

# Asset Pricing Implications of Firms' Financing Constraints.

Joao F. Gomes\*, Amir Yaron† and Lu Zhang‡

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## Abstract

We use a production-based asset pricing model to investigate whether financial market imperfections are quantitatively important for pricing the cross-section of returns. Specifically, we use GMM to explore the stochastic Euler equation restrictions imposed on asset returns by optimal investment behavior. Our methodology allows us to identify the impact of financial frictions on the stochastic discount factor with cyclical variations in cost of external funds. We find evidence that financing frictions provide an important common factor for the cross section of stock returns. In addition, we find that the shadow price of external funds exhibits strong procyclical variation, so that financial frictions are more important when economic conditions are relatively good. These findings seem consistent with models emphasizing the importance of agency conflicts between insiders and outsiders.

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\*Finance Department, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104, and CEPR. E-mail: gomesj@wharton.upenn.edu.

†Finance Department, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104, and NBER. E-mail: yaron@wharton.upenn.edu.

‡William E. Simon Graduate School of Business Administration, University of Rochester, Rochester, NY 14627. E-mail: zhanglu@simon.rochester.edu.

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# 1 Introduction

We investigate whether financial frictions are quantitatively important in determining the cross-section of expected stock returns. Specifically, we construct a production based asset pricing framework in the presence of financial market imperfections and use GMM to explore the stochastic Euler equation restrictions imposed on asset returns by the optimal investment decisions of firms.

Our results are as follows. First, we find evidence that financial frictions provide an important common factor for the cross-section of expected returns. Second, our findings suggest that the shadow price of external funds is strongly procyclical i.e., financial market imperfections are more important when economic conditions are relatively good. These results are robust to the use of alternative measures of fundamentals such as profits and investment, alternative assumptions about the forms of the stochastic discount factor, and alternative measures of the shadow price of external funds.

The intuition behind our results is simple. The empirical success of production based asset pricing models lies in the alignment between the theoretical returns on capital investment and stocks returns. Given the forward looking nature of the firms' dynamic optimization decisions, the returns to capital accumulation will be positively correlated with expected future profitability. Accordingly, the model generates a series of investment returns that is procyclical and leads the business cycle. This pattern accords well with the observed cyclical behavior of stock returns documented by Fama (1981) and Fama and Gibbons (1982).

Financial frictions create an important additional source of variation in investment returns. Specifically, financial market imperfections introduce a wedge, driven by the shadow cost on external funds, between investment returns and fundamentals such as profitability. All else equal, a countercyclical wedge generally lowers the correlation between the theoretical investment returns and the observed stock returns. This will weaken the performance of the standard production based asset pricing model. Conversely, a procyclical shadow cost of external funds significantly strengthens the empirical success of the model.

Our work has important connections to the existing literature on empirical asset pricing. Our findings that financing frictions provide an important risk factor for the cross section

of expected returns are consistent with recent research by Lamont, Polk, and Saa-Requejo (2001) and Whited and Wu (2003). However, by explicitly modelling the effect of financial market imperfections on optimal investment and returns, our structural approach helps to shed light on the precise nature of the underlying financial market imperfections.

By identifying the role of cyclical fluctuations in the shadow price of external funds our results also have important implications for the various theories of financial market imperfections. Specifically, our findings are generally consistent with models that emphasize the importance of frictions generated by the presence of agency issues between insiders and outsiders. These models imply that frictions are more important when economic conditions are good and managers have too many funds available.<sup>1</sup>

Our research builds on Cochrane (1991, 1996) who first explores the asset pricing implications of optimal production and investment decisions by firms. Our work is also closely related to recent research by Li (2003) and by Whited and Wu (2003). Li (2003) builds directly on our approach to investigate implications of financial frictions at the firm level. Whited and Wu (2003) use a less general model of financial frictions to focus on variations in *ex-post* returns at the firm level. They assume a constant pricing kernel and explore instead the differences in average returns across various firms. Their work also provides an important empirical link between the shadow cost of external funds and firm specific variables. Both sets of authors find that financing frictions are particularly important for the subset of firms a priori classified as financially constrained.<sup>2</sup>

The role of cyclical variations in financial market imperfections also plays an important role in two recent general equilibrium settings. Gomes, Yaron, and Zhang (2003a) use a fully specified general equilibrium model with costly external finance to highlight the tension between the model's ability to match typical business cycle facts and the cyclical variation in the cost of external funds. In contrast to the very stylized example (suitable for a general equilibrium setting) in Gomes, Yaron, and Zhang (2003a), the current paper allows for

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<sup>1</sup>Dow, Gorton, and Krishnamurthy (2003) use a stylized equilibrium model with imperfect corporate control to explain the term structure of interest rates. A crucial element of their analysis is also the fact that financing frictions become more important when economic conditions are relatively good.

<sup>2</sup>Other related papers in this area include Whited (1992), Bond and Meghir (1994), Restoy and Rockinger (1994), and Li, Vassalou and Xing (2001).

a much more general characterization of the role of financial market imperfections - thus providing a more suitable framework for empirical analysis.

The remainder of this paper is organized as follows. Section 2 shows how financial market imperfections affect firm investment and asset prices under fairly general conditions. This section derives the expression for returns to physical investment, the key ingredient in the stochastic discount factor in this economy. Section 3 describes our empirical methodology, while Section 4 discusses the results of our GMM estimation and tests. Finally, Section 5 offers some concluding remarks.

## 2 Production Based Asset Pricing with Financial Frictions

In this section we incorporate financial frictions in a production based asset pricing framework in the tradition of Cochrane (1991, 1996) and derive the expression for the behavior of investment returns, the key ingredient in our stochastic discount factor.

### 2.1 Modelling Financial Frictions

Several theoretical foundations of financial market imperfections are available in the literature. Rather than offering another rationalization for their existence we seek instead to summarize the common ground across the existing literature with a representation of financial constraints that is both parsimonious and empirically useful.

While exact assumptions and modelling strategies often differ quite significantly across authors, the key feature of this literature is the simple idea that external funds (new equity or debt), are not perfect substitutes for internal cash flows. It is this crucial property that we explore in our analysis below by assuming that any form of financial market imperfection can be usefully summarized by adding a distortion to the relative price between internal and external funds.

Consider the case of new equity finance. Suppose that a firm issues  $N_t$  dollars in new equity and let  $W_t$  denote the reduction on the claim of existing shareholders per dollar of new equity issued. In a frictionless world it must be the case that  $W_t = 1$ , since the value

of the firm is not affected by financing decisions. However, the presence of any financial market imperfections such as transaction costs, agency problems or market timing issues will cause  $W_t$  to differ from 1. Characterizing the exact form of  $W_t$  requires a detailed model of the precise nature of the distortion but it is not necessary to derive the key asset pricing restrictions below. Similarly we need not take a stand about whether new issues add or lower firm value.

Suppose now that the firm also uses debt financing,  $B_t$ , and let  $R_t$  denote the gross (interest plus principal) repayment per dollar of debt raised. As before, without any financial frictions, the cost of this debt will be equal to the return on savings and the opportunity cost of internal funds, say  $R_{ft}$ . The presence of any form of imperfection, such as asymmetric information or moral hazard problems, will again distort relative prices and will cause  $R_t$  to differ from  $R_{ft}$ , at least when  $B_t > 0$ . As in the case of equity issues, this basic idea is sufficient to derive the asset pricing results below.

For the sake of generality we will allow both  $W_t$  and  $R_t$  to vary with the state of the economy and with the total amount of financing actually raised. Accordingly, we will generally write  $W_t = W(N_t, S_t)$  and  $R_t = R(B_t, S_t)$ , where  $S_t$  summarizes all forms of uncertainty.

Focusing on the common ground across much of the existing literature on financial market imperfections allows us to provide a fairly general test of the role that these frictions play in determining asset prices.

## 2.2 Investment Returns

Consider the problem of a firm seeking to maximize the value to existing shareholders, denoted  $V$ . This firm makes investment decisions by choosing the optimal amount of capital to have at the beginning of the next period,  $K_{t+1}$ . Investment spending,  $I_t$ , as well as dividends,  $D_t$ , can be financed with internal cash flows  $\Pi_t$ , new equity issues,  $N_t$ , or new one-period debt  $B_{t+1}$ .<sup>3</sup>

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<sup>3</sup>Assuming one period debt simplifies the notation significantly but it does not change the basic asset pricing implications of the model.

