Introduction

- A contingent claim’s price process can be transformed into a *martingale* process by
  1. Adjusting its Brownian motion by the market price of risk.
  2. Deflating by a riskless asset price.
- The claim’s value equals the expectation of the transformed process’s future payoff.
- We derive the continuous-time state price deflator that transforms actual probabilities into risk-neutral probabilities.
- Valuing a contingent claim might be simplified by deflating the contingent claim’s price by that of another risky asset.
- We consider applications: options on assets that pay a continuous dividend; the term structure of interest rates.
Let $S$ be the value of a risky asset that follows a general scalar diffusion process

$$dS = \mu S dt + \sigma S dz$$

(1)

where both $\mu = \mu (S, t)$ and $\sigma = \sigma (S, t)$ may be functions of $S$ and $t$ and $dz$ is a Brownian motion.

Itô’s lemma gives the process for a contingent claim’s price, $c(S, t)$:

$$dc = \mu_c c dt + \sigma_c c dz$$

(2)

where $\mu_c c = c_t + \mu Sc_S + \frac{1}{2} \sigma^2 S^2 c_{SS}$ and $\sigma_c c = \sigma Sc_S$, and the subscripts on $c$ denote partial derivatives.

Consider a hedge portfolio of $-1$ units of the contingent claim and $c_S$ units of the risky asset.
The value of this hedge portfolio, $H$, satisfies

$$H = -c + c_S S$$ \hspace{1cm} (3)$$

and the change in its value over the next instant is

$$dH = -dc + c_S dS$$

$$= -\mu_c c dt - \sigma_c c dz + c_S \mu S dt + c_S \sigma S dz$$

$$= \left[c_S \mu S - \mu_c c\right] dt$$ \hspace{1cm} (4)$$

In the absence of arbitrage, the riskless portfolio change must be $H(t)r(t)dt$:

$$dH = \left[c_S \mu S - \mu_c c\right] dt = rH dt = r[-c + c_S S] dt$$ \hspace{1cm} (5)$$
Arbitrage and Martingales cont’d

- This no-arbitrage condition for $dH$ implies:

$$c_S \mu S - \mu_c c = r[-c + c_SS] \quad (6)$$

- Substituting $\mu_c c = c_t + \mu Sc_S + \frac{1}{2} \sigma^2 S^2 c_{SS}$ into (6) leads to the Black-Scholes equation:

$$\frac{1}{2} \sigma^2 S^2 c_{SS} + rSc_S - rc + c_t = 0 \quad (7)$$

- However, a different interpretation of (6) results from substituting $c_S = \frac{\sigma_c c}{\sigma S}$ (from $\sigma_c c = \sigma Sc_S$):

$$\frac{\mu - r}{\sigma} = \frac{\mu_c - r}{\sigma_c} \equiv \theta(t) \quad (8)$$

- No-arbitrage condition (8) requires a unique market price of risk, say $\theta(t)$, so that $\mu_c = r + \sigma_c \theta(t)$. 
A Change in Probability

- Substituting for $\mu_c$ in (2) gives

$$ dc = \mu_c c dt + \sigma_c c dz = [rc + \theta \sigma_c c] dt + \sigma_c c dz \quad (9) $$

- Next, consider a new process $\hat{z}_t = z_t + \int_0^t \theta (s) ds$, so that $d\hat{z}_t = dz_t + \theta (t) dt$.

- Then substituting $dz_t = d\hat{z}_t - \theta (t) dt$ in (9):

$$ dc = [rc + \theta \sigma_c c] dt + \sigma_c c [d\hat{z} - \theta dt] $$

$$ = r c dt + \sigma_c c d\hat{z} \quad (10) $$

- If $\hat{z}_t$ were a Brownian motion, future values of $c$ generated by $d\hat{z}$ occur under the $Q$ or “risk-neutral” probability measure.

- The actual or “physical” distribution, $P$, is generated by the $dz$ Brownian motion.
Girsanov’s Theorem

- Let $dP_T$ be the instantaneous change in the cumulative distribution at date $T$ generated by $dz_t$ (the physical pdf).
- $dQ_T$ is the analogous risk-neutral pdf generated by $d\hat{z}_t$.
- Girsanov’s theorem says that at date $t < T$, the two probability densities satisfy

$$
dQ_T = \exp \left[ - \int_t^T \theta(u) \, dz - \frac{1}{2} \int_t^T \theta(u)^2 \, du \right] \, dP_T
$$

$$
= \left( \frac{\xi_T}{\xi_t} \right) \, dP_T
$$

(11)

where $\xi_t$ is a positive random process depending on $\theta(t)$ and $z_t$:

$$
\xi_T = \exp \left[ - \int_0^T \theta(u) \, dz - \frac{1}{2} \int_0^T \theta(u)^2 \, du \right]
$$

