	$97.48
5)	Number of Shares After Swap	61.12	69.12	75.34	80.37	84.51	87.94	90.79	93.13
6)	Change in Share Price	$0.2607	$0.2107	$0.1691	$0.1344	$0.1057	$0.0820	$0.0626	$0.0467
	Debt:								
7)	Face Value of Debt After Swap	$49.01	$37.38	$28.82	$22.26	$17.12	$13.04	$9.79	$7.19
8)	Value of Debt After Swap	$49.01	$37.38	$28.82	$22.26	$17.12	$13.04	$9.79	$7.19
9)	Coupon After Swap	$2.5935	$2.0055	$1.5734	$1.2392	$0.9726	$0.7557	$0.5775	$0.4310
10)	**Bankruptcy Costs After Swap**	$0.9230	$1.4952	$1.9800	$2.3028	$2.4449	$2.4154	$2.2386	$1.9502
11)	**Tax Benefit After Swap**	$26.9954	$22.5675	$18.8860	$15.7447	$13.0176	$10.6198	$8.4969	$6.6210
12)	**Change in Utility**	0.2681	0.2055	0.1557	0.1166	0.0859	0.0622	0.0440	0.0302
13)	**Firm Value At Which E=K After Swap**	$85.9412	$86.7723	$88.1444	$89.7141	$91.3151	$92.8539	$94.2763	$95.5506

Table IV

Model Output for Firms with Different Tax Rates Where Objective is to Maximize Share Value

Values are for a manager with risk aversion parameter of 2, 0.32 shares, 0.38 options.

		Tax Rate							
Row	**Variable**	**1%**	**12%**	**23%**	**34%**	**45%**	**56%**	**67%**	**78%**
1)	**Debt/Total Capital After Swap**	4.5%	11.8%	16.1%	19.7%	22.9%	25.9%	29.1%	32.9%
	Equity:								
2)	Value of Equity Before Swap	$100	$100	$100	$100	$100	$100	$100	$100
3)	Number of Shares Before Swap	100	100	100	100	100	100	100	100
4)	Value of Equity After Swap	$95.55	$90.38	$89.10	$89.17	$90.15	$91.80	$93.89	$95.91
5)	Number of Shares After Swap	95.46	88.17	83.87	80.32	77.13	74.09	70.91	67.15
6)	Change in Share Price	$0.0009	$0.0251	$0.0624	$0.1102	$0.1687	$0.2391	$0.3240	$0.4284
	Debt:								
7)	Face Value of Debt After Swap	$4.54	$12.13	$17.13	$21.85	$26.72	$32.11	$38.51	$46.93
8)	Value of Debt After Swap	$4.54	$12.13	$17.13	$21.85	$26.72	$32.11	$38.51	$46.93
9)	Coupon After Swap	$0.2376	$0.6588	$0.9736	$1.3078	$1.6958	$2.1783	$2.8302	$3.8265
10)	**Bankruptcy Costs After Swap**	$0.0269	$0.9049	$2.4214	$4.4442	$6.9505	$10.0057	$13.8028	$18.7770
11)	**Tax Benefit After Swap**	$0.1203	$3.4117	$8.6564	$15.4646	$23.8210	$33.9167	$46.2053	$61.6123
12)	**Change in Utility**	-0.0014	0.0159	0.0448	0.0806	0.1213	0.1651	0.2097	0.2505
13)	**Firm Value At Which E=K After Swap**	$99.9015	$97.6922	$94.8135	$91.7863	$88.9116	$86.4217	$84.5281	$83.5236

Table V

Values are for a manager with risk aversion parameter of 2, 0.32 shares, 0.38 options.