The value-maximization problem of the firm can then be summarized as follows:

$$V(K_t, B_t, S_t) = \max_{\substack{D_t, B_{t+1}, \\ K_{t+1}, N_t}} \{D_t - W(N_t, S_t)N_t + E_t[M_{t+1}V(K_{t+1}, B_{t+1}, S_{t+1})]\} \quad (1)$$

subject to

$$D_t = \Pi(K_t, S_t) - I_t - \frac{a}{2} \left[ \frac{I_t}{K_t} \right]^2 K_t + N_t + B_{t+1} - R(B_t, S_t)B_t \quad (2)$$

$$I_t = K_{t+1} - (1 - \delta)K_t \quad (3)$$

$$D_t \geq \bar{D}, \quad N_t \geq 0 \quad (4)$$

where  $M_{t+1}$  is the stochastic discount factor (of the owners of the firm) between  $t$  to  $t + 1$  and  $\bar{D}$  is the firm's minimum, possibly zero, dividend payment. Note that we allow a firm to accumulate financial assets, in which case debt,  $B_t$ , is negative. We also assume that investment is subject to convex (quadratic) adjustment costs, the magnitude of which is governed by the parameter  $a$ .<sup>4</sup> The form of the internal cash flow function  $\Pi(\cdot)$ , is not important. For simplicity we assume only that it exhibits constant returns scale.

Equation (2) is the resource constraint for the firm. It implies that dividends must equal internal funds  $\Pi(K_t, S_t)$ , net of investment spending  $I_t$ , plus new external funds  $N_t + B_{t+1}$ , net of debt repayments  $R(B_t, S_t)B_t$ . Equation (3) is the standard capital accumulation equation, relating current investment spending,  $I_t$ , to future capital,  $K_{t+1}$ . We assume that old capital depreciates at the rate  $\delta$ .

Letting  $\mu_t$  denote the Lagrange multiplier associated with the inequality constraint on dividends, the optimal first-order condition with respect to  $K_{t+1}$  (derived in Appendix A) implies that:

$$E_t[M_{t+1}R_{t+1}^I] = 1 \quad (5)$$

where  $R_{t+1}^I$  denotes the returns to investment in physical capital and is given by:

$$R_{t+1}^I = R_{t+1}^I(\pi, i, \mu) \equiv \frac{(1 + \mu_{t+1})(\pi_{t+1} + \frac{a}{2}i_{t+1}^2 + (1 + ai_{t+1})(1 - \delta))}{(1 + \mu_t)(1 + ai_t)} \quad (6)$$

Here  $i \equiv (I/K)$  is the investment-to-capital ratio, and  $\pi \equiv (\Pi/K)$  is the profits-to-capital

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<sup>4</sup>Without adjustment costs the price of capital is always one and the capital gains component of returns is always zero, which is clearly counterfactual.

ratio.<sup>5</sup>

To gain some intuition on the role of the financial frictions, we can decompose (6) as:

$$R_{t+1}^I(\pi, i, \mu) = \frac{1 + \mu_{t+1}}{1 + \mu_t} \tilde{R}_{t+1}^I \quad \text{and} \quad \tilde{R}_{t+1}^I(\pi, i) \equiv \frac{\pi_{t+1} + \frac{a}{2}i_{t+1}^2 + (1 + ai_{t+1})(1 - \delta)}{1 + ai_t} \quad (7)$$

where  $\tilde{R}_{t+1}^I$  denotes the investment return with no financial constraints, i.e.,  $\mu_{t+1} = \mu_t = 0$ , which is entirely driven by fundamentals,  $i$  and  $\pi$ . The role of the financial market imperfections is completely captured by the term  $\frac{1 + \mu_{t+1}}{1 + \mu_t}$ , which depends only on the shadow price of external funds.

The decomposition in equation (7) provides important intuition on the effects of financial market imperfections on returns. This result shows that if  $\mu_t = \mu_{t+1}$  financing frictions do not affect returns at all. They will simply have a permanent effect on the value of the firm without producing time series variation in returns. This highlights the crucial role of cyclical variation in the shadow price of external funds. From the standpoint of asset returns however, the exact level of  $\mu$  is irrelevant.

## 3 Empirical Methodology

### 3.1 Testing Framework

The essence of our empirical strategy is to use the information contained in asset prices to formally evaluate the effects of financial constraints. Specifically, we test

$$E_t[M_{t+1}\mathbf{R}_{t+1}] = 1 \quad (8)$$

where  $\mathbf{R}_{t+1}$  is a vector of returns, that may include stocks and bonds as well as the returns to physical investment (from equation (5)).

Following Cochrane (1996) we ask whether investment returns are factors for asset returns. Formally, we parameterize the stochastic discount factor as a linear function of

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<sup>5</sup>Equation (6) is very general and it is also valid in the presence of quantity constraints such those created by credit rationing (see Whited (1992) and Whited and Wu (2003)). For details see Gomes, Yaron, and Zhang (2003b).

the returns to physical investment:<sup>6</sup>

$$M_{t+1} = l_0 + l_1 R_{t+1}^I \quad (9)$$

The role of financial market imperfections in explaining the cross-section of expected returns as a factor is captured by their impact on  $R^I$  in the pricing kernel (9). Thus, financial frictions will be relevant for the pricing of expected returns only to the extent that they provide a *common* factor or a source of systematic risk, which can influence the stochastic discount factor. In this sense, our formulation is essentially a structural version of an APT-type framework such as those proposed in Fama and French (1993, 1996) and Lamont, Polk, and Saá-Requejo (2001), in which one of the factors proxies for aggregate financial conditions.

### 3.2 The Shadow Price of External Funds

Empirically, our characterization of investment returns is extremely appealing since it requires only data on the two fundamentals,  $i$  and  $\pi$ , as well as a measure of the shadow cost of external funds to be implemented. Formally, we parameterize the shadow price of external funds  $\mu_t$  with the following form:

$$\mu_t = b_0 + b_1 \cdot f_t \quad (10)$$

where  $b_0$  and  $b_1$  are parameters and  $f_t$  is some aggregate index of financial frictions.

Recall that the key element for asset returns is the time series variation in  $\mu_t$ . Here this is captured by the cyclical properties of the financing factor  $f_t$ . Thus, the estimated value of  $b_1$  will summarize all information about the impact of financial market imperfections on returns.<sup>7</sup>

Cyclicality plays an important role in the various theories of financial market imperfections. For example models emphasizing the importance of agency issues between

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<sup>6</sup>With financing frictions, equation (9) provides only a reasonable approximation to the exact pricing kernel for this economy. Gomes, Yaron, and Zhang (2003b) use a more general expression which is also used in our empirical work in Section 4.

<sup>7</sup>Since levels do not affect returns  $b_0$  is irrelevant. As a practical matter for our empirical estimation we fix  $b_0$ . Later we show that our results are not affected by this choice.

insiders and outsiders suggest that frictions are more important when economic conditions are good and managers have too many funds available. Conversely, models that focus on costly external finance typically emphasize the role of credit market constraints and rely on the fact that the cost of external funds rises when economic conditions are adverse.<sup>8</sup> By isolating the dynamic properties of the shadow price of external funds we can do more than just assess the overall impact of financing frictions on asset returns. Our methodology also allows us to distinguish between the various theories of financial market imperfections.

As a first measure of aggregate financial frictions we use the default premium, defined as the yield spread between Baa and Aaa rated corporate bonds. Bernanke (1990), and Stock and Watson (1989, 1999) show that the default premium is one of the most powerful predictors of aggregate economic conditions. The default premium is also a frequent measure of the premium of external funds in the literature, (e.g., Kashyap, Stein, and Wilcox (1993), Kashyap, Lamont, and Stein (1994), Bernanke and Gertler (1995), and Bernanke, Gertler, and Gilchrist (1996, 1999))

In our tests we also use two additional measures of the marginal cost of external finance. The first is the aggregate return factor of financial constraints constructed in Lamont, Polk, and Saa-Requejo (2001). The other measure is the aggregate distress likelihood constructed by Vassalou and Xing (2003). Both of these measures are described in more details below.

### 3.3 Implementation

We use GMM to estimate the factor loadings,  $\mathbf{l}$ , as well as the parameters,  $a$  and  $b_1$ , by utilizing  $M$  as specified in (9) in conjunction with moment conditions (8). Specifically, three alternative sets of moment conditions in implementing (8) are examined (see also Cochrane (1996)). First, we look at the relatively weak restrictions implied by the unconditional moments. We then focus on the conditional moments by scaling returns with instruments, and finally we look at time variation in the factor loadings, by scaling the factors.

For the unconditional factor pricing we use standard GMM procedures to minimize a weighted average of the sample moments (8). Letting  $\sum_T$  denote the sample mean, we

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<sup>8</sup>Jensen (1986) is an example of the former while Bernanke and Gertler (1989) provide an example of the latter. Stein (2003) offers a detailed survey of this literature.

rewrite these moments,  $\mathbf{g}_T$  as:

$$\mathbf{g}_T \equiv \mathbf{g}_T(a, b_0, b_1, \mathbf{l}) \equiv \sum_T [M\mathbf{R} - \mathbf{p}]$$

where  $\mathbf{R}$  is the menu of asset returns being priced and  $\mathbf{p}$  is a vector of prices. We then choose  $(a, b_1, \mathbf{l})$  to minimize a weighted sum of squares of the pricing errors across assets:

$$J_T = \mathbf{g}'_T \mathbf{W} \mathbf{g}_T \tag{11}$$

A convenient feature of our setup is that, given the cost parameters, the criterion function above is linear in  $\mathbf{l}$ , the factor loading coefficients. Standard  $\chi^2$  tests of over-identifying restrictions follow from this procedure. This also provides a natural framework to assess whether the loading factors or technology parameters are important for pricing assets.

