Regulation and Return: The Role of Ambiguity

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Abstract

In this paper we examine the role of regulation of financial markets in an economy with both expected utility maximizing and ambiguity averse traders. Both types of traders are rational, but ambiguity averse traders do not have enough information about asset markets to form a prior over the set of possible models of return determination. Instead, they have a set of possible models of returns and their investment behavior is greatly affected by the most pessimistic model they consider. In particular, in the CARA-normal setting we examine their choice of whether or not to participate in the market is determined by the lowest mean return they believe to be possible, and when they do participate the amount of the asset they hold depends on the maximum variance of returns they consider possible. Because of these effects, regulations that affect these most pessimistic values can have surprisingly large effects on equilibrium asset prices. We show how the presence of ambiguity averse traders provides new rationale for deposit insurance, suitability rules, insider “short swing” trading profit restrictions and mutual fund restrictions.

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1. Introduction

Research in finance has increasingly turned to investigating alternatives to the standard model of investor behavior. In the standard model, investors are assumed to be extraordinarily rational: they act as if they maximize expected utility using rational, or correct, expectations.[1] But a growing body of empirical evidence suggests that asset pricing behavior is not well described by this traditional paradigm. One direction taken in the literature is behavioral finance, where the rationality of the investor is replaced by any number of psychology-based alternatives, such as over-confidence, under-reaction, loss aversion, and the like.[2] Another direction focuses on model uncertainty or learning problems. This literature retains rationality, but replaces rational expectations with beliefs updated through a learning rule. In this approach, investment decisions can differ from the standard outcome due to the difficulty of learning the true state when the actual state process can be complex.[3] A third approach focuses on alternative models of rational decision-making. Of particular importance here is the role of ambiguity aversion.[4]

The model of ambiguity aversion allows individuals to distinguish between risk and ambiguity in their decision-making. This distinction is best illustrated by the famous Ellsberg Paradox (Ellsberg [1961]). In a simple two urn version of this paradox subjects are offered a choice between a series of bets on the draw of a ball from an urn with a known mix of red and black balls, or on the draw of a ball from an urn with an unspecified mix of red and black balls. Most subjects make choices that are inconsistent with having any prior on the number of balls of each type in the second urn.[5] That is, they do not act as if they have a prior on the odds of red versus black in the second urn, compute predicted probabilities of red and black balls and chose

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[1] For a discussion of the distinction between rationality and rational expectations, and the implications for asset pricing, see Blume and Easley [2004].
[2] The behavior finance literature is extensive, but representative important papers are Daniel, Hirshleifer, and Subrahmanyam [2001];Gervais and Odean [2000]; Hirshleifer [2001].
[3] The literature here is diverse and includes models with symmetric information and asymmetric information. For analysis featuring symmetric information see Brav and Heaton [2002]; Brennan and Xia [2001; 2002]; Lewellen and Shanken [2002]; Xia [2001]. Asset pricing with asymmetric information is considered by Easley and O'Hara [2004]; Easley, Hvidkjaer, and O'Hara [2002]; and O'Hara [2003].
[4] This topic is referred to in the literature sometimes as uncertainty aversion and other times as ambiguity aversion.
[5] Here is a simple version of the Ellsberg experiment. Urn one has 50 red and 50 black balls. Urn two has 100 balls which are some mix of red and black. First, subjects are offered a choice between two gambles: $1 if the ball drawn from urn one is red and nothing if it is black or $1 if the ball drawn from urn two is red and nothing if it is black. Most subjects chose the first gamble so if they have a prior on urn two the predicted probability of red in urn two is

a bet to maximize expected utility. Ellsberg's interpretation is that people are averse to the ambiguity about the odds for the second urn. They prefer to bet on events with known, rather than ambiguous, odds.

Knight [1921] originally developed the notion of individuals making a distinction between known odds and uncertain or ambiguous odds. This distinction was noted by Savage [1954], but in his model of subjective expected utility it plays no role. The standard model of asset pricing is based on Savage's foundation for expected utility maximization. So the distinction between known odds and ambiguous ones plays no role in the standard asset pricing model. The distinction between ambiguity and risk has seen a resurgence due to the work of Schmeidler [1989] and Gilboa and Schmeidler [1989]. Particular applications of ambiguity aversion to asset pricing can be found in recent papers by Dow and Werlang [1992]; Epstein and Wang [1994], Routledge and Zin [2002]; Liu, Longstaff, and Pan [2003]; Liu, Pan, and Wang [2003], and Cao, Wang and Zhang [2003].

In this paper, we investigate the implications of ambiguity aversion for the performance and regulation of markets. Our explicit goal is to demonstrate that regulation, particularly regulation of unlikely events, can have welfare-improving effects on the economy. We develop our results in a multi-asset model that includes both standard expected utility maximizing agents and ambiguity averse agents. In our economy, the parameters determining the distribution of returns to risky assets are unknown. Expected utility maximizers have a prior on the set of parameters and treat the uncertainty about parameters in the same way that they treat the uncertainty about returns given parameters. Ambiguity averse agents would select a portfolio to maximize expected utility if they knew the correct value of the parameters, but they do not know the parameters, and unlike expected utility maximizers they do not aggregate across parameters with a prior. Instead, they act as if they had a set of distributions on returns; one distribution for each possible value of the unknown parameters. They select a portfolio to maximize the minimum expected utility over this set of distributions.

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less than 0.5. Next subjects are offered a choice between two new gambles: $1 if the ball drawn from urn one is black and nothing if it is red or $1 if the ball drawn from urn two is black and nothing if it is red. Most subjects again chose the first gamble so if they have a prior on urn two the predicted probability of black in urn two is less than 0.5. This cannot be so they do not act as if they have any one prior on urn two.

We show that ambiguity aversion can affect the risk premium through its influence on the participation of investors in financial markets.[6] An ambiguity averse investor facing a choice between investing in a safe asset such as cash, and a risky asset with an unknown distribution of returns, evaluates investment in the risky asset using the worst case return distribution that he perceives to be possible. So if the ambiguity averse investor considers an unfavorable return distribution to be possible he may choose not to invest in the risky asset. An expected utility maximizer places a prior on the set of distributions and invests according to his predicted distribution of returns. He will chose to invest in the risky asset if its risk-return tradeoff is favorable where risk and return are evaluated using his prior over distributions. Thus an economy with some ambiguity averse investors may price assets as if the ambiguity averse investors did not exist. Because the aggregate risk has to be held by the subpopulation of expected utility maximizers, the per capita risk is greater and in equilibrium a greater risk premium is required.[7]

We consider an economy with a safe asset and two risky assets. The risky assets have normally distributed returns, but the mean and variance of these distributions are unknown. We show that reducing the perceived maximum variance of returns or increasing the perceived minimum mean return can (depending on the specification of the economy) induce large equilibrium effects, the most important of which is to the lower the equilibrium risk premium. In our model, these changes are first examined holding the prior mean of mean returns and variances (used by the expected utility maximizers) constant, demonstrating the distinct effects that arise from risk and ambiguity. We then show that allowing these prior means to vary in natural ways reinforces our results.