		g										
Row	**Variable**	**0%**	**2%**	**4%**	**6%**	**8%**	**10%**	**12%**	**14%**	**16%**	**18%**	**20%**
1)	**Debt/Total Capital After Swap**	15.8%	17.3%	18.8%	20.3%	21.7%	23.2%	24.6%	25.9%	27.2%	28.6%	29.9%
	Equity:											
2)	Value of Equity Before Swap	$100	$100	$100	$100	$100	$100	$100	$100	$100	$100	$100
3)	Number of Shares Before Swap	100	100	100	100	100	100	100	100	100	100	100
4)	Value of Equity After Swap	$92.26	$91.10	$89.90	$88.67	$87.42	$86.16	$84.90	$83.64	$82.39	$81.13	$79.85
5)	Number of Shares After Swap	84.20	82.70	81.21	79.72	78.26	76.84	75.44	74.08	72.75	71.44	70.15
6)	Change in Share Price	$0.0957	$0.1016	$0.1071	$0.1122	$0.1170	$0.1214	$0.1254	$0.1290	$0.1324	$0.1355	$0.1384
	Debt:											
7)	Face Value of Debt After Swap	$17.31	$19.05	$20.81	$22.55	$24.28	$25.97	$27.63	$29.26	$30.86	$32.43	$33.98
8)	Value of Debt After Swap	$17.31	$19.05	$20.81	$22.55	$24.28	$25.97	$27.63	$29.26	$30.86	$32.43	$33.98
9)	Coupon After Swap	$0.9910	$1.1098	$1.2330	$1.3592	$1.4872	$1.6163	$1.7459	$1.8757	$2.0060	$2.1371	$2.2699
10)	**Bankruptcy Costs After Swap**	$3.3544	$3.7768	$4.1974	$4.6099	$5.0101	$5.3957	$5.7660	$6.1217	$6.4645	$6.7968	$7.1216
11)	**Tax Benefit After Swap**	$12.9200	$13.9330	$14.9071	$15.8340	$16.7092	$17.5318	$18.3031	$19.0266	$19.7069	$20.3494	$20.9600
12)	**Change in Utility**	0.0739	0.0768	0.0793	0.0814	0.0830	0.0843	0.0852	0.0858	0.0860	0.0860	0.0857
13)	**Firm Value At Which E=K After Swap**	$92.1685	$91.9891	$91.8513	$91.7485	$91.6751	$91.6262	$91.5980	$91.5873	$91.5921	$91.6110	$91.6432

Table VI

Model Output for Firms with Different Bankruptcy Costs Where Objective is to Maximize Share Value

		Bankruptcy Costs								
Row	**Variable**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**	**45%**	**50%**
1)	**Debt/Total Capital After Swap**	31.3%	28.7%	26.6%	24.9%	23.4%	22.2%	21.1%	20.2%	19.4%
	Equity:									
2)	Value of Equity Before Swap	$100	$100	$100	$100	$100	$100	$100	$100	$100
3)	Number of Shares Before Swap	100	100	100	100	100	100	100	100	100
4)	Value of Equity After Swap	$79.05	$81.48	$83.38	$84.90	$86.14	$87.17	$88.03	$88.77	$89.41
5)	Number of Shares After Swap	68.67	71.29	73.41	75.14	76.60	77.83	78.89	79.81	80.62
6)	Change in Share Price	$0.1511	$0.1429	$0.1359	$0.1299	$0.1246	$0.1200	$0.1159	$0.1122	$0.1090
	Debt:									
7)	Face Value of Debt After Swap	$36.07	$32.81	$30.21	$28.08	$26.32	$24.83	$23.56	$22.45	$21.49
8)	Value of Debt After Swap	$36.07	$32.81	$30.21	$28.08	$26.32	$24.83	$23.56	$22.45	$21.49
9)	Coupon After Swap	$2.2004	$1.9909	$1.8261	$1.6930	$1.5832	$1.4911	$1.4126	$1.3449	$1.2858
10)	**Bankruptcy Costs After Swap**	$2.4331	$3.1278	$3.6056	$3.9308	$4.1486	$4.2905	$4.3785	$4.4280	$4.4499
11)	**Tax Benefit After Swap**	$17.5479	$17.4191	$17.1952	$16.9161	$16.6083	$16.2888	$15.9682	$15.6530	$15.3473
12)	**Change in Utility**	0.0964	0.0946	0.0924	0.0901	0.0879	0.0857	0.0836	0.0817	0.0799
13)	**Firm Value At Which E=K After Swap**	$90.8881	$90.9762	$91.0903	$91.2158	$91.3446	$91.4722	$91.5963	$91.7157	$91.8300