It is straightforward to include the effects of conditioning information by scaling the returns by instruments. The essence of this exercise lies in extracting the conditional implications of (8) since, for a time-varying conditional model, these implications may not be well captured by a corresponding set of unconditional moment restrictions as noted by Hansen and Richard (1987).

To test conditional predictions of (8), we expand the set of returns to include returns scaled by instruments to obtain the moment conditions:

$$\mathbf{E} [\mathbf{p}_t \otimes \mathbf{z}_t] = \mathbf{E} [M_{t,t+1} (\mathbf{R}_{t+1} \otimes \mathbf{z}_t)]$$

where  $\mathbf{z}_t$  is some instrument in the information set at time  $t$  and  $\otimes$  denotes Kronecker product.

A more direct way to extract the potential non-linear restrictions embodied in (8) is to let the stochastic discount factor be a linear combination of factors with weights that vary over time. That is, the vector of factor loadings  $\mathbf{l}$  is a function of instruments  $\mathbf{z}$  that vary over time. With sufficiently many powers of  $\mathbf{z}$ , the linearity of  $l$  can actually accommodate nonlinear relationships. Therefore, to estimate and test a model in which factors are expected to price assets only conditionally, we simply expand the set of factors to include factors scaled

by instruments. The stochastic discount factor utilized in estimating (8) is then,

$$M_{t+1} = [l_0 + l_1 R_{t+1}^I] \otimes \mathbf{z}_t$$

## 4 Findings

This section is organized as follows. Section 4.1 describes our data. Section 4.2 reports the results from GMM estimation and tests of our benchmark specifications, while Section 4.3 discusses and interprets our empirical findings regarding the role of financing frictions. Finally, Section 4.4 includes a wide array of robustness checks on our results.

### 4.1 Data

This section provides an overview of the data used in our study. A more detailed description is provided in Appendix B. Macroeconomic data comes from National Income and Product Accounts (NIPA) published by the Bureau of Economic Analysis, and the Flow of Funds Accounts available from the Federal Reserve System. These data are cross-referenced and mutually consistent, so they form, for practical purposes, a unique source of information. The construction of investment returns requires data on profits, investment, and capital. Capital consumption data is used to compute the time series average of the depreciation rate and pin down the value of  $\delta$ , the only technology parameter not formally estimated. To avoid measurement problems due to chain weighting in the earlier periods our sample starts in the first quarter of 1954 and ends in the last quarter of 2000. Since models of financing frictions are usually applied to non-financial firms we focus mainly on data from the Non-Financial Corporate Sector. For comparison purposes however, we also report results for the aggregate economy.

Information about financial assets is obtained from CRSP and Ibbotson. In order to implement the estimation procedure, we require a reasonable number of moment conditions. Here we focus on the ten size portfolios of NYSE stock returns. In addition, we also provide results for the 25 Fama and French (1993) size and book-to-market portfolio. Bond data comes from Ibbotson's index of Long Term Corporate Bonds. In all cases we report real returns, constructed using the Consumer Price Index for all Urban Households, as reported

**Table 1 : Summary Statistics of the Assets Returns in GMM**

	Decile Returns										$R^S$	$R^f$	$R^B$
	1	2	3	4	5	6	7	8	9	10			
mean	11.80	9.49	9.03	9.07	8.50	8.57	7.67	8.16	7.34	6.64	7.10	1.86	0.72
std	19.61	17.49	16.73	16.16	15.49	15.19	14.51	13.80	12.90	11.35	11.87	1.32	7.11
Sharpe	0.60	0.54	0.53	0.55	0.54	0.56	0.52	0.58	0.56	0.57	0.58	0.00	0.10
$\rho(1)$	0.26	0.29	0.29	0.31	0.29	0.28	0.32	0.27	0.27	0.36	0.33	0.67	0.30

This table reports the means, volatilities, Sharpe ratios, and first-order autocorrelations of excess returns of deciles 1–10, excess value-weighted market return ( $R^S$ ), real t-bill rate ( $R^f$ ), and excess corporate bond return ( $R^B$ ). These returns are used in GMM estimation and tests. The sample period is from 1954:2Q to 2000:3Q. Means and volatilities are in annualized percentage points.

by the Bureau of Labor Statistics. Table 1 summarizes the basic properties of these asset returns.

Investment data are quarterly averages, while asset returns are reported from the beginning to the end of the quarter. As a correction, we follow Cochrane (1991, 1996) and average monthly asset returns over the quarter and then adjust them so that they go from approximately the middle of the initial quarter to the middle of the next quarter.<sup>9</sup>

The default premium is defined as the difference between the yields on Baa and Aaa corporate bonds, both obtained from the Federal Reserve System. As an alternative we also use the spread between Baa and long term government bonds yields.

Finally, conditioning information comes from two sources: the term premium, defined as the yield on ten year notes minus that on three-month Treasury Bills, and the dividend-price ratio of the equally weighted NYSE portfolio. We limit the number of moment conditions and scaled factors in three ways: (i) we do not scale the Treasury-Bill return by the instruments since we are more interested in the time-variation of risk premium than that of risk-free rate; (ii) instruments themselves are not included as factors; and (iii) we use a smaller number of stock portfolios in the conditional estimates.

<sup>9</sup>Lamont (2000) also discusses the importance of aligning investment and asset returns.

## 4.2 GMM Estimates and Tests

Table 2 reports iterated GMM estimates and tests for the unconditional, conditional, and the scaled factor models. When the default premium is the instrument for the shadow price of external funds in equation (10). In all cases we report the value of the parameters  $a$  and  $b_1$  as well as estimated loadings,  $\mathbf{l}$ , and corresponding  $t$ -statistics. Also included are the results of  $J$  tests on the model's overall ability to match the data, and the corresponding  $p$ -values.

Overall the model is reasonably successful in pricing the cross-section of returns. In spite of the inclusion of the last few years of stock market data, the model cannot be rejected using the test of over-identification,  $J_T$ . The root mean squared errors (RMSE, mean minus predicted mean) are all low, suggesting the statistical significance of the  $J$  tests is not due to an excessively large covariance matrix. They are also cut in half if we truncate our sample in 1997. Figure 1 confirms this good fit by showing the close alignment between actual and predicted mean excess returns from first stage estimation. In addition, the hypothesis that all factor loadings are zero is almost always rejected at the standard 5% significance level.

Although we use a single aggregate investment return as a pricing factor these results are generally comparable to Cochrane's (1996) findings. The reason for this empirical success is that our construction of investment returns,  $R^I$ , uses independent information on variations in the marginal productivity of capital,  $\pi_t$ , and investment,  $i_t$ . Cochrane (1996), on the other hand, abstracts from the variation in the marginal productivity of capital in constructing investment returns and must instead use two separate investment series (residential and non-residential) to construct two investment returns.

Gomes, Yaron, and Zhang (2003b) show that in the presence of financing frictions the return to physical investment is a linear combination of stock and bond returns, with the weights given by the leverage ratio. This motivates us to augment the basic pricing kernel (9) with the return on corporate bonds,  $R^B$ . Nevertheless, as Table 3 shows, this modification has very minor effects on our results. The loadings on bond returns are not statistically significant and most point estimates for the coefficients  $a$  and  $b_1$  change only slightly.

Finally, Table 4 confirms that our benchmark results are not affected by our choice of the constant  $b_0$ .