We argue that changing these perceived minimum mean returns and maximum variances can be accomplished by particular regulations. We also demonstrate that for some specifications of the economy the equilibrium is one in which such regulations will not affect investor behavior. Our analysis thus provides an explanation of why regulations on ostensibly unlikely,

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[6] Merten [1987] first proposed the importance of limited participation for influencing market behavior, and numerous authors (see Shapiro [2000]; Basak and Cuoco [1998]) have extended this research. These models generally rely on incomplete information to generate participation constraints. Our ambiguity-based analysis provides an alternative explanation for such participation effects.

[7] Cao, Wang, and Zhang [2003] demonstrate similar risk premium effects in a model in which agents have heterogeneous uncertainty aversion.

or improbable, events, can, but do not always, have large effects; these effects provide one explanation for why markets seem to "over-react" to seemingly small changes.

We illustrate the importance of these effects by considering a range of applications including deposit insurance, suitability rules, insider "short-swing" trading profit restrictions (SEC Rule 16(b)), and mutual fund restrictions. A more general application is to explain the perplexing upside-down structure of securities regulations, in which firms with the most available public information are subject to the greatest regulatory scrutiny. As we show here, regulations can induce alternative equilibrium outcomes by either reducing or increasing perceived ambiguity. Our results suggest that for some securities limiting participation through greater ambiguity may be a more cost-effective means of investor protection than direct regulatory supervision of firms or markets.

Apart from providing important insights into the structure and impact of market regulation, our analysis of ambiguity aversion provides implications for broader concepts such as contracting and the role of disclosure. Contracting cannot solve the participation problems induced by ambiguity aversion because the problem involves low probability specifications of the world (minimum possible mean and maximum possible variance in our model) whose very existence would make complete contracting prohibitively expensive. Similarly, disclosure cannot solve ambiguity -induced problems because the difficulty is the probability to attach to unlikely states, not their existence. Indeed, with ambiguity averse investors, disclosure can exacerbate problems by scaring away potential market participants. Further, the palliative role of arbitrage is limited because ambiguity aversion induces non-participation, and not merely mis-pricing. These difficulties underscore why market solutions may be incapable of removing the influences of ambiguity aversion, even when all market participants act rationally.

This paper is organized as follows. In the next section, we develop a multi-asset model involving both risk averse and ambiguity averse investors. We show how ambiguity aversion affects individual asset demands, and we solve for market equilibria. Section 3 then investigates how changing perceptions of extreme models of returns influences investor participation, and how this in turn affects equilibrium outcomes. We show here that changing the perception of extreme models from being unlikely to being impossible can have large effects in equilibrium. In Section 4, we consider the implications of our model by examining a number of rules and regulations designed to limit unlikely events. Section 5 then considers generalizations and extensions by looking at the role of market solutions such as contracting, disclosure, and arbitrage in dealing with ambiguity. Section 6 is a conclusion

2. **The Model**

We analyze an economy with three assets. There is one risk free asset, money, which is has a constant price of 1. There are two risky assets with independent, normally distributed returns r^i, $i = 1,2$. All investors know that returns are independent and normal, but they do not know the mean or variance of the return distributions. The set of possible means for asset i is $\{\overline{r}_1^i,...,\overline{r}_N^i\}$; the set of possible variances is $\{s_1^i,...,s_N^i\}$. All pairs of mean and variance are possible and we let $\Theta^i = \{q_1^i,...,q_n^i\}$, with $n = N^2$ elements, be the set of possible return parameters.

There are J investors indexed by $j = 1,...,J$. All investors have CARA utility for wealth, with the risk aversion parameter set equal to 1:

$$u_j(w) = -\exp(-w). \qquad (1)$$

There are two types of investors in the economy, denoted EU investors and AA investors. Fraction $1-m$ of investors are standard expected utility maximizers (EU) with common beliefs on the sets of possible return parameters. For these investors all that matters about beliefs is the expected value of means and variances of the return distributions. Let $(\hat{r}^i, \hat{s}^i)$ denote the common expected value of the mean return and variance for asset i.

Fraction m of the investors are ambiguity averse (AA). Ambiguity averse investors also care about means and variances, but they differ from more standard EU investors in that the AA investors do not have a prior on the set of return parameters.[8] Instead, they consider each normal distribution of returns, $N(q^i)$, as a possible return distribution. Consequently, these investors do not average over the possible distributions of outcomes the way an EU investor does, but rather they evaluate each possible distribution. Following Gilboa and Schmeidler's (1989) axiomatic foundation for ambiguity aversion, we model ambiguity averse investors as choosing a portfolio to maximize their minimum expected utility over the set of possible return distributions.

[8] These investors can be thought of as inexperienced potential investors who do not have enough experience in the market to reliably access return distributions. Perhaps they have not yet participated in the asset market, and although they can imagine many possible return distributions, they are unable to place a prior on this set of distributions. They know that holding cash is safe, but are just not sure how to think about stocks.

The per capita endowments of money and assets are $(\overline{m}, \overline{x}^1, \overline{x}^2)$. The exact distribution of this endowment over investors does not affect their demands for risky assets because of the CARA-Normal structure, so we do not specify it.[9] We denote a typical investor's wealth by w. Where no confusion would occur, we will drop the investor index. The investor's budget constraint is

$$w = m + p^1 x^1 + p^2 x^2 \tag{2}$$

where p^i is the price of asset i , m is the quantity of money and x^i is the quantity of risky asset i. Investors are allowed to go long or short in each asset. If the investor chooses portfolio (m, x^1, x^2) his random next period wealth will be

$$\tilde{w} = m + \tilde{r}^1 x^1 + \tilde{r}^2 x^2. \tag{3}$$

For a standard EU maximizer, with CARA utility of wealth and expected value of return parameters $(\hat{r}^i, \hat{s}^i)$, the expected utility of this random wealth is a strictly increasing transformation of

$$(\hat{r}^1 - p^1)x^1 + (\hat{r}^2 - p^2)x^2 - 1/2\,\hat{s}^1 (x^1)^2 - 1/2\,\hat{s}^2 (x^2)^2 + w. \tag{4}$$