(12)
Girsanov’s Theorem cont’d

- Thus, multiplying the physical pdf at $T$ by $\xi_T/\xi_t$ leads to the risk-neutral pdf at $T$.
- Since $\xi_T/\xi_t > 0$, equation (11) implies that whenever $dP_T$ has positive probability, so does $dQ_T$, making them equivalent measures.
- Rearranging (11) gives the Radon-Nikodym derivative of $Q$ wrt $P$:
  \[ \frac{dQ_T}{dP_T} = \xi_T/\xi_t \]  \hspace{1cm} (13)
  Later we will relate this derivative to the continuous-time pricing kernel.
Money Market Deflator

- Let $B(t)$ be the value of an instantaneous-maturity riskless “money market fund” investment:

$$dB/B = r(t)dt$$  \hspace{1cm} (14)

- Note that $B(T) = B(t) e^\int_t^T r(u)du$ for any date $T \geq t$.

- Now define $C(t) \equiv c(t)/B(t)$ as the deflated price process for the contingent claim and use Itô’s lemma:

$$dC = \frac{1}{B} dc - \frac{c}{B^2} dB$$

$$= \frac{rc}{B} dt + \frac{\sigma_c c}{B} d\hat{z} - r \frac{c}{B} dt$$

$$= \sigma_c C d\hat{z}$$

since $dc dB = 0$ and we substitute for $dc$ from (10).
Money Market Deflator cont’d

- An implication of (15) is

$$C(t) = \hat{E}_t[C(T)] \quad \forall T \geq t$$

(16)

where $\hat{E}_t[\cdot]$ denotes the expectation operator under the probability measure generated by $d\hat{Z}$.

- Thus, $C(t)$ is a martingale (random walk) process.

- Note that (16) holds for any deflated non-dividend-paying contingent claim, including $C = \frac{S}{B}$.

- Later, we will consider assets that pay dividends.
Feynman-Kac Solution

- Rewrite (16) in terms of the undeflated contingent claims price:

\[
\begin{align*}
    c(t) &= B(t) \hat{E}_t \left[ c(T) \frac{1}{B(T)} \right] \\
    &= \hat{E}_t \left[ e^{-\int_t^T \theta(u)du} c(T) \right]
\end{align*}
\]

Equation (17) is the “Feynman-Kac” solution to the Black-Scholes PDE and does not require knowledge of \( \theta(t) \).

- This is the continuous-time formulation of risk-neutral pricing: risk-neutral (or \( Q \) measure) expected payoffs are discounted by the risk-free rate.
Arbitrage and Pricing Kernels

- Recall from the single- or multi-period consumption-portfolio choice problem with time-separable utility:

\[
c(t) = E_t [m_{t,T}c(T)] = E_t \left[ \frac{M_T}{M_t} c(T) \right]
\]  (18)

where date \( T \geq t \), \( m_{t,T} \equiv M_T/M_t \) and \( M_t = U_c(C_t, t) \).

- Rewriting (18):

\[
c(t) M_t = E_t [c(T) M_T]
\]  (19)

which says that the deflated price process, \( c(t) M_t \), is a martingale under \( P \) (not \( Q \)).
Assume that the state price deflator, $M_t$, follows a strictly positive diffusion process of the general form

$$dM_t = \mu_m dt + \sigma_m dz$$

(20)

Define $c^m = cM$ and apply Itô’s lemma:

$$dc^m = cdM + Mdc + (dc)(dM)$$

(21)

$$= [c\mu_m + M\mu_c c + \sigma_c c\sigma_m] dt + [c\sigma_m + M\sigma_c c] dz$$

If $c^m = cM$ satisfies (19), that is, $c^m$ is a martingale, then its drift in (21) must be zero, implying

$$\mu_c = -\frac{\mu_m}{M} - \frac{\sigma_c \sigma_m}{M}$$

(22)
Arbitrage and Pricing Kernels cont’d

- Consider the case in which $c$ is the instantaneously riskless investment $B(t)$; that is, $dc(t) = dB(t) = r(t)Bdt$ so that $\sigma_c = 0$ and $\mu_c = r(t)$.
- From (22), this requires

$$r(t) = -\frac{\mu_m}{M} \quad (23)$$

- Thus, the expected rate of change of the pricing kernel must equal minus the instantaneous risk-free interest rate.
- Next, consider the general case where the asset $c$ is risky, so that $\sigma_c \neq 0$. Using (22) and (23) together, we obtain

$$\mu_c = r(t) - \frac{\sigma_c \sigma_m}{M} \quad (24)$$

or
Comparing (25) to (8), we see that

\[ -\frac{\sigma_m}{M} = \theta(t) \]  