**Table 2 : GMM Estimates and Tests in the Benchmark Case**

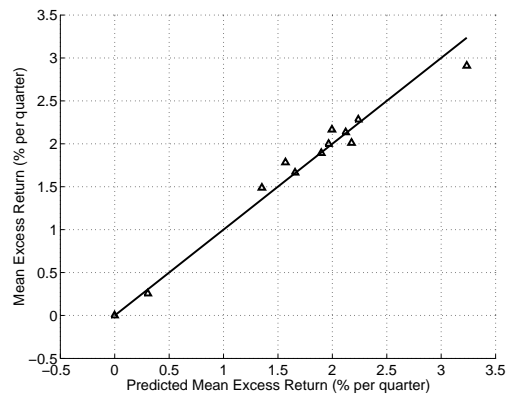
	Unconditional		Conditional		Scaled Factor	
	Parameters					
$a$	0.00		8.64	( 3.84)	9.52	( 3.82)
$b_1$	-0.23	(-7.20)	-0.09	(-2.18)	-0.10	(-2.13)
	Loadings					
$l_0$	154.92	( 2.66)	60.75	( 4.66)	49.81	( 3.31)
$l_1$	-151.40	(-2.65)	-58.80	(-4.59)	-47.83	(-3.21)
$l_2$					-0.07	(-0.39)
$l_3$					-0.13	( -0.76)
	$J_T$ Test					
$\chi^2$	5.65		11.84		10.61	
$p$	0.69		0.46		0.39	
	Wald Test ( $b_1=0$ )					
$\chi^2_{(1)}$	4.07		13.78		7.78	
$p$	0.04		0.00		0.01	

This table reports GMM estimates and tests for the benchmark model. The shadow price of external funds is  $\mu_t = b_0 + b_1 \cdot d_t$ , where  $d_t$  is the default premium, defined as the difference between the yields on Baa and Aaa corporate bonds.  $b_0$  is chosen so that the implied share of financing costs to investment expenditure is 3%. We report the estimates for  $a$ ,  $b_1$ , the pricing kernel loadings,  $l$ 's, the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and the  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The unconditional model uses as moment conditions the excess returns of ten CRSP size decile portfolios and one excess investment return (all over corporate bond returns) and the corporate bond returns. The unscaled and scaled conditional models use the excess returns of size deciles 1, 3, 8, 10, and excess investment returns (over corporate bond returns), all scaled by instruments, and the corporate bond return. Instruments are the constant, term premium ( $tp$ ), and equally weighted dividend-price ratio ( $dp$ ). The pricing kernel is  $M = l_0 + l_1 R^I$  for the unconditional and conditional models and  $M = l_0 + l_1 R^I + l_2 R^I \cdot tp + l_3 R^I \cdot dp$  for the scaled factor model.  $R^I$  is real investment return and is constructed from the flow-of-fund accounts using data from the nonfinancial corporate sector with before-tax profits.

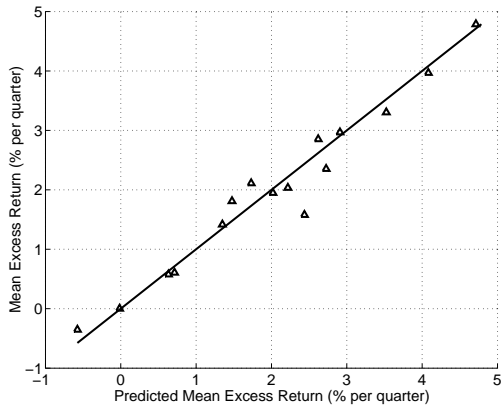
### Figure 1 : Predicted Versus Actual Mean Excess Returns

This figure plots the mean excess returns against predicted mean excess return, both of which are in % per quarter, for conditional model (Panel A), conditional model (Panel B), and scaled factor model (Panel C). All plots are from first-stage GMM estimates.

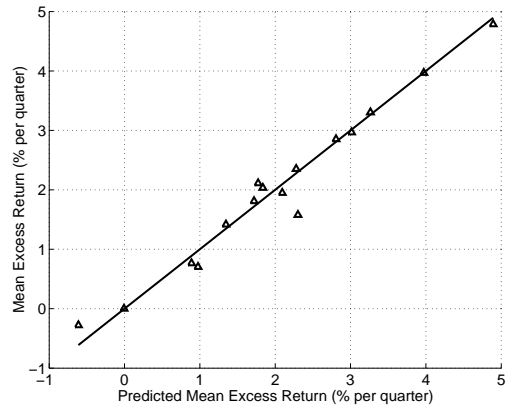
Panel A: Unconditional Estimates



Panel B: Conditional Estimates



Panel C: Scaled Factor



**Table 3 : GMM Estimates and Tests with Bond Returns in the Pricing Kernel**

	Unconditional		Conditional		Scaled Factor	
	Parameters					
$a$	0.00		7.04	( 1.38)	6.34	( 1.24)
$b_1$	-0.05	(-1.33)	-0.19	(-3.26)	-0.17	(-2.57)
	Loadings					
$l_0$	150.64	( 1.97)	45.46	( 4.83)	47.91	( 3.21)
$l_1$	-150.33	(-2.00)	-42.76	(-3.98)	-59.21	(-3.23)
$l_2$	-3.27	(-1.06)	-1.00	(-0.25)	13.40	( 1.18)
$l_3$					4.56	( 1.68)
$l_4$					3.11	( 1.09)
$l_5$					-4.84	(-1.69)
$l_6$					-3.02	(-1.01)
	$J_T$ Test					
$\chi^2$	6.07		8.19		6.54	
$p$	0.53		0.70		0.48	
	Wald Test ( $b_1=0$ )					
$\chi^2_{(1)}$	4.33		11.30		5.73	
$p$	0.04		0.00		0.02	

This table reports GMM estimates and tests for the benchmark model using an augmented pricing kernel. Both  $\mu_t$  and  $b_0$  are the same as in Table 2. We report the estimates for  $a$ ,  $b_1$ , and the loadings,  $l$ 's, in the pricing kernel, the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1=0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel is  $M=l_0+l_1R^I+l_2R^B$  for the unconditional and conditional models and  $M=l_0+l_1R^I+l_2R^B+l_3R^I.tp+l_4R^I.dp+l_5R^B.tp+l_6R^B.dp$  for the scaled factor model.  $R^I$  and  $R^B$  are real investment and bond returns respectively. Moment conditions, instruments, and data, are the same as those reported in Table 2.

**Table 4 : GMM Estimates and Tests with Alternative Levels of Financing Cost**

	Panel A: Low Share 1%			Panel B: High Share 10%		
	Unconditional	Conditional	Scaled Factor	Unconditional	Conditional	Scaled Factor
Parameters						
$a$	0.00	7.03 ( 1.38)	6.37 ( 1.25)	0.00	7.04 ( 1.37)	6.33 ( 1.24)
$b_1$	-0.05 (-1.33)	-0.18 (-3.26)	-0.16 (-2.58)	-0.06 (-1.32)	-0.22 (-3.26)	-0.19 (-2.57)
$J_T$ Test						
$\chi^2$	6.07	8.19	6.52	6.08	8.20	6.54
$p$	0.53	0.70	0.48	0.53	0.70	0.48
Wald Test ( $b_1=0$ )						
$\chi^2_{(1)}$	4.33	11.29	5.74	4.33	11.31	5.73
$p$	0.04	0.00	0.02	0.04	0.00	0.02

This table reports GMM estimates and tests with alternative levels of financing cost. Panel A reports the GMM estimates and tests when  $b_0$  is chosen so that the implied share of financing cost in investment is 1%. Panel B does the same for the case where the implied share is 10%. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The data, pricing kernel and the moment conditions are the same as those reported in Table 3.

### 4.3 The Effects of Financial Frictions

The focus of our analysis however is on the impact of financial frictions on returns. Here the results in Tables 2-4 can be summarized as follows:

- financial market imperfections play an important role in the cross section of expected returns; and
- the shadow price of external funds exhibits an important procyclical variation.

Tables 2-4 show a consistently negative and generally significant point estimate for the slope coefficient  $b_1$ . The included Wald tests confirm that financial market imperfections provide an important common factor for the pricing kernel in this economy.

Given the strongly countercyclical nature of the default premium, our finding that  $b_1$  is negative also implies that the shadow price of external funds is quite procyclical. Intuitively,

financing distortions are more important when aggregate economic conditions are relatively good.

What drives these results? Mechanically, our GMM estimation seeks to minimize a weighted average of the price errors associated with (8). Broadly speaking, this requires aligning the dynamic properties of the stochastic discount factor (essentially driven by investment return) and those of asset returns (basically driven by the large volatility in stock returns). A successful estimation procedure will then choose parameter values for  $a$  and  $b_1$  so that the investment return has similar dynamic properties to those of the targeted stock returns.

Therefore, to gain more intuition on our results, we now examine the dynamic properties of the investment returns generated under alternative values of  $b_1$ , and compare those with the behavior of stock returns.