Calculation shows that the EU maximizing investor's demand function for asset i is given by:

$$x_U^{i*}(p^i) = \frac{\hat{r}^i - p^i}{\hat{s}^i}. \tag{5}$$

An ambiguity averse investor evaluates the expected utility of wealth for each parameter vector and chooses the portfolio that maximizes the minimum of these expected utilities. In effect, the AA investor tries to avoid the worst case outcomes, and so chooses a portfolio that explicitly limits exposure to such adverse outcomes. The expected utility of random wealth, given parameters $(q^1 = (\overline{r}^1, s^1), q^2 = (\overline{r}^2, s^2))$, is a strictly increasing transformation of

$$(\overline{r}^1 - p^1)x^1 + (\overline{r}^2 - p^2)x^2 - 1/2\,s^1 (x^1)^2 - 1/2\,s^2 (x^2)^2 + w. \tag{6}$$

Thus, the AA investor's decision problem can be written as

$$\underset{(x^1,x^2)}{Max}\ \underset{(q^1,q^2)}{Min} \left((\overline{r}^1 - p^1)x^1 + (\overline{r}^2 - p^2)x^2 - 1/2\,s^1 (x^1)^2 - 1/2\,s^2 (x^2)^2 + w \right) \tag{7}$$

[9] For ambiguity averse investors the most natural interpretation is that they have no endowment of risky assets. But regardless of their endowments what they care about is their final asset position. So in an equilibrium in which ambiguity averse investors choose not to hold a risky asset they will trade if necessary in order to achieve a zero asset position.

Examining the minimization problem reveals that for any portfolio the minimum occurs at the maximum possible variance for each asset. Denote these variances by $s^i_{\max}$. Consequently, what matters to the AA investor is not the "expected" variance, but rather the largest variance. Whether the minimum occurs at the maximum or minimum mean return depends on whether the investor is long or short in the asset. The minimum occurs at minimum mean return for asset i if the investor is long in asset i and at maximum mean return for asset i if the investor is short in asset i. Denote these mean returns by $\overline{r}^i_{\min}$ and $\overline{r}^i_{\max}$, respectively. Calculation shows that the ambiguity averse investor's demand function for asset i is

$$x^{i*}_A(p^i) = \begin{bmatrix} \frac{\overline{r}^i_{\min} - p^i}{s^i_{\max}} \ \ if \quad \overline{r}^i_{\min} > p^i \\ 0 \ \ if \quad \overline{r}^i_{\min} \le p^i \le \overline{r}^i_{\max} \\ \frac{\overline{r}^i_{\max} - p^i}{s^i_{\max}} \ \ if \quad \overline{r}^i_{\max} < p^i \end{bmatrix}. \tag{8}$$

There are several properties of this demand function that will be important for our analysis. First, note that if the price of asset i is above the minimum possible mean return and below the maximum possible mean return, then the ambiguity averse investor will not participate in the market for asset i.[10] This occurs because an ambiguity averse investor is heavily influenced by the worst possible state and what is worst depends on the investor's asset position. If the investor holds a positive quantity of the asset he evaluates it using the lowest possible mean return, $\overline{r}^i_{\min}$, and the highest possible variance, $s^i_{\max}$. If the investor goes short, the worst possible mean switches to $\overline{r}^i_{\max}$ and the worst variance stays at $s^i_{\max}$. So unless the price of the asset is above $\overline{r}^i_{\max}$ or below $\overline{r}^i_{\min}$, an ambiguity averse investor will not participate in the asset market.

Second, note that the AA investor's decision about whether to hold the asset is independent of the set of variances he believes to be possible. All that matters for the participation decision is the price, the minimum mean return and the maximum mean return. If the AA investor decides to hold the asset, then variance matters, just as it does for the EU investor. But note that only the maximum possible variance affects the quantity to be held. The

[10] Here by not participating we mean that his final asset position will be zero. This interpretation is most natural if ambiguity averse investors do not initially hold the risky asset.

other variances the AA investor believes to be possible do not affect his decision about whether to participate or his decision about how much to hold if he chooses to participate.

Graphs of the demand for asset i for EU and AA investors are shown in **Figure 1**. Note that the EU investor always holds a larger amount (in absolute value) of the risky asset than does the ambiguity averse investor. This is because for any given return parameters these investors evaluate the tradeoff between mean and variance equivalently. They both avoid risk and require compensation in expected return in order to hold risk. But the AA investor also avoids ambiguity in the distribution of returns, and so as long as the set of possible means and variances is non-degenerate he further reduces his position in the risky asset.

In equilibrium per capita demand for asset i must equal its per capita supply. Equating the demands from equations (5) and (8) to this supply then yields

$$m x_U^{i*}(p^i) + (1-m) x_A^{i*}(p^i) = \bar{x}^i . \tag{9}$$

Because these demands are complex, the equilibrium may also be complex. In particular, depending on the parameters of the economy, there are two possible types of solutions to this equation.

First, if at a price between $\bar{r}^i_{\min}$ and $\bar{r}^i_{\max}$ the EU investors are willing to hold the entire supply of the asset, then in equilibrium the ambiguity averse investors will not participate in the market. If only EU investors participate in the market the market clearing price must be

$$\hat{p}^i = \hat{r}^i - \tfrac{\hat{s}^i \bar{x}^i}{1-m} \tag{10}$$

Thus, $\hat{p}^i$ will be the market clearing price for asset i if $\bar{r}^i_{\max} \geq \hat{p}^i \geq \bar{r}^i_{\min}$. Note that $\bar{r}^i_{\max} \geq \hat{p}^i$ as $\bar{r}^i_{\max} \geq \hat{r}^i \geq \hat{p}^i$, so the binding condition is $\hat{p}^i \geq \bar{r}^i_{\min}$.

Second, it is possible that both types of investors participate in the market for asset i. If we conjecture that both types of investors participate, then the market clearing price must be

$$p^{i*} = \frac{m\hat{s}^i \bar{r}^i_{\min} + (1-m) s^i_{\max} \hat{r}^i - \bar{x}^i s^i_{\max} \hat{s}^i}{m\hat{s}^i + (1-m) s^i_{\max}} . \tag{11}$$

This can be an equilibrium price only if ambiguity averse investors are willing to participate, i.e. only if $p^{i*} < \bar{r}^i_{\min}$. Calculation shows that this constraint is met if and only if $\hat{p}^i < \bar{r}^i_{\min}$.

As the binding condition for a non-participation equilibrium is $\hat{p}^i \geq \bar{r}^i_{\min}$, one and only one of these equilibria will prevail for any economy. Thus, there is a unique equilibrium. This equilibrium is either one in which ambiguity averse investors do not participate, a Non-Participating Equilibrium, or one in which they do participate, a Participating Equilibrium. These results are summarized in the proposition below.