Table VII
Individual Firm Estimates

Company	Volatility of Asset Value	Actual Debt/Total Capital	Estimated Debt/Total Capital	
			Value Maximizing	Utility Maximizing
Panel A: Wholesale Distribution Firms				
Hughes Supply	0.171	56.36%	34.39%	28.90%
Stewart & Stevenson	0.564	10.98%	10.07%	8.57%
Airgas	0.223	56.67%	28.05%	21.01%
Valley National	0.206	63.44%	29.89%	22.91%
Speizman	0.129	65.06%	40.01%	34.95%
Panel B: Beer and Wine Manufacturing				
Mondavi	0.273	29.21%	23.33%	15.63%
Willamette	0.220	32.76%	28.37%	18.82%
Pyramid Breweries	0.290	31.74%	21.94%	17.07%
Golden State	0.291	35.52%	21.87%	14.53%
Ravenswood	0.362	8.77%	17.10%	8.76%
Panel C: Paper and Allied Products Manufacturing				
Boise Cascade	0.184	48.19%	32.69%	29.42%
Kimberly-Clark	0.469	6.24%	11.89%	12.55%
Mead	0.214	30.75%	28.99%	28.19%
P H Glatfelter	0.293	37.01%	21.66%	18.08%
Wausau-Mosinee	0.300	30.90%	21.15%	21.92%

Appendix A

In this appendix, we develop the dynamic model in more detail. To this end, we first consider a static model with a finite-maturity coupon bond.

The value of the firm's assets (unleveraged firm) is assumed to follow a geometric Brownian motion,

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dZ(t), \tag{A1}$$

where μ, δ, and σ are constants.[8]

At time zero, a coupon bond with annual coupon rate C_S and maturity T is issued at par. The bankruptcy boundary V_B is assumed to be an exponential function during [0,*T*],

$$V_B(t) = F_S e^{-g(T-t)}, \tag{A2}$$

where F_S is the par value of the bond and g a constant.

To price such a risky bond, let *f(t)* and *G(t)* be the density function and the cumulative distribution function, respectively, of the first-passage time to reach V_B from the initial firm asset value, *V*. The value of the bond at issue is given by

$$\begin{aligned} D_S(0) &= C_S \int_0^T e^{-rt}(1 - G(t))dt + \int_0^T e^{-rt}(1 - \alpha_{BC})F_S e^{-g(T-t)} f(t)dt + F_S(1 - G(T))e^{-rT} \\ &= \frac{C}{r}\left(1 - (1 - G(T))e^{-rT} - H(T)\right) + (1 - \alpha_{BC})F_S e^{-gT} I(T) + F_S(1 - G(T))e^{-rT}, \end{aligned} \tag{A3}$$

where α_{BC} is the portion of the firm's asset value that is lost in bankruptcy and

$$G(T) = \int_0^T f(t)dt, \qquad H(T) = \int_0^T e^{-rt} f(t)dt, \qquad I(T) = \int_0^T e^{-(r-g)t} f(t)dt.$$

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[8] Note that under risk-neutral measure for security pricing, μ will be replaced by the riskless interest *r*. The drift μ under the physical measure will be used in calculating the manager's utility.