(26)

Thus, the no-arbitrage condition implies that the form of the pricing kernel must be

\[ \frac{\mu_c - r}{\sigma_c} = -\frac{\sigma_m}{M} \]  

(25)

\[ dM/M = -r(t)\,dt - \theta(t)\,dz \]  

(27)
Arbitrage and Pricing Kernels cont’d

- Define $m_t \equiv \ln M_t$ so that $dm = -[r + \frac{1}{2}\theta^2]dt - \theta dz$.
- We can rewrite (18) as

$$c(t) = E_t [c(T) M_T / M_t] = E_t [c(T) e^{m_T - m_t}] \quad (28)$$

$$= E_t [c(T) e^{-\int_t^T \left[r(u) + \frac{1}{2}\theta^2(u)\right]du - \int_t^T \theta(u)dz}]$$

- Since the price under the money-market deflator ($Q$ measure) and the SDF ($P$ measure) must be the same, equating (17) and (28) implies

$$\hat{E}_t \left[e^{-\int_t^T r(u)du} c(T)\right] = E_t [c(T) M_T / M_t] \quad (29)$$

$$= E_t \left[e^{-\int_t^T r(u)du} c(T) e^{-\int_t^T \frac{1}{2}\theta^2(u)du - \int_t^T \theta(u)dz}\right]$$
Linking Valuation Methods

- Substituting the definition of $\xi_T$ from (12) leads to

$$
\hat{E}_t \left[ e^{\int_t^T r(u)du} c(T) \right] = E_t \left[ e^{\int_t^T r(u)du} c(T) (\xi_T/\xi_t) \right]
$$

$$
\hat{E}_t [C(T)] = E_t [C(T) (\xi_T/\xi_t)]
$$

$$
\int C(T) dQ_T = \int C(T) (\xi_T/\xi_t) dP_T
$$

where $C(t) = c(t)/B(t)$. Thus, relating (29) to (30):

$$
M_T/M_t = e^{\int_t^T r(u)du} (\xi_T/\xi_t)
$$

- Hence, $M_T/M_t$ provides both discounting at the risk-free rate and transforming the probability distribution to the risk-neutral one via $\xi_T/\xi_t$. 

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Arbitrage, Martingales, and Pricing Kernels
Multivariate Case

- Consider a multivariate extension where asset returns depend on an $n \times 1$ vector of independent Brownian motion processes, $d\mathbf{Z} = (dz_1 \ldots dz_n)'$ where $dz_i \, dz_j = 0$ for $i \neq j$.
- A contingent claim whose payoff depended on these asset returns has the price process

$$dc/c = \mu_c \, dt + \Sigma_c \, d\mathbf{Z}$$

(32)

where $\Sigma_c$ is a $1 \times n$ vector $\Sigma_c = (\sigma_{c1} \ldots \sigma_{cn})$.
- Let the corresponding $n \times 1$ vector of market prices of risks associated with each of the Brownian motions be $\Theta = (\theta_1 \ldots \theta_n)'$. 
Then the no-arbitrage condition (the multivariate equivalent of (8)) is

$$\mu_c - r = \sum_c \Theta$$  \hspace{1cm} (33)

Equations (16) and (17) would still hold, and now the pricing kernel’s process would be given by

$$dM/M = -r(t) \, dt - \Theta(t)' \, dZ$$  \hspace{1cm} (34)
Consider an option written on the difference between two securities’ (stocks’) prices. The date $t$ price of stock 1, $S_1(t)$, follows the process

$$dS_1/S_1 = \mu_1 dt + \sigma_1 dz_1 \quad (35)$$

and the date $t$ price of stock 2, $S_2(t)$, follows the process

$$dS_2/S_2 = \mu_2 dt + \sigma_2 dz_2 \quad (36)$$

where $\sigma_1$ and $\sigma_2$ are assumed to be constants and $dz_1 dz_2 = \rho dt$.

Let $C(t)$ be the date $t$ price of a European option written on the difference between these two stocks’ prices.
Alternative Price Deflators cont’d

- At this option’s maturity date, $T$, its value equals
  \[ C(T) = \max[0, S_1(T) - S_2(T)] \]  \hspace{1cm} (37)

- Now define $c(t) = C(t)/S_2(t)$, $s(t) ≡ S_1(t)/S_2(t)$, and $B(t) = S_2(t)/S_2(t) = 1$ as the deflated price processes, where the prices of the option, stock 1, and stock 2 are all normalized by the price of stock 2.