Proposition: There is a unique equilibrium in the market for asset i. It is one of two types:

1. Non-Participating: If $\hat{p}^i = \hat{r}^i - \frac{\hat{s}^i \overline{x}^i}{1-m} > \overline{r}^i_{\min}$ then in the equilibrium $x_A^{i*} = 0$, $x_U^{i*} = \frac{\overline{x}^i}{1-m}$ and $\hat{p}^i$ is the market clearing price.
2. Participating: If $\hat{p}^i = \hat{r}^i - \frac{\hat{s}^i \overline{x}^i}{1-m} < \overline{r}^i_{\min}$ then in the equilibrium both $x_A^{i*} > 0$ and $x_U^{i*} > 0$, and p^{i*} is the market clearing price.

One important point to note about this equilibrium is the crucial role played by the minimum possible mean return. Two risky assets that have identical supplies, fractions of ambiguity averse investors, and expected values for the mean and variance of returns can nonetheless have different equilibrium prices, and thus different excess returns, if their minimum possible mean returns differ.

This property implies that asset prices may appear to be biased if viewed from the standard metrics usually employed in asset pricing. In particular, an outside observer might be able to assess per capita supplies for active investors and would typically measure expected means and variances by historical averages. If the EU investors have rational expectations, their beliefs would match these historical averages. So an asset in which ambiguity averse investors do not participate will seem to the outside observer to be priced correctly.[11] But an asset in which equilibrium requires participation by ambiguity averse investors will seem to be priced incorrectly, with prices too low and risk premia too high relative to what is expected. Only if the outside observer can also assess the minimum mean return and account for the effect of ambiguity averse investors on equilibrium prices will these biases be revealed.

[11] This, of course, requires the outside observer to correctly access per capita supply for the active, risk averse investors. If instead, per capita supply for the total potential investor population is used, then the asset will seem to earn an excess return.

3. Characterization of Equilibrium

In this section we analyze how changes in the economy affect equilibrium asset prices. We are particularly interested in changes in the fraction of investors who are ambiguity averse and in the set of means and variances they consider possible. We want to analyze the effects of changes in the return distribution parameters while keeping the behavior of risk averse investors constant. So, for much of our analysis, we hold the beliefs of EU investors constant while we change the set of models the ambiguity averse investors consider. We then show that allowing changes in the set of return distributions to affect the beliefs of EU investors reinforces the effects we find for ambiguity averse investors.

A. Fraction of Ambiguity Averse Investors

We first consider how the distribution of investors between EU and AA types influences the equilibrium outcome. In an economy with only EU investors the equilibrium price of asset i is $\hat{r}^i - \hat{s}^i \bar{x}^i$. It is straightforward to show that an increase in the fraction of investors who are ambiguity averse decreases the price of risky assets. In the limit with only AA investors the equilibrium price of asset i is $\bar{r}_{\min}{}^i - s^i{}_{\max} \bar{x}^i$. The equilibrium price is a continuous and decreasing function of the fraction of AA investors, m, in both the non-participating and in the participating equilibrium. At the critical level of $m = 1 - \frac{\hat{s}^i \bar{x}^i}{\hat{r}^i - \bar{r}_{\min}{}^i}$ the economy switches from a non-participating to a participating equilibrium, which causes a further decrease in asset prices.

To understand why these effects occur, consider first a market for asset i that is in a non-participating equilibrium and stays there after an increase in the fraction of ambiguity averse investors. More AA investors means fewer EU investors, so the price of asset i decreases as the aggregate risk must be held by fewer investors. In order to induce investors to hold this increased per capita risk they must be offered an increase in mean return which requires a fall in asset prices. This effect is similar to Merton's (1987) incomplete information effect. In his analysis, some investors are unaware of some assets. If there is a decrease in the number of investors who are aware of an asset, then the asset price must fall. The difference in our model is that all investors are aware of all assets, but some investors, the ambiguity averse, are unwilling to hold assets about which they have too little information to form beliefs.[12]

Conversely, in a participating equilibrium, the equilibrium price of the asset, p^{i*}, is identical to the price that would occur in a CARA-Normal economy with fraction m of investors with beliefs $(\overline{r}^{i}_{\min}, s^{i}_{\max})$ and fraction $1-m$ of investors with beliefs $(\hat{r}^{i}, \hat{s}^{i})$. Since $\overline{r}^{i}_{\min} < \hat{r}^{i}$ and $s^{i}_{\max} > \hat{s}^{i}$, increasing the fraction of ambiguity averse investors has the same effect as increasing the fraction of pessimistic investors in a standard asset pricing framework. It reduces the equilibrium price of the asset.

Finally, since increasing the fraction of ambiguity averse investors can decrease $\hat{p}^{i}$, substituting AA investors for EU investors can cause $\hat{p}^{i}$ to fall below $\overline{r}^{i}_{\min}$. If this happens, the equilibrium shifts from a non-participating to a participating equilibrium. In the new equilibrium, the asset price thus decreases from a price which was initially above $\overline{r}^{i}_{\min}$ to $p^{i*} < \overline{r}^{i}_{\min}$.

B. Maximum Possible Variance

Whether equilibrium prices are sensitive to changes in the set of variances considered by ambiguity averse investors depends on whether the economy is a participating or a non-participating equilibrium. First, note that only the maximum possible variance matters. Changes in the set of variances considered by ambiguity averse investors have no effect unless the maximum possible variance changes. Second, if the economy is a non-participating equilibrium then changes in $s^{i}_{\max}$ have no effect as ambiguity averse investors do not participate in the asset market. Further, changes in $s^{i}_{\max}$ cannot move the economy from a non-participating equilibrium to a participating equilibrium, or vice versa. Finally, note that if the economy is a participating equilibrium then increases in $s^{i}_{\max}$ cause the price of asset i to fall. This occurs because an increase in $s^{i}_{\max}$ reduces the demand by ambiguity averse investors.

[12] Brav, Constantinidis, and Geczy [2002] provide empirical evidence linking non-participation to increased equity market premia. See also Vissing-Jorgenson for an analysis of participation effects.