## Correlation Structure

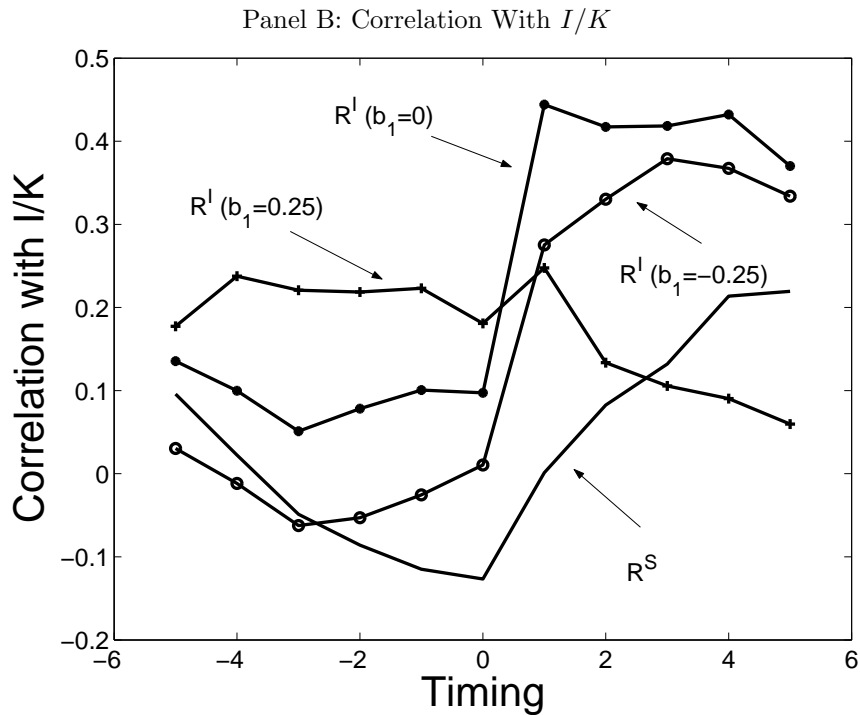
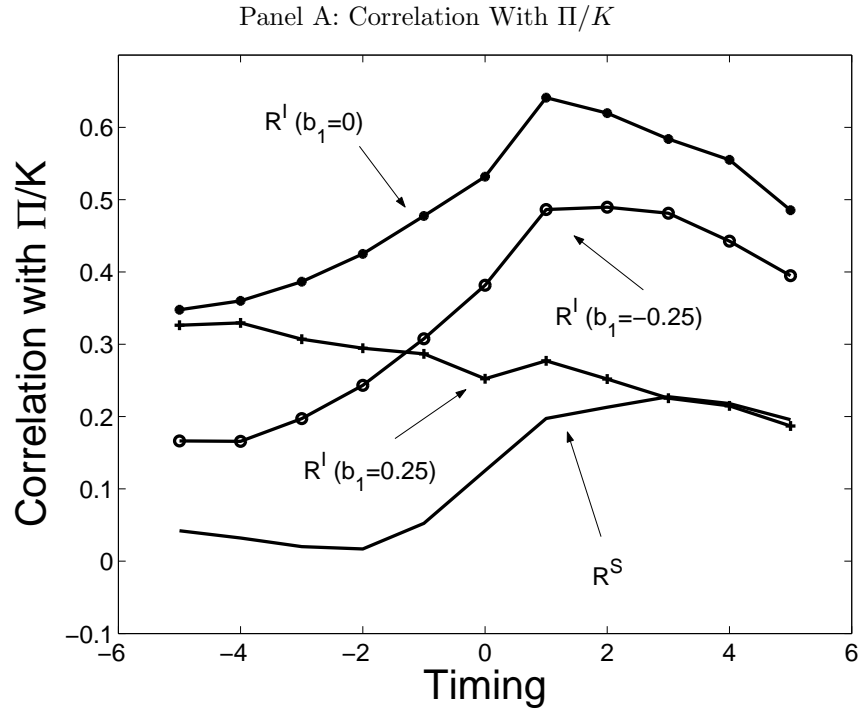
We start by focusing on the correlation structure of stock and investment returns with the two economic fundamentals, aggregate investment/capital ratio  $i$ , and aggregate profits/capital ratio  $\pi$ . Recall that equation (6) decomposes investment returns into a frictionless component,  $\tilde{R}^I$ , that is driven by the fundamentals  $i$  and  $\pi$ , and a financing component, captured by the dynamics in the shadow price of external funds,  $\mu$ .

Figure 2 displays the correlation structure between returns and various leads and lags of the fundamentals  $\pi$  (Panel A) and  $i$  (Panel B). In both panels, the dynamic pattern of the frictionless returns,  $\tilde{R}^I$  ( $b_1 = 0$ ), is very similar to that of the observed  $R^S$ . In particular, both returns *lead* future economic activity, while their contemporaneous correlations with fundamentals are somewhat low. As Cochrane (1991) notes, this is to be expected if firms adjust current investment in response to an anticipated shocks on future productivity.

Figure 2 also shows how the effect of financing on investment returns depends on the cyclical nature of the premium on external funds, here measured by the default premium. As the figure shows, financing frictions improve the model's ability to match the underlying pattern of stock returns only when  $b_1 < 0$ .

**Figure 2 : Correlation Structure of Stock and Investment Returns with Leads of Lags of  $i$  and  $\pi$**

This figure presents the correlations of investment returns  $R^I$  and real value-weighted market returns  $R^S$  with the various leads and lags of  $I/K$  and  $\Pi/K$ . Panel A plots the correlation structure of the above series with  $\Pi/K$  and Panel B plots that with  $I/K$ . In the figures,  $b_1$  is the slope term in the specification of financing premium (10).



The economic intuition is the following. Suppose for a moment that the shadow price of external funds was countercyclical, so that  $b_1 > 0$ . In this case, a rise in expected future productivity is also associated with an expected decline in the marginal cost of external financing. Productivity and financial constraints provide two competing forces for the response of investment returns to business cycle conditions. An increase in expected future productivity implies that firms should respond by investing immediately. However, since the shock also entails lower marginal cost of external funds in the future, firms prefer to delay investment. Relative to a frictionless world, equation (6) implies a reduction in investment returns, and thus *lower* correlations with future economic activity. Figure 2 shows however that this reaction is not consistent with observed asset return data.

Finally, Figure 2 also indicates that there is no obvious phase shift between any of the series, suggesting that our results are not likely to be sensitive to timing issues such as those created by the existence of time-to-plan, or perhaps time-to-finance in this context. What seems crucial is the cyclical pattern of the shadow price of external funds.

### Properties of the Pricing Kernel

Further intuition can be obtained by looking directly at effect of the financing premium on the properties of the pricing kernel. Table 5 describes the effects of imposing  $b_1 > 0$  in each set of moment conditions (unconditional, conditional, and scaled factor), while keeping the value of the adjustment cost parameter  $a$  is set at its optimal level reported Table 3.

The left panel of the table indicates that a countercyclical shadow price of external funds,  $b_1 \geq 0$ , lowers the absolute magnitude of the correlation between the stochastic discount factor and value-weighted returns (as well as the price of risk  $\sigma(M)/E(M)$ ) for all three sets of momentum conditions, thus deteriorating the performance of the stochastic discount factor considerably.

A more direct way to evaluate the effect of a positive  $b_1$  on the pricing kernel is perhaps to examine the implied pricing errors. A simple way is to use the beta representation, which is equivalent to the stochastic discount factor representation in (8), e.g., Cochrane (2001):

$$R^p - R^f = \alpha_i + \beta_{1i}(R^I - R^f) + \beta_{2i}(R^B - R^f)$$

**Table 5 : Properties of Pricing Kernels, Jensen’s  $\alpha$ , and Investment Returns**

$b_1$	Pricing Kernel		Jensen’s $\alpha$				Investment Return			
	$\frac{\sigma[M]}{E[M]}$	$\rho_{M,R^S}$	$\alpha^{vw}$	$t_\alpha^{vw}$	$\alpha^{d1}$	$t_\alpha^{d1}$	mean	$\sigma_{R^I}$	$\rho(1)$	$\rho_{R^I,R^S}$
Unconditional Model										
0.00	0.82	-0.28	0.26	0.35	1.02	0.78	6.55	0.97	0.76	0.30
0.15	0.57	-0.03	3.03	4.94	5.69	5.45	6.56	1.70	0.38	-0.31
0.30	0.58	-0.07	3.07	6.22	5.58	6.66	6.58	2.98	0.31	-0.41
Conditional Model										
0.00	0.75	-0.29	0.16	0.30	0.68	0.77	5.91	2.24	0.09	0.35
0.15	0.37	0.39	1.46	2.70	3.01	3.25	5.92	2.23	0.00	-0.01
0.30	0.79	0.17	2.22	4.51	4.21	5.02	5.93	3.05	0.10	-0.24
Scaled Factor Model										
0.00	0.81	-0.36	0.03	0.06	0.51	0.55	6.02	1.99	0.14	0.36
0.25	0.67	-0.06	1.63	2.92	3.35	3.48	6.03	2.06	0.06	-0.05
0.50	0.61	0.01	2.38	4.79	4.48	5.30	6.04	2.98	0.15	-0.27

This table reports, for each combination of parameters  $a$  and  $b_1$ , properties of the pricing kernel, including market price of risk ( $\sigma[M]/E[M]$ ), the contemporaneous correlation between pricing kernel and real market return ( $\rho_{M,R^S}$ ), Jensen’s  $\alpha$  and its corresponding  $t$ -statistic ( $t_\alpha$ ), summary statistics of investment return, including mean, volatility ( $\sigma_{R^I}$ ), first-order autocorrelation ( $\rho(1)$ ), and correlation with the real value-weighted market return ( $\rho_{R^I,R^S}$ ). Jensen’s  $\alpha$  is defined from the following regression:

$$R^p - R^f = \alpha + \beta_1(R^I - R^f) + \beta_2(R^B - R^f)$$

where  $R^p$  is either the real value-weighted market return ( $R^{vw}$ ) or the real decile one return ( $R^1$ ),  $R^f$  is the real interest rate proxied by the real treasury-bill rate,  $R^I$  is the real investment return, and  $R^B$  is the real corporate bond return. In each case the cost parameters  $a$ ’s are held fixed at the GMM estimates.

for any portfolio  $p$ . Given the assumed structure of the pricing kernel this representation exists, with  $\alpha_p=0$ . Therefore, large values of  $\alpha$  indicate poor performance of the model.