- Under this normalized price system, the payoff (37) is
  \[ c(T) = \max[0, s(T) - 1] \]  \hspace{1cm} (38)

- Applying Itô’s lemma, the process for $s(t)$ is
  \[ ds/s = \mu_s dt + \sigma_s dz_3 \]  \hspace{1cm} (39)
Alternative Price Deflators cont’d

- Here \( \mu_s \equiv \mu_1 - \mu_2 + \sigma_2^2 - \rho \sigma_1 \sigma_2 \), \( \sigma_s dz_3 \equiv \sigma_1 dz_1 - \sigma_2 dz_2 \), and \( \sigma_s^2 = \sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2 \).

- Further, when prices are measured in terms of stock 2, the deflated price of stock 2 becomes the riskless asset with \( dB/B = 0dt \) (the deflated price never changes).

- Using Itô’s lemma on \( c \),

\[
dc = \left[ c_s \mu_s s + c_t + \frac{1}{2} c_{ss} \sigma_s^2 s^2 \right] dt + c_s \sigma_s s \, dz_3 \quad (40)
\]

- The familiar Black-Scholes hedge portfolio can be created from the option and stock 1. The portfolio’s value is

\[
H = -c + c_s s \quad (41)
\]
Alternative Price Deflators cont’d

- The instantaneous change in value of the portfolio is

\[
\begin{align*}
\frac{dH}{dt} &= -dc + c_s ds \\
&= - \left[ c_s \mu_s s + c_t + \frac{1}{2} c_{ss} \sigma_s^2 s^2 \right] dt - c_s \sigma_s s dz_3 \\
&\quad + c_s \mu_s s dt + c_s \sigma_s s dz_3 \\
&= - \left[ c_t + \frac{1}{2} c_{ss} \sigma_s^2 s^2 \right] dt
\end{align*}
\]

which is riskless and must earn the riskless return \( dB/B = 0 \):

\[
\begin{align*}
\frac{dH}{dt} &= - \left[ c_t + \frac{1}{2} c_{ss} \sigma_s^2 s^2 \right] dt = 0 \\
\end{align*}
\]

which implies

\[
c_t + \frac{1}{2} c_{ss} \sigma_s^2 s^2 = 0
\]
Alternative Price Deflators cont’d

- This is the Black-Scholes PDE with the risk-free rate, \( r \), set to zero. With boundary condition (38), the solution is

\[
c(s, t) = s \, N(d_1) - N(d_2)
\]  

(45)

where

\[
d_1 = \frac{\ln(s(t)) + \frac{1}{2} \sigma_s^2 (T - t)}{\sigma_s \sqrt{T - t}}
\]  

(46)

\[
d_2 = d_1 - \sigma_s \sqrt{T - t}
\]

- Multiply by \( S_2(t) \) to convert back to the undeflated price system:

\[
C(t) = S_1 \, N(d_1) - S_2 \, N(d_2)
\]  

(47)

- \( C(t) \) does not depend on \( r(t) \), so that this formula holds even for stochastic interest rates.
Continuous Dividends

- Let $S(t)$ be the date $t$ price per share of an asset that continuously pays a dividend of $\delta S(t)$ per unit time. Thus,

$$dS = (\mu - \delta) Sdt + \sigma Sdz$$

where $\sigma$ and $\delta$ are assumed to be constants.

- Note that the asset’s total rate of return is $dS/S + \delta dt = \mu dt + \sigma dz$, so that $\mu$ is its instantaneous expected rate of return.

- Consider a European call option written on this asset with exercise price of $X$ and maturity date of $T > t$, where we define $\tau \equiv T - t$.

- Let $r$ be the constant risk-free interest rate.
Continuous Dividends cont’d

- Based on (17), the date $t$ price of this option is
  \[
  c(t) = \hat{E}_t [e^{-rT} c(T)] = e^{-rT} \hat{E}_t [\max [S(T) - X, 0]]
  \]  
  (49)

- As in (10), convert from the physical measure generated by $dz$ to the risk-neutral measure generated by $d\hat{z}$, which removes the risk premium from the asset’s expected rate of return so that:
  \[
  dS = (r - \delta) Sdt + \sigma Sd\hat{z}
  \]  
  (50)

- Since $r - \delta$ and $\sigma$ are constants, $S$ is a geometric Brownian motion process and is lognormally distributed under $Q$. 
Continuous Dividends cont’d

Thus, the risk-neutral distribution of $\ln[S(T)]$ is normal:

$$\ln[S(T)] \sim N\left(\ln[S(t)] + (r - \delta - \frac{1}{2}\sigma^2)\tau, \sigma^2\tau\right) \quad (51)$$

Equation (49) can now be computed as

$$c(t) = e^{-r\tau} \hat{E}_t \left[ \max[S(T) - X, 0] \right] \quad (52)$$

$$= e^{-r\tau} \int_{X}^{\infty} (S(T) - X) g(S(T)) \, dS(T)$$

where $g(S_T)$ is the lognormal probability density function.