C. Minimum Mean Return

Clearly only the minimum possible mean return affects the demands of ambiguity averse investors. If AA investors do not participate in the asset market, then changes in $\overline{r}^i_{\min}$ have no effect on the price of asset i as long the market remains in a non-participating equilibrium after the change. If ambiguity averse investors participate in the market for asset i, then increasing $\overline{r}^i_{\min}$ causes the price of asset i to increase. This occurs as an increase in $\overline{r}^i_{\min}$ increases the demand for asset i by ambiguity averse investors. Finally, an increase in $\overline{r}^i_{\min}$ can cause the market to switch from a non-participating to a participating equilibrium. The critical value of $\overline{r}^i_{\min}$ for this switch is $\hat{r}^i - \frac{\hat{s}^i \overline{x}^i}{1-m}$. When $r^i_{\min}$ equals this value the prices in the non-participating and the participating equilibria are equal. As $\overline{r}^i_{\min}$ increases, the market moves to a participating equilibrium and the equilibrium price of asset i increases smoothly. This effect is illustrated in **Figure II**.

D. EU Investors Beliefs

These comparative static results have been obtained holding the expected values of the mean and variance constant. If we assume that the beliefs of EU investors are formed by computing expected values according to some distribution over the set of values considered possible by ambiguity averse investors, then changes in $s^i_{\max}$ or $\overline{r}^i_{\min}$ also affect the demands of EU investors. These effects are small. The derivative of the expected value of the mean return with respect to the minimum possible return is the prior probability on the minimum mean return. Increasing $\overline{r}^i_{\min}$ increases the expected mean which increases the demand by EU investors. This reinforces the effect on price obtained from ambiguity averse investors.

The derivative of the expected value of the variance with respect to the maximum possible variance is the prior probability on the maximum variance. Increasing the maximum possible variance increases the expected value of the variance which reduces the demand by risk avers investors. This also reinforces the effect on price obtained from ambiguity averse investors.

These results are summarized in **Table I.**

4. **Ambiguity and Regulation**

The model developed in the previous sections demonstrates that changing the perception of extreme models of returns can potentially have large effects on equilibrium outcomes.[13] These effects arise because ambiguity averse individuals attach great importance to worst case models, with the result that they can chose not to participate in the market. These participation effects, in turn, limit the risk-sharing abilities of the market, and so induce an increase in the equilibrium risk premium.

Our analysis suggests that increasing the perceived minimum mean return or lowering the perceived maximum variance could have large effects on these investors' demands, even when the mean of mean returns and variances remain the same. In effect, what is needed is to convince ambiguity averse investors that perceived worst case models are in fact impossible. In this section we consider several applications of regulations designed specifically to apply to unlikely models. Our analysis shows why regulation designed to exclude some previously possible models can have large beneficial effect on asset prices, and correspondingly why seemingly irrelevant rules can play an important role in the economy.

A. Deposit Insurance and Guarantees

Perhaps the most direct approach to raising the minimum mean return on an investment is to have a government guarantee that no matter what happens the investor will always receive at least their money back. Such governmental guarantees can be found in many settings, but few play a more important role than the government guarantee of bank deposits.[14] Numerous researchers have shown (see, for example, Diamond and Dybvig [1983]) that the introduction of deposit insurance dramatically enhanced the stability of the banking system by reducing the risk of bank runs. In a world with some ambiguity averse investors, however, deposit insurance can play an augmented role by inducing these individuals to participate in the banking system in the first place.

[13] The effects are large relative to those that could be expected in an economy with only EU traders. In such an economy the effect of extreme models on asset prices is multiplied by the prior on the model which would most naturally be small. With AA investors the effect of extreme models is direct; it is multiplied by any prior belief.

[14] For example, the government guarantees the payment of principle and interest for GNMA securities, through the PBGC it guarantees the payment of minimum pension benefits, and through the SIPC it guarantees funds held in broker/dealer accounts.

Consider, for the moment, the decision-problem confronting an ambiguity averse investor who is considering whether to invest in a risky venture such as a deposit in an uninsured bank or an investment in a money market fund, or to hold cash. The return on investments is random and the distribution of these returns may be ambiguous for an unsophisticated investor. If the investor considers several models of how banks work and one of these models has bank runs (or money market fund defaults) occurring with high probability, then the AA investor refuses to put any money in the bank, preferring instead to "keep it under the mattress". In the correct model of the economy, bank failures are extremely unlikely, but an ambiguity averse individual's portfolio decisions are greatly influenced by the worst case scenario. Thus, while an EU investor may be willing to make a bank deposit or, for a higher interest rate, invest in a money market fund (which holds very low risk assets, but is not insured), an ambiguity averse investor would not do so even for implausibly high interest rates.

Suppose the government introduces a guarantee on deposits. Now losses to the investor from bank runs do not occur and the worse case scenario of returns disappears.[15] The ambiguity averse investor optimally deposits money in the bank (while still eschewing the potentially lethal money market fund). By inducing participation in the banking system, deposit insurance enhances individual welfare by replacing a non-interest bearing investment with a comparably safe but interest-bearing one. Perhaps more important, this regulatory approach expands the resources available to the banking system and facilitates greater-risk sharing in the economy.

Is there any evidence that such an ambiguity effect ever arises? The market effects surrounding the "Too Big To Fail" doctrine suggest that it does. During testimony to Congress in April 1985, the Comptroller of the Currency announced that henceforth the 11 largest banks in the U.S. would not be allowed to fail, in effect extending 100% deposit insurance to this select group of banks. O'Hara and Shaw [1991] document that this announcement elicited a statistically significant market effect, with the prices of included banks rising an average of almost 2% relative to non-included banks. Yet, it was widely presumed that such banks would have been bailed out in any case (and only one large US bank had actually failed in the previous 50 years). Such a large market reaction, however, is perfectly consistent with ambiguity aversion; removing even an unlikely event from consideration can induce large effects.

[15] We assume that the AA investor believes the government guarantee. Otherwise, he might maintain the same set of models and still not make a deposit.

Conversely, recent events suggest that the absence of a presumed guarantee can have similarly large effects. The IMF's refusal to bail out Russia following its bond market default in 1998 set off a veritable "free fall" in emerging market debt prices as investors fled to higher quality issues. Intriguingly, this "flight to quality" affected virtually all emerging market debt, even though conditions in Russia were hardly representative of other emerging markets. Yet, such behavior is consistent with ambiguity aversion: investors now perceived that outcomes they previously believed precluded by IMF guarantees were possible. With this new lower $r^i_{\min}$, investors opted not to participate in these markets even with very high returns.

IMF guarantees and the "Too Big To Fail" policy have both been criticized for creating moral hazard problems in markets. What may not have been appreciated, however, is that such absolute guarantees may also promote stability in markets by enhancing participation by ambiguity averse investors. Whether such positive participation effects could be large enough to offset the negative moral hazard effects depends, in part, on the prevailing equilibrium. If the market remains in a non-participating equilibrium, then providing guarantees only exacerbates moral hazard and is surely the wrong policy. But if there is a participatory equilibrium, or the guarantee moves the markets to such an equilibrium, then the guarantee may be welfare-enhancing. This suggests a proactive role for regulatory guarantees in settings where ambiguity is likely to be important.