The middle panel of Table 5 reports the implied  $\alpha$ s for the regressions on both small firms (NYSE decile 1) and value-weighted returns. The panel displays a clear pattern of rising  $\alpha$  as we increase the magnitude of  $b_1$ . Indeed, while we cannot reject that  $\alpha=0$  when  $b_1=0$ , this is no longer true for most positive values of  $b_1$ .

Finally, we also report the implications of financial constraints for the moments of investment returns and their correlations with market returns. While both the mean and the variance of investment returns are not affected much as  $b_1$  increases, the correlation with

stock returns lowers significantly. Indeed, while the correlation between the two returns is about 30 percent with  $b_1=0$ , the correlation becomes negative with a positive  $b_1$ . Since the overall performance of a factor model hinges on its covariance structure with stock returns, it is not surprising that financial constraints are important only when the shadow price of external funds is  $b_1 < 0$ .

## Implications

Our findings on the procyclical properties of the shadow cost of external funds effectively impose a restriction on the nature of these costs. Thus our results can also be viewed as providing a test for the various theories of financial market imperfections.

In this sense our estimates are consistent with models that emphasize the importance of frictions generated by the presence of agency issues between insiders and outsiders. These are much more likely to be important when economic conditions are good and managers have too many funds available.<sup>10</sup>

Conversely our results are at odds with theories of financial frictions based on costly external finance, where adverse liquidity shocks are magnified by a rising cost of external funds. As we have seen, this interpretation of the data significantly worsens the ability of investment returns to match the observed data on asset returns.<sup>11,12</sup>

## 4.4 Robustness

We now examine the robustness of these results by exploring several alternatives to our benchmark setup.

### Alternative Sets of Moment Conditions

Many authors interpret the cross-sectional variation in the Fama and French (1993) size and book-to-market portfolio returns as proxies for aggregate financial distress. Panel A in Table

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<sup>10</sup>Dow, Gordon and Krishnamurthy (2003) consider one example and study the implications of free cash flows for the behavior of investment and returns.

<sup>11</sup>Gomes, Yaron, and Zhang (2003a) study a general equilibrium version of a one simple model of costly external finance and show the potentially counterfactual implications for asset prices.

<sup>12</sup>It is also possible to interpret our findings that  $b_1 < 0$  as evidence that external funds are less expensive than internal cash flows. The evidence is inconclusive however since we cannot identify the overall level of the constant  $b_0$ .

**Table 6 : GMM Estimates and Tests with Alternative Moment Conditions**

	Panel A: Fama-French Portfolios						Panel B: Small Firm Deciles					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
	Parameters											
$a$	0.00		1.35	(0.10)	0.00		1.26	(0.55)	14.40	(1.80)	2.21	(0.41)
$b_1$	-0.13	(-1.82)	-0.26	(-2.23)	-0.22	(-1.62)	-0.05	(-1.24)	-0.18	(-2.42)	-0.11	(-0.58)
	$J_T$ Test											
$\chi^2$	33.57		25.47		16.10		32.73		20.17		9.99	
$p$	0.00		0.01		0.02		0.00		0.04		0.19	
	Wald Test ( $b_1=0$ )											
$\chi^2_{(1)}$	4.41		13.60		3.95		2.19		6.60		2.20	
$p$	0.04		0.00		0.05		0.14		0.01		0.14	

Panel A reports GMM estimates and tests using Fama-French 25 portfolios in the moment conditions. Specifically, the unconditional model uses the excess returns of portfolios 11, 13, 15, 23, 31, 33, 35, 43, 51, 53, 55 of the Fama and French (1993) 25 portfolios, one investment excess return (over real corporate bond return), and real corporate bond return. The Fama-French portfolios are numbered such that the first digit denotes the size group and the second digit denotes the book-to-market group, both of which are in ascending order. The conditional and scaled model use excess returns of Fama-French portfolio 11, 15, 51, and 55, scaled by instruments, excess investment return and the real corporate bond return. Panel B reports GMM estimates and tests using small firm portfolios. Specifically, the unconditional model uses the excess returns of 11, 12, 13, 14, 15, 21, 22, 23, 24, and 25 of the Fama and French 25 portfolios, one investment excess return, and real corporate bond return. The conditional and scaled model use excess returns of Fama-French portfolios 11, 12, 13, and 15, scaled by instruments, excess investment return and the real corporate bond return. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1=0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and data are the same as those reported in Table 3.

6 investigates this possibility by using our model to price the 25 Fama and French (1993) portfolio returns. Specifically, the unconditional model uses the excess returns of portfolios 11, 13, 15, 23, 31, 33, 35, 43, 51, 53, 55 of the Fama and French (1993) 25 portfolios, one investment excess return (over real corporate bond returns), and the real corporate bond return.<sup>13</sup> The conditional and scaled model use excess returns of Fama-French portfolios 11, 15, 51, and 55, scaled by instruments, excess investment returns (over corporate bond returns) and the real corporate bond returns.

The results from using the Fama and French (1993) portfolio returns confirm our basic

<sup>13</sup>Following the convention in the literature, the first digit denotes the size group and the second digit denotes the book-to-market group, both of which are in ascending order. Thus, portfolio 15 is formed by the intersection of smallest size and highest book-to-market ratio firms.

**Table 7 : GMM Estimates and Tests with Alternative Default Premium Measured As the Yield Spread Between Baa and Ten-year Treasury Bond**

	Panel A: Size Deciles						Panel B: Fama-French Portfolios					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
	Parameters											
$a$	0.00		7.48	(1.74)	4.61	(0.81)	0.00		3.35	(0.17)	0.00	
$b_1$	-0.03	(-0.83)	-0.10	(-2.26)	-0.10	(-1.85)	-0.09	(-1.74)	-0.22	(-2.09)	-0.17	(-1.07)
	$J_T$ Test											
$\chi^2$	7.69		11.63		6.85		28.77		21.60		8.53	
$p$	0.36		0.39		0.44		0.00		0.03		0.29	
	Wald Test ( $b_1=0$ )											
$\chi^2_{(1)}$	3.55		8.88		6.01		5.63		10.52		2.89	
$p$	0.06		0.00		0.01		0.02		0.00		0.09	

This table reports GMM estimates and tests using the difference between yields of Baa and long-term government bonds as a measure of default premium. Panel A uses the 10 CRSP size deciles as in the benchmark Table 3. Panel B uses the Fama-French portfolios as in Panel A of Table 6. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and data the same as those reported in Table 3.

findings. In all cases the estimated values of  $b_1$  are both negative and significant at the 5% level.

Several studies on firm-level financial constraints (e.g., Gertler and Gilchrist (1994)), emphasize that these constraints are more likely to be detected when looking only at the behavior of small firms. An easy way to assess the model's implications for different firms is to test the moment conditions (8) for portfolios of small firms only. Panel B of Table 6 reports GMM estimates and tests using small firm portfolios in the moment conditions. Specifically, the unconditional model uses the excess returns of 11, 12, 13, 14, 15, 21, 22, 23, 24, and 25 of the Fama and French 25 portfolios, one investment excess return (over real corporate bond return), and real corporate bond return. The conditional and scaled model use excess returns of Fama-French portfolios 11, 12, 13, and 15, scaled by instruments, excess investment return (over corporate bond return) and the real corporate bond return

**Table 8 : GMM Estimates and Tests with an Alternative Measure of the Shadow Price of External Funds As a Linear Function of Aggregate Default Measure**

	Panel A: Size Deciles						Panel B: Fama-French Portfolios					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
	Parameters											
$a$	4.59	(0.65)	1.50	(1.00)	1.17	(0.34)	35.70	(1.14)	20.00	(0.55)	4.89	(0.72)
$b_1$	-0.02	(-1.78)	-0.02	(-5.48)	-0.02	(-1.96)	0.00		0.06	(0.95)	-0.01	(-0.32)
	$J_T$ Test											
$\chi^2$	4.32		26.10		5.72		13.70		43.82		11.14	
$p$	0.74		0.01		0.57		0.06		0.00		0.13	
	Wald Test ( $b_1=0$ )											
$\chi^2_{(1)}$	2.24		35.07		1.27				2.77		0.17	
$p$	0.13		0.00		0.26				0.10		0.68	

This table reports GMM estimates and tests where the shadow price of external funds is a linear function of the aggregate default likelihood indicator constructed in Vassalou and Xing (2003). Panel A uses the ten CRSP size deciles as in the benchmark Table 3. Panel B uses the Fama-French portfolios as in Panel A of Table 6. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1=0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and data are the same as those reported in Table 3.

(16 moment conditions). Nevertheless, our basic conclusions are not altered qualitatively for this subset of firms.