To obtain *G(T)*, *H(T)*, $I(T)$, we need the first passage time density function. To this end, we define

$$x(t) = \log\left(\frac{V(t)}{F_s e^{-g(T-t)}}\right). \tag{A4}$$

One simple application of Ito's lemma yields (under risk-neutral measure)

$$dx = \left(r - \delta - g - \sigma^2/2\right)dt + \sigma dZ^Q(t). \tag{A5}$$

Therefore *x(t)* is a Brownian motion with drift $m = r - \delta - g - \sigma^2/2$ and diffusion σ, starting at $x_0 = \log\left(\frac{V}{F_s e^{-gT}}\right)$. From Ingersoll (1987), the first-passage time density function *f(t)* for crossing the origin is given by

$$f(t) = \frac{x_0}{\sigma t^{3/2}} n\left(\frac{x_0 + mt}{\sigma t^{1/2}}\right), \tag{A6}$$

where $n(\cdot)$ is the standard normal density function.

Now lengthy but straightforward calculations yield, [9]

$$G(T) = N\left[h_1(T)\right] + \left(\frac{V}{F_s e^{-gT}}\right)^{-2a} N\left[h_2(T)\right], \tag{A7}$$

$$H(T) = \left(\frac{V}{F_s e^{-gT}}\right)^{-a+z} N\left[q_1(T)\right] + \left(\frac{V}{F_s e^{-gT}}\right)^{-a-z} N\left[q_2(T)\right], \tag{A8}$$

$$I(T) = \left(\frac{V}{F_s e^{-gT}}\right)^{-a+\bar{z}} N\left[\bar{q}_1(T)\right] + \left(\frac{V}{F_s e^{-gT}}\right)^{-a-\bar{z}} N\left[\bar{q}_2(T)\right], \tag{A9}$$

$$h_1(T) = \left(\frac{-x_0 - a\sigma^2 T}{\sigma\sqrt{T}}\right), \qquad h_2(T) = \left(\frac{-x_0 + a\sigma^2 T}{\sigma\sqrt{T}}\right)$$

[9] Explicit derivation is available upon request.

$$q_1(T) = \left(\frac{-x_0 - z\sigma^2 T}{\sigma\sqrt{T}} \right), \qquad q_2(T) = \left(\frac{-x_0 + z\sigma^2 T}{\sigma\sqrt{T}} \right)$$

$$\bar{q}_1(T) = \left(\frac{-x_0 - \bar{z}\sigma^2 T}{\sigma\sqrt{T}} \right), \qquad \bar{q}_2(T) = \left(\frac{-x_0 + \bar{z}\sigma^2 T}{\sigma\sqrt{T}} \right)$$

$$a = \frac{\left(r - \delta - g - \sigma^2/2\right)}{\sigma^2}, \quad z = \frac{\left[\left(a\sigma^2\right)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}, \quad \bar{z} = \frac{\left[\left(a\sigma^2\right)^2 + 2(r-g)\sigma^2\right]^{1/2}}{\sigma^2},$$

where $N(\cdot)$ is the cumulative standard normal distribution function.

Similarly, the bankruptcy cost $BC_S(0)$ and tax benefit $TB_S(0)$ are given by

$$BC_S(0) = \int_0^T \alpha_{BC} F_S e^{-g(T-t)} e^{-rt} f(t) dt = \alpha_{BC} F_S e^{-gT} I(T), \tag{A9}$$

$$TB_S(0) = \tau C_S \int_0^T e^{-rt}\left(1 - G(t)\right) dt = \frac{\tau C_S}{r}\left(1 - \left(1 - G(T)\right)e^{-rT} - H(T)\right), \tag{A10}$$

where τ is the corporate tax rate.

The total firm value in the static model, when only one debt is ever issued, is given by the firm's asset plus the tax shield (A10) minus the bankruptcy cost (A9),

$$V_S(0) = V + TB_S(0) - BC_S(0) . \tag{A11}$$

Now we turn our attention to the dynamic model. In this model the firm optimally issues debt every *T* years. Obviously, the optimal coupon for the second issue will depend on the firm value V_T at time *T*. We note the following scaling property: if the optimal coupon in the first period is C_S, then the optimal coupon in the second period will be $C_S V_T / V$. That is, the coupon is scaled by V_T / V. The reason is that the firm at time *T* is identical to itself at time zero, except that it is V_T / V times as large. In fact, all future issues of debt will be scaled by the ratio of the firm's asset value when the debt matures over that when the debt is issued.