B. Suitability Rules

Earlier we showed that greater participation by ambiguity averse investors results in a lower risk premium. But how can ambiguity averse investors be induced to invest? Because an ambiguity averse investor fears the worst, he might not entrust his assets to a financial advisor or broker/dealer for fear that the broker would simply steal the money, or extract it in other ways (such as excessive commissions or fees generated on excessively risky investments).

Suitability rules provide one solution to this problem. Suitability rules are imposed by all major U.S. securities industry self-regulatory organizations.[16] These rules require financial advisors to select only suitable investments for their clients; failure to do so can result in the advisor having to compensate the investor for all losses. Thus, while the investor is still subject

[16] The NYSE's suitability requirement is Rule 405, also known as the "know your customer" rule. The NASD suitability requirements are found in Rule 2310 of the NASD Manual. See also the SEC web site for a discussion of suitability requirements.

to the risk of the underlying investment, suitability rules are designed to rule out losses due to broker misbehavior, essentially raising $\overline{r}^{i}_{\min}$. Note that by having these rules in force, brokers are much less likely to act in unsuitable ways, and so the rules themselves may rarely come into play. Nonetheless, their existence can induce large changes in participation by simply ruling out particular models of returns.

C. Rule 16(b): Insider "Short Swing" Trading Profit Rules

Another risk of concern to investors is posed by insider trading in a stock. An investor may be unwilling to buy stock for fear that insiders will profit at the expense of the investor. A particular concern is that insiders will take advantage of their access to information by engaging in short-term trading in the company's stock. An ambiguity averse investor who is unsure of how returns are generated may consider possible a model of returns in which insiders frequently take advantage of him by engaging in short-term trading, and so may opt not to invest in the stock.

Rule 16(b) of the Securities Exchange Act is designed to remove this ambiguity by requiring an insider (defined as an officer, director, or 10% shareholder) to return any profits made on a trading position held for less than six months.[17] Note that unlike insider trading prohibitions in general, Rule 16(b) does not require showing that the insider actually traded on any specific information; simply earning a profit on a trade closed out within a six month period is enough to trigger the forfeiture rules. But for an ambiguity averse investor, such an over-arching rule provides the exact reassurance needed to rule out a model of returns determined by such behavior. With the insiders precluded from short-term profits, the investor now perceives a higher $\overline{r}^{i}_{\min}$, and so may be willing to participate in the market.

D. Mutual Fund Diversification Restrictions

The examples given above all concern regulations affecting the perceived minimum mean return, $\overline{r}^{i}_{\min}$. As demonstrated in the previous sections, however, an ambiguity averse investor is also greatly influenced by the maximum variance, $s^{i}_{\max}$. An example of a variance-related regulation can be found in the Investment Act of 1940. In particular, among other rules,

[17] See Securities Exchange Act of 1934, 15 USC.

this act limits a mutual fund to holding no more than 5% of its assets in any one issue. This rule necessitates diversification, and this in turn limits the variance of the fund's returns.

In a world with only expected utility maximizing investors, it is hard to see why such a rule is necessary; investors seeking to reduce their risk could simply hold larger proportions of risk-free assets in their portfolios, and thereby reach their desired risk-return trade-off. But in a world with ambiguity averse investors this may not occur. The AA investor who does not know how the fund's portfolio is formed, may consider various models of how the return to investment in a mutual fund is determined. If, in one of these models, the mutual fund places all his investment in a very risky asset, the AA investor may choose not to invest in the fund. This can occur even if, in most worlds the investor considers possible, investing in the fund is advantageous. Thus, diversification cannot play the traditional role envisioned in standard asset pricing models because AA traders will simply not hold the risky asset in his portfolio.

This distinction in behavior highlights an important difference between hedge funds and mutual funds, and their appropriate regulation. Unlike mutual funds, hedge funds do not have restrictions on holding size, and consequently are free to take on virtually any level of risk. While an EU investor may choose to hold such a fund, an AA investor is unlikely to invest in a hedge fund . Thus, mutual fund diversification requirements will promote a separating equilibrium in which only EU investors will invest in hedge funds, while both EU and ambiguity averse investors will be found holding mutual funds. Such an outcome seems desirable in that it induces investors to participate in mutual funds, but does not limit investments in more speculative offerings.

E. Unlisted Securities Regulation

The examples given above demonstrate how regulations that change the perceived minimum mean returns and maximum variances can induce greater participation by investors, thereby changing the resulting equilibrium in the market. Such a proactive role for regulation is consistent with traditional arguments for regulatory action, although to our knowledge our paper is the first to demonstrate a role for the regulation of unlikely events. The presence of ambiguity aversion, however, suggests another regulatory approach, one that purposely exploits the existence of ambiguity to restrict participation in risky assets. Such an approach can provide investor protection at virtually no explicit cost.

To see how this would work, consider an asset in which at least some investors face difficulty in determining the mean and variance of returns. Such difficulties could arise from inexperience on the part of investors, or it may reflect a more fundamental problem such as a paucity of information about the underlying asset. In our model, depending on the structure of regulations, two alternative equilibria could arise. If the ambiguity is large enough, a non-participating equilibrium will occur in which the AA investors hold none of the asset. Alternatively, to try and reach a participatory equilibrium, the regulators could require extensive disclosure, impose various limitations on the assets' issuers, mandate various educational initiatives and the like, all in an attempt to reduce the underlying ambiguity.

Which equilibrium is optimal? We would argue that the answer depends, in part, on the cost of these regulatory initiatives. For high enough cost levels, the reduction in the equilibrium equity premium may be overwhelmed by the expense needed to induce participation. In this case, the optimal regulatory approach would be to retain the ambiguity and protect investors through their non-participation.

We suggest that just such an outcome may explain the paradoxical world of unlisted securities regulation. An equity security listed and traded on a U.S. exchange or on the Nasdaq faces a plethora of SEC regulations, as well as listing requirements, disclosure rules and even corporate governance restrictions imposed by the exchange. Yet such listed firms are also amongst the most actively followed by analysts, investment banks, ratings agencies, the financial press, and even internet chat rooms. In contrast, unlisted securities (generally defined as securities which trade on the Pink Sheets or on the OTCBB) face much lower regulatory requirements. The SEC requires substantially less disclosure, and there are few, if any, requirements imposed by their trading locales. Moreover, since unlisted firms tend to be small and tightly-held, there is often little information available on these firms from other sources.[18] Thus, securities regulation appears to be upside down: firms with the most information face the largest regulatory scrutiny, while firms with the least information face the least.