### Alternative Factors in Financing Frictions

Several authors (Bernanke (1990), and Stock and Watson (1989, 1999)) show that the default premium is an extremely good predictor of aggregate economic activity. Nevertheless we investigate whether our results are sensitive to this specific measure of the shadow cost of external funds. Table 7 shows that our main results are unaffected when we use the spread between the yields on Baa bonds and those on ten year government notes as an alternative measure of financial market frictions.

In addition we also look at two more elaborate measures of the marginal cost of external funds. The first one is the aggregate default likelihood measure constructed in Vassalou and Xing (2003), who use firm-level equity data to estimate default likelihood indicators for

**Table 9 : GMM Estimates and Tests with the Common Factor of Financial Constraints As An Measure of the Shadow Price of External Funds**

	Panel A: Size Deciles			Panel B: Fama-French Portfolios		
	Unconditional	Conditional	Scaled Factor	Unconditional	Conditional	Scaled Factor
	Parameters					
$a$	0.00	0.00	0.00	21.65 (0.76)	3.67 (0.87)	12.20 (0.89)
$b_1$	-0.59 (-0.77)	-0.29 (-2.69)	0.21 (1.80)	0.68 (1.23)	0.00	0.06 (0.34)
	$J_T$ Test					
$\chi^2$	61.06	26.18	5.35	16.16	91.63	6.11
$p$	0.00	0.01	0.62	0.02	0.00	0.53
	Wald Test ( $b_1=0$ )					
$\chi^2_{(1)}$	1.05	23.44	2.77	2.44		0.16
$p$	0.31	0.00	0.10	0.12		0.67

This table reports GMM estimates and tests where the shadow price is a linear function of the common factor of financing constraints constructed by Lamont, Polk, and Saa-Requejo (2001). Panel A uses the 10 CRSP size deciles as in the benchmark Table 3. Panel B uses the Fama-French portfolios as in Panel A of Table 6. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and data are the same as those reported in Table 3.

individual firms using a contingent claims methodology. The aggregate default likelihood measure is then defined as a simple average of the default likelihood indicators of all firms. Vassalou and Xing (2003) show that this aggregate default measure varies greatly with the business cycle and increases substantially during recessions. This measure is available at monthly frequency between January of 1971 and December of 1999. We construct the corresponding quarterly measure by averaging the likelihoods of the three months within a given quarter and use this as a factor in the expression for financial market distortions (10).

Table 8 reports our GMM results for this case. We find that  $b_1$  is mostly negative and significant when we use ten size deciles as target asset returns (Panel A), and  $b_1$  is not significantly different from zero when we use Fama-French portfolios (Panel B). This is perhaps not surprising, since the aggregate default measure is also quite countercyclical.

Finally, we also use the common factor of financial constraints constructed in Lamont, Polk, and Saa-Requejo (2001). This is defined as the return spread between more financially

**Table 10 : GMM Estimates and Tests with Alternative Macroeconomic Data**

	Panel A: Nonfinancial After Tax						Panel B: Aggregate Profits					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
	Parameters											
$a$	0.79	(0.65)	4.17	(0.84)	2.77	(0.62)	0.00		5.14	(0.45)	7.76	(1.23)
$b_1$	-0.04	(-1.20)	-0.17	(-3.84)	-0.12	(-2.21)	-0.14	(-0.96)	-0.20	(-3.18)	-0.12	(-2.46)
	$J_T$ Test											
$\chi^2$	4.70		8.78		5.93		11.12		12.00		13.41	
$p$	0.70		0.64		0.55		0.13		0.36		0.06	
	Wald Test ( $b_1 = 0$ )											
$\chi^2_{(1)}$	2.42		12.60		6.38		4.88		11.63		5.48	
$p$	0.12		0.00		0.01		0.03		0.00		0.02	

This table reports GMM estimates and tests using alternative measures of profits. Panel A uses data on nonfinancial profits after tax and Panel B uses data for the aggregate economy (not just the non-financial corporate sector). We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. The pricing kernel is the same as those reported in Table 3.

constrained firms and less constrained firms. This series is also available at monthly frequency between July of 1968 and December of 1997. Quarterly data is constructed with time aggregation and used as a factor in the shadow price equation (10).

Table 9 also confirms our earlier results. In this case most of the estimates of  $b_1$  are not significantly different from zero. Again, this is not surprising since the common factor of financial constraint does not seem to covary with credit conditions or business cycles, as documented in Lamont, Polk, and Saa-Requejo (2001).

### Alternative Macroeconomic Series

Table 10 shows the our results are not sensitive to the use of alternative macroeconomic data in the construction of the investment returns. Panel A reports the results of using after tax profits, while Panel B shows similar results when data for the entire economy (not just the non-financial corporate sector) is used.

**Table 11 : GMM Estimates and Tests with Alternative Pricing Kernels**

	Panel A: $M=l_0 + l_1R^I + l_2(R^I)^2$						Panel B: $M=l_0 + l_1R^I + l_2R^B + l_3(R^I)^2 + l_4(R^B)^2$					
	Unconditional		Conditional		Scaled Factor		Unconditional		Conditional		Scaled Factor	
	Parameters											
$a$	1.00	(0.15)	6.16	(1.82)	4.90	(1.11)	1.49	(1.48)	8.29	(1.25)	3.72	(0.68)
$b_1$	-0.26	(-3.79)	-0.19	(-3.67)	-0.18	(-3.31)	-0.03	(-1.66)	-0.20	(-3.34)	-0.08	(-0.80)
	$J_T$ Test											
$\chi^2$	11.64		8.33		4.21		1.33		6.56		1.43	
$p$	0.11		0.68		0.76		0.93		0.68		0.15	
	Wald Test ( $b_1=0$ )											
$\chi^2_{(1)}$	1.36		16.72		12.28		2.35		8.40		4.94	
$p$	0.24		0.00		0.00		0.13		0.00		0.03	

This table reports GMM estimates and tests with alternative specifications of the pricing kernel. Panel A uses the pricing kernel:  $M = l_0 + l_1R^I + l_2(R^I)^2$  for the unconditional and conditional model, and  $M=l_0+l_1R^I+l_2(R^I)^2+l_3(R^I \cdot tp)+l_4(R^I \cdot dp)+l_5((R^I)^2 \cdot tp)+l_6((R^I)^2 \cdot dp)$  for the scaled factor model. Panel B uses the pricing kernel:  $M=l_0+l_1R^I+l_2R^B+l_3(R^I)^2+l_4(R^B)^2$  for the unconditional and conditional model and  $M=l_0+l_1R^I+l_2R^B+l_3(R^I)^2+l_4(R^B)^2+l_5(R^I \cdot tp)+l_6(R^I \cdot dp)+l_7(R^B \cdot tp)+l_8(R^B \cdot dp)+l_9((R^I)^2 \cdot tp)+l_{10}((R^I)^2 \cdot dp)+l_{11}((R^B)^2 \cdot tp)+l_{12}((R^B)^2 \cdot dp)$  for the scaled factor model.  $R^I$  and  $R^B$  are the real investment and corporate bond returns, respectively. We report the estimates for  $a$  and  $b_1$ , the  $\chi^2$  statistic and corresponding  $p$ -value for the  $J_T$  test on over-identification, and  $\chi^2$  statistic and  $p$ -value of the Wald test on the null hypothesis that  $b_1 = 0$ .  $t$ -statistics are reported in parentheses to the right of parameter estimates. Moment conditions and data are the same as those reported in Table 3.

### Alternative Pricing Kernels

Finally, we relax our simple linear factor representation of the pricing kernel (9). Several alternative approaches modelling nonlinear pricing kernels have been recently advanced in the literature, (e.g., Bansal and Vishwanathan (1993)). Here we explore this possibility by re-estimating the moment conditions using some nonlinear pricing kernels. Here, we consider examples where the pricing kernel is quadratic in either  $R^I$  alone or in both  $R^I$  and  $R^B$ . As Table 11 shows, none of these cases changes our original findings qualitatively.

## 5 Conclusion

By concentrating on optimal firm behavior, the investment-based asset pricing model (Cochrane (1991, 1996)) provides a natural way of integrating new developments in the theory of corporate finance into an asset pricing framework. In this paper we pursue this line of research by incorporating financial frictions into a production based asset pricing model and ask whether they help in pricing the cross-section of expected returns. Our methodology allows us to identify the impact of financial frictions on the stochastic discount factor with cyclical variations in cost of external funds. We find evidence that financing frictions provide an important common factor for the cross section of stock returns. In addition, we also find that the shadow price of external funds exhibits strong procyclical variation, so that financial frictions are more important when economic conditions are relatively good. These findings are robust to several alternative formulations of our empirical design, particularly the measures of the shadow price of external funds, the specific macroeconomic data used, and the set of returns used in our GMM implementations. Our results seem consistent with theories of financial frictions emphasizing the importance of agency conflicts between insiders and outsiders. Conversely, this evidence is at odds with models based on costly external finance, where the cost of external funds rises during recessions.