Though at time zero, only the current issue of debt is outstanding, the tax shield and bankruptcy cost reflect all future debt issues. Let $TB_S^{Dynamic}(0)$ denote the total tax shield in the dynamic model. Then the total tax shield at time T will be $TB_S^{Dynamic}(0)\ V_T/V$ according to the scaling argument in the previous paragraph. Now if we let $TB_S(0)$ denote the tax shield from the current outstanding debt, we have

$$TB_S^{Dynamic}(0) = TB_S(0) + e^{-rT} E^Q\left[\frac{V_T}{V} TB_S^{Dynamic}(0) \mid NBC\right] = TB_S(0) + \phi TB_S^{Dynamic}(0), \tag{A12}$$

where

$$\phi = e^{-rT} E^Q\left[\frac{V_T}{V} \mid NBC\right] \tag{A13}$$

and `*NBC*' denotes `No Bankruptcy yet by time *T*'.

The above equation states that the total tax shield is the tax shield for the next period, plus the risk-neutral discounted total tax shield at the end of the first period. Solving for $TB_S^{Dynamic}(0)$, we have

$$TB_S^{Dynamic}(0) = \frac{TB_S(0)}{1-\phi}. \tag{A14}$$

The total tax shield has an intuitive series expansion. Each term in the expansion

$$TB_S^{Dynamic}(0) = TB_S(0)\left(1 + \phi + \phi^2 + \phi^3 + \cdots\right)$$

evidently represents the present value of the tax benefit from the debt issue in each succeeding period.

To find ϕ, we need the conditional distribution of V_T (i.e., $x(T)$) such that the firm has not gone bankrupt at time T. Again, from Ingersoll (1987) we have the following conditional density function for $x(T)$:

$$g(x) = \frac{1}{\sigma\sqrt{T}} n\left(\frac{x - x_0 - mT}{\sigma\sqrt{T}}\right) - \frac{e^{-\frac{2mx_0}{\sigma^2}}}{\sigma\sqrt{T}} n\left(\frac{x + x_0 - mT}{\sigma\sqrt{T}}\right). \tag{A15}$$

Using the above density function, tedious but straightforward derivations yield the following closed form for ϕ,

$$\phi = e^{-\delta T}\left(N\left(d_1\right)-\left(\frac{F_s e^{-gT}}{V}\right)^{2\lambda} N\left(d_2\right)\right), \tag{A16}$$

where $\lambda = 1 + m/\sigma^2$ and

$$d_1 = \frac{-\log\left(F_s e^{-gT}/V\right)+\left(r-\delta-g+\sigma^2/2\right)/T}{\sigma\sqrt{T}}, \quad d_2 = \frac{\log\left(F_s e^{-gT}/V\right)+\left(r-\delta-g+\sigma^2/2\right)/T}{\sigma\sqrt{T}}.$$

Similarly, the total bankruptcy cost in the dynamic model, $BC_S^{Dynamic}(0)$, is given by

$$BC_S^{Dynamic}(0) = \frac{BC_S(0)}{1-\phi}. \tag{A17}$$

The total leveraged firm value, $V_S^{Dynamic}(0)$, in the dynamic model equals the unleveraged firm value *V*, plus the total tax benefit $TB_S^{Dynamic}$, less the total bankruptcy cost $BC_S^{Dynamic}(0)$,

$$V_S^{Dynamic}(0) = V + TB_S^{Dynamic}(0) - BC_S^{Dynamic}(0) = V + \frac{TB_S(0) - BC_S(0)}{1-\phi}. \tag{A18}$$

The optimal capital structure is obtained by maximizing either the total firm value or the manager's utility.