Such a regulatory structure makes sense, however, in a world in which ambiguity aversion plays a role. To induce participation by ambiguity averse investors in markets for unlisted stocks, the regulators would have to require vast amounts of information, and even then

[18] Even internet chat sites often restrict discussion of such stocks. Motley Fool, for example, prohibits chat rooms for stocks with prices below $5.00, a price level much more typical of unlisted stocks. For more discussion of unlisted stocks see Macey, O'Hara, and Pompilio [2003].

such securities would remain very risky investments. It may be far better to foster participation in stocks with greater information, and leave the ambiguity to restrict participation in more unknown ventures.

5. Generalizations and Extensions: Market Forces and Ambiguity

Our analysis suggests that regulation can play an important role when at least some traders exhibit ambiguity aversion. Such a pro-active role for direct interference in markets is not generally advocated in the finance literature, where the ameliorative role of market forces is often presumed sufficient to allow optimal outcomes to arise. As we discuss in this section, however, when some traders care both about risk and ambiguity, standard market solutions such as contracting, disclosure, or even arbitrage cannot operate as they do in the more standard paradigm. Consequently, the market cannot "solve" the problems induced by ambiguity aversion, underscoring the robustness of our results.

Consider first the role of contracting. Why can't rational traders simply contract between themselves to rule out undesirable outcomes? Certainly, for many specific issues, they can. But the problems here arise from unlikely outcomes, or events that occur with very low probability. Contracting is costly, and writing contracts to cover every possible state of the world, let alone every possible model generating such states, is prohibitively expensive. This general difficulty led to the legal concept of fiduciary duties, which essentially "fill in" the holes in contracts by requiring agents to act in particular ways when unspecified events occur.[19] Yet, fiduciary duties are themselves subject to ambiguity, and this problem is particular severe when the events in question are improbable in nature.[20] Thus, for an ambiguity averse agent, the inability to contract on every possible state of the world amplifies the ambiguity, rather than resolves it.

A related difficulty attaches to the enforcement of optimal contracts. Even if complete contracting is feasible, enforcement of optimal contracts is not automatic. Tort enforcement is typically a civil action, and so enforcement relies upon bringing legal action for breach of contract. The ambiguity attached to such a legal process, both in terms of timing and outcome, may be enough to induce non-participation. The greater enforcement of laws versus contracts is

[19] In particular, an agent owes a principal the duties of care, trust, and loyalty. For an insightful analysis of the economic role of fiduciary duties see Hart [1994].
[20] This problem is exemplified by the role of fiduciary duties in banking. As discussed by Macey and O'Hara [2003], the courts have vacillated in their view towards the duty of care of bank directors, with a greater standard enforced when instability problems in banking are high.

a particular benefit when ambiguity aversion is a factor, and it underscores why laws may succeed where contracts would fail.

Alternatively, disclosure is often viewed as a market solution when investors face uncertain outcomes. When traders care simply about risk, providing them information on the various potential outcomes can ameliorate many problems by allowing agents to quantify correctly the risks they face. But when they also care about ambiguity, disclosure can actually exacerbate difficulties by making investors aware of obscure outcomes that they otherwise might not have thought possible. For ambiguity averse traders, it is the possibility of outcome, and not its expected occurrence, that is of greater relevance. Disclosure of risks may thus have the opposite effect of inducing non-participation of investors, rather participation.

Of course, the specific effects of disclosure may depend upon whether the market is in a participatory or non-participatory equilibrium. If AA investors are not participating, then disclosing the panoply of potential risks has no effect on these traders. However, by reducing the perceived risk faced by EU agents, disclosure could reduce the equilibrium risk premium, thereby exhibiting the positive role often ascribed to disclosure remedies. The more interesting case is when AA investors are participating, but shift to non-participation upon learning of outcomes heretofore deemed infeasible. Now disclosure induces non-participation in the market and increases the equilibrium risk premium. Consequently, disclosure can exacerbate, rather than resolve, the difficulties induced by ambiguity aversion.

Finally, the role of arbitrage is generally viewed as a panacea for a wide variety of market difficulties. Certainly, if some agents act irrationally, then rational traders can profit from their mistakes, and in the process push markets to the correct equilibrium. In our model, however, all traders are rational. Problems arise not because traders chose sub-optimal trades, but rather because some traders choose not to trade at all. The traders remaining in the market will all rationally choose optimal trades, but they cannot completely offset the loss of the non-participating traders: the aggregate risk must still be borne, albeit now by a smaller number of traders. Thus, while arbitrage can ameliorate mis-pricing in markets, it cannot eliminate the problems caused by non-participation. This result highlights another limit to the role of arbitrage in markets.

6. Conclusions

We have a developed a model of asset pricing in which all agents are rational, but their decision-making may incorporate both risk and ambiguity. Our model demonstrates that ambiguity aversion can induce large equilibrium effects by affecting the participation of traders in markets. These participation effects arise not from some presumed ignorance of trading opportunities, or an imposed inability to enter the markets, but rather from the rational decision by some traders to avoid ambiguity. In equilibrium, these participation decisions can affect the equilibrium risk premium, and distort the performance of the market when viewed from the perspective of traditional asset pricing models.

A particularly important result of our analysis is that alternate equilibria can arise when at least some traders are ambiguity averse. This sets the stage for a number of important findings. We show why it is that seemingly irrelevant events can have large effects on markets, providing a decision-making based explanation for why markets appear to "over react". We also show that regulations designed to rule out unlikely outcomes can play a very important role in moving the economy to a preferred equilibrium. This positive role for regulation provides insights into a wide range of issues such as deposit insurance, securities regulation, and the role of guarantees. Perhaps equally important, we show why it is that market forces are not capable of achieving these same outcomes.

Our analysis here suggests that ambiguity aversion may play an important role in explaining asset pricing behavior. We hope to expand on this role in future research, as well as investigate other linkages with the developing literature in law and finance.

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Figure I
Demand Functions for Asset i

This Figure shows the demands (x) for asset i by Expected Utility Investors (EU) and Ambiguity Averse Investors (AA). The prior mean return is given by $\hat{r}^i$ and $r^i_{\min}$ and $\overline{r}^i_{\max}$ are the minimum and maximum mean returns perceived by ambiguity averse investors.