## A Derivation of Investment Returns

We start by rewriting the firms' value-maximization problem as:

$$\max V(K_0, B_0, S_0) \equiv E_0 \left[ \sum_{t=0}^{\infty} M_{0t} (D_t - W_t N_t) \right]$$

where  $M_{0t}$  is the common stochastic discount factor from time 0 to time  $t$ . Dividend is given by:

$$D_t = \Pi(K_t, S_t) - I_t - \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 K_t + N_t + B_{t+1} - R_t B_t$$

Capital accumulation is:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

and the dividend constraint:

$$D_t \geq \bar{D}$$

Letting  $\mu_t$  denote the Lagrange multiplier associated with the dividend constraint, the Lagrange function conditional on the information set at time  $t$  is:

$$\begin{aligned} \mathcal{L}_t = & \dots + M_{0t}(1 + \mu_t) \left[ \Pi(K_t, S_t) - \frac{a}{2} \left( \frac{K_{t+1} - (1 - \delta)K_t}{K_t} \right)^2 K_t - K_{t+1} + (1 - \delta)K_t \right] + \\ & E_t \left[ M_{0t+1}(1 + \mu_{t+1}) \left[ \Pi(K_{t+1}, S_{t+1}) - \frac{a}{2} \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 K_{t+1} - I_{t+1} + N_{t+1} + B_{t+2} - R_{t+1}B_{t+1} \right] \right] + \dots \end{aligned}$$

After using the capital accumulation to substitute  $I_{t+1}$ , the first-order condition for investment is given by:

$$0 = \frac{\partial \mathcal{L}_t}{\partial K_{t+1}} = M_{0t}(1 + \mu_t)[1 + ai_t] + E_t \left[ M_{0t+1}(1 + \mu_{t+1}) \left[ \Pi_K(K_{t+1}, S_{t+1}) + \frac{a}{2} i_{t+1}^2 + (1 + ai_{t+1})(1 - \delta) \right] \right]$$

where  $i_t \equiv I_t/K_t$ . Equations (5) and (6) then follow by noting that  $M_{t+1} = M_{0t+1}/M_{0t}$  and  $\Pi_K = \Pi/K$  with constant return to scale.

## B Data Construction

Macroeconomic data comes from National Income and Product Accounts (NIPA) published by the Bureau of Economic Analysis, and the Flow of Funds Accounts available from the Federal Reserve System. These data are cross-referenced and mutually consistent, so they form, for practical purposes, a unique source of information. Most of our experiments use data for the Nonfinancial Corporate Sector. Specifically Table F102 from the Flow of Funds Accounts is used to construct measures of profits before (item FA106060005) and after tax accruals (item FA106231005). To these measures we add both consumption of capital goods (item FA106300015) and inventory valuation adjustments (item FA106020601) to obtain

a better indicator of actual cash flows. Investment spending is gross investment (item 105090005). The capital stock comes from Table B102 (Item FL102010005). Since stock valuations include cash flows from operations abroad, we also include in our measures of profits the value of foreign earnings abroad (item FA266006003) and that of net foreign holdings to the capital stock (items FL103092005 minus FL103192005, from Table L230) and investment (the change in net holdings). Financial liabilities come from Table B102. They are constructed by subtracting financial assets, including trade receivables, (Item FL104090005) from liabilities in credit market instruments (Item FL104104005) plus trade payables (Item FL103170005). Interest payments come from NIPA Table 1.16, line 35. All these are available at quarterly frequency and require no further adjustments. All data for the aggregate economy come from NIPA.

Financial data come from CRSP and Ibbotson. We use ten size portfolios of NYSE stocks (CRSP series DECRET1 to DECRET10). Corporate bond data comes from Ibbotson's index of Long Term Corporate Bonds. The default premium is defined as the difference between the yields on Aaa and Baa corporate bonds, from CRSP. Term premium, defined as the yield on 10 year notes minus that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio (constructed from CRSP EWRETD and EWRETX).<sup>14</sup>

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<sup>14</sup>Dividend-price ratios are also normalized so that scaled and non-scaled returns are comparable.

## References

- [1] Bansal Ravi, and S. Viswanathan, 1993, No Arbitrage and Arbitrage Pricing: A New Approach, *Journal of Finance*, 48, 1231–1261.
- [2] Bernanke, Ben and Mark Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review*, 79 (1), 14–31.
- [3] Bernanke, Ben and Mark Gertler, 1995, Inside the Black Box: The Credit Channel of Monetary Policy transmission, *Journal of Economic Perspectives*, 9, 27–48.
- [4] Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 1996, The Financial Accelerator and the Flight to Quality, *Review of Economics and Statistics*, 78, 1–15.
- [5] Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 1999, The Financial Accelerator in a Quantitative Business Cycle Framework, in *Handbook of Macroeconomics*, Edited by Michael Woodford and John Taylor, North Holland.
- [6] Bond, Stephen and Costas Meghir, 1994, Dynamic Investment Models and the Firm’s Financial Policy, *Review of Economic Studies*, 61, 197–222
- [7] Cochrane, John H., 1991, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *Journal of Finance*, 46 (1), 209–237.
- [8] Cochrane, John H., 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy*, 104 (3), 572–621.
- [9] Dow, James, Gary Gorton, and Arvind Krishnamurthy, 2003, Equilibrium Asset Prices with Imperfect Corporate Control, Working Paper, Kellogg School of Management, Northwestern University.
- [10] Fama, Eugene F., 1981, Stock Returns, Real Activity, Inflation, and Money, *American Economic Review*, 71, 545–565.
- [11] Fama, Eugene F., and Michael R. Gibbons, 1982, Inflation, Real Returns and Capital Investment, *Journal of Monetary Economics*, 9, 297–323.
- [12] Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Finance*, 33, 3–56.
- [13] Fama, Eugene F., and Kenneth R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance*, 51, 55–84.

- [14] Gertler, Mark, and Simon Gilchrist, 1994, Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms, *Quarterly Journal of Economics*, CIX (2), 309–340.
- [15] Gomes, Joao F., Amir Yaron, and Lu Zhang, 2003a, Asset Prices and Business Cycles with Costly External Finance, forthcoming *Review of Economic Dynamics*.
- [16] Gomes, Joao F., Amir Yaron, and Lu Zhang, 2003b, Asset Pricing Implications of Firms' Financing Constraints - Technical Appendix, unpublished manuscript, University of Pennsylvania.
- [17] Hansen, Lars Peter, and S. Richard, 1987, The role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models, *Econometrica*, 55, 587–613.
- [18] Jensen, Michael, 1986, The Agency Costs of Free Cash Flow: Corporate Finance and Takeovers, *American Economic Review*, 76 (2), 323–330.
- [19] Kashyap, Anil K., Jeremy C. Stein, and David W. Wilcox, 1993, Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance, *American Economic Review*, 83, 78–98.
- [20] Kashyap, Anil K., Owen A. Lamont, and Jeremy C. Stein, 1994, Credit Conditions and the Cyclical Behavior of Inventories, *Quarterly Journal of Economics*, 109, 565–592.
- [21] Lamont, Owen, Investment Plans and Stock Returns, 2000, *Journal of Finance*, LV (6), pp. 2719–2745
- [22] Lamont, Owen, Christopher Polk and Jesús Saá-Requejo, 2001, Financial Constraints and Stock Returns, forthcoming, *Review of Financial Studies*.
- [23] Li, Xiangyang, 2003, Asset Pricing and Financing Constraints: A Firm-Level Study, Working Paper, Economics Department, Yale University.
- [24] Li, Qing, Maria Vassalou, and Yuhang Xing, 2001, An Investment Growth Asset Pricing Model, Working Paper, Columbia University.
- [25] Restoy, Fernando, and G. Michael Rockinger, 1994, On Stock Market Returns and Returns on Investment, *Journal of Finance*, 49 (2), 543–556.

- [26] Stein, Jeremy, 2003, Agency, Information and Corporate Investment, forthcoming in *Handbook of Economics and Finance*, edited by George Constantinides, Milton Harris, and Rene Stulz.
- [27] Stock, James H. and Mark W. Watson, 1989, New Indexes of Coincident and Leading Economic Indicators, in *NBER Macroeconomics Annual*, edited by Oliver J. Blanchard and Stanley Fischer, 352–394.
- [28] Stock, James H. and Mark W. Watson, 1999, Business Cycle Fluctuations in U.S. Macroeconomic Time Series, in *Handbook of Macroeconomics*, edited by James B. Taylor and Michael Woodford, 1, 3–64.
- [29] Vassalou, Maria, and Yuhang Xing, 2003, Default Risk in Equity Returns, Forthcoming, *Journal of Finance*.
- [30] Whited, Toni M., 1992, Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data, *Journal of Finance*, 47 (4), 1425–1460.
- [31] Whited, Toni M., and Guojun Wu, 2003, Financial Constraints Risk, Working Paper, University of Michigan Business School.