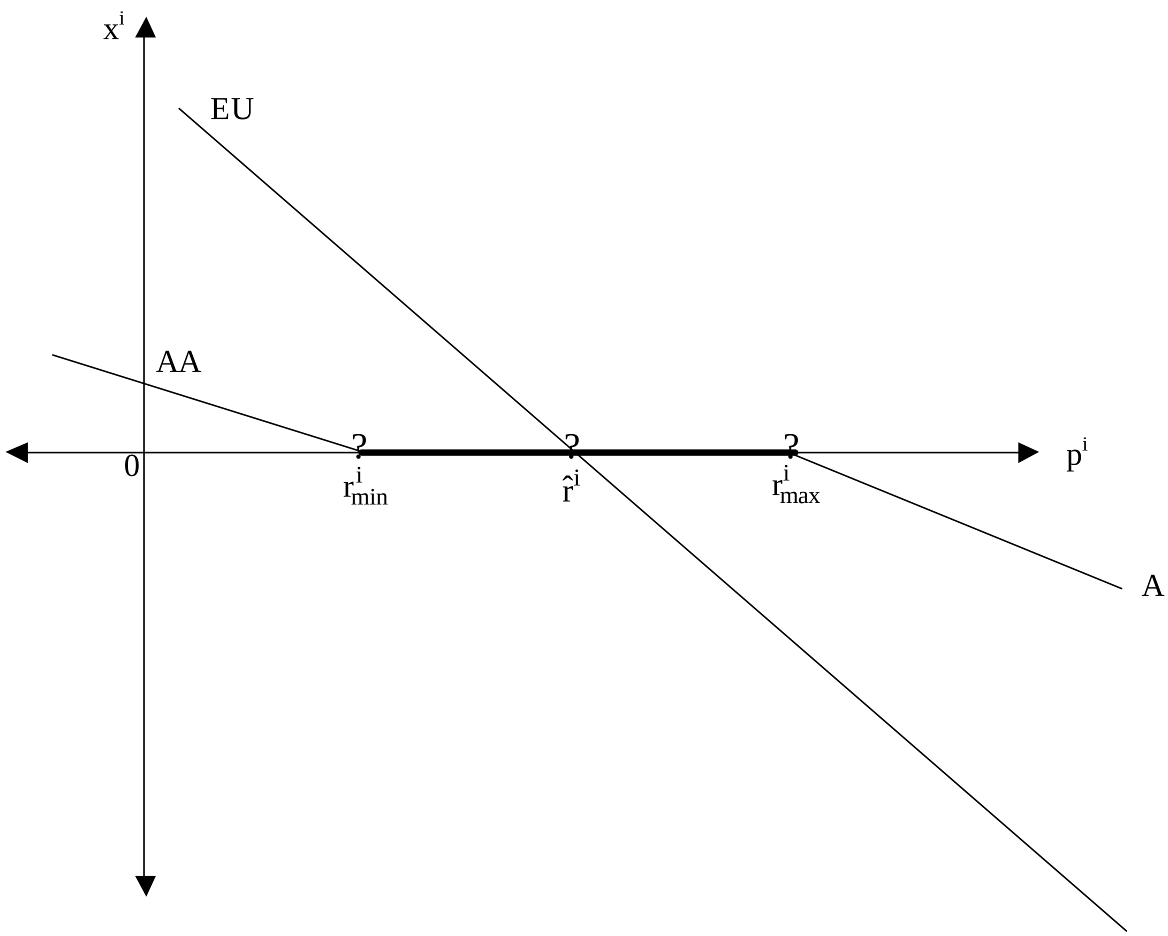

Figure II
Equilibrium Price of Asset i
(Holding $\hat{r}^i$ constant)

This Figure demonstrates the equilibrium price as a function of the perceived minimum mean return. In the non-participating equilibrium, AA investors hold none of the asset. The kink occurs at the switch point between the non- participating equilibrium and the participating equilibrium. In the participating equilibrium, increases in the perceived minimum return induce greater holdings by the AA investors, causing prices to increase.

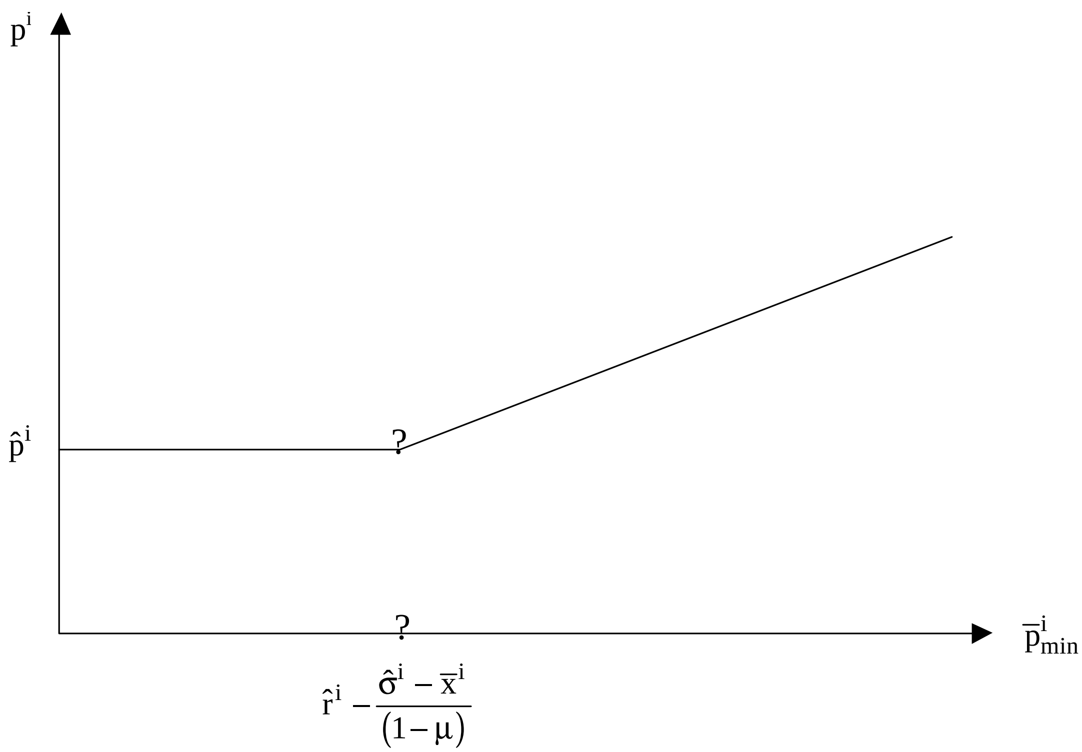

Table I
Comparative Statics Results

This Table shows the effect on the equilibrium price of changing the fraction of ambiguity averse investor (μ), the maximum variance (σ^i_{max}), and the minimum mean return (r^i_{min}).

Variable	**Price Effect**
$\mu \uparrow$	$p^i \downarrow$
$\sigma^i_{max} \uparrow$	$p^i \downarrow$
$r^i_{min} \uparrow$	$p^i \uparrow$