Consider the change in variable

$$Y = \frac{\ln[S(T)/S(t)] - (r - \delta - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \quad (53)$$
Continuous Dividends cont’d

- \( Y \sim \mathcal{N}(0, 1) \) and allows (52) to be evaluated as

\[
c = Se^{-\delta \tau} N(d_1) - Xe^{-r \tau} N(d_2)
\]

where

\[
d_1 = \frac{\ln (S/X) + (r - \delta + \frac{1}{2} \sigma^2) \tau}{\sigma \sqrt{\tau}}
\]

\[
d_2 = d_1 - \sigma \sqrt{\tau}
\]

- If contingent claims have more complex payoffs or the underlying asset has a more complex risk-neutral process, a numeric solution to \( c(t) = \hat{E}_t [e^{-r \tau} c(S(T))] \) can be obtained, perhaps by Monte Carlo simulation.
Continuous Dividends cont’d

- Compared to an option written on an asset that pays no dividends, the non-dividend-paying asset’s price, $S(t)$, is replaced with the dividend-discounted price of the dividend-paying asset, $S(t) e^{-\delta \tau}$ (to keep the total expected rate of return at $r$).

- Thus, the risk-neutral expectation of $S(T)$ is

\[
\hat{E}_t [S(T)] = S(t) e^{(r-\delta)\tau}
\]

\[
= S(t) e^{-\delta \tau} e^{r \tau} = \overline{S}(t) e^{r \tau}
\]

where we define $\overline{S}(t) \equiv S(t) e^{-\delta \tau}$. 
Foreign Currency Options

- Define $S(t)$ as the domestic currency value of a unit of foreign currency (spot exchange rate).
- Purchase of a foreign currency allows the owner to invest at the risk-free foreign currency interest rate, $r_f$.
- Thus the dividend yield will equal this foreign currency rate, $\delta = r_f$ and $\hat{E}_t [S(T)] = S(t) e^{(r-r_f)\tau}$.
- This expression is the no-arbitrage value of the date $t$ forward exchange rate having a time until maturity of $\tau$, that is, $F_{t,\tau} = Se^{(r-r_f)\tau}$.
- Therefore, a European option on foreign exchange is

$$c(t) = e^{-r\tau} [F_{t,\tau}N(d_1) - XN(d_2)]$$  \hspace{1cm} (57)

where $d_1 = \frac{\ln[F_{t,\tau}/X] + \sigma^2\tau}{\sigma\sqrt{\tau}}$, and $d_2 = d_1 - \sigma\sqrt{\tau}$. 
Options on Futures

- Consider an option written on a futures price $F_{t,t^*}$, the date $t$ futures price for a contract maturing at date $t^*$.

- The undiscounted profit (loss) earned by the long (short) party over the period from date $t$ to date $T \leq t^*$ is simply $F_{T,t^*} - F_{t,t^*}$.

- Like forward contracts, there is no initial cost for the parties who enter into a futures contract. Hence, in a risk-neutral world, their expected profits must be zero:

$$\hat{E}_t [F_{T,t^*} - F_{t,t^*}] = 0$$  \hspace{1cm} (58)

so under the $Q$ measure, the futures price is a martingale:

$$\hat{E}_t [F_{T,t^*}] = F_{t,t^*}$$  \hspace{1cm} (59)
Options on Futures cont’d

- Since an asset’s expected return under $Q$ must be $r$, a futures price is like the price of an asset with a dividend yield of $\delta = r$.
- The value of a futures call option that matures in $\tau$ periods where $\tau \leq (t^* - t)$ is

$$c(t) = e^{-r\tau} \left[ F_{t,t^*} N(d_1) - X N(d_2) \right]$$

(60)

where $d_1 = \frac{\ln[F_{t,t^*}/X] + \frac{\sigma^2}{2} \tau}{\sigma \sqrt{\tau}}$, and $d_2 = d_1 - \sigma \sqrt{\tau}$.
- Note that this is similar in form to an option on a foreign currency written in terms of the forward exchange rate.
Term Structure Revisited

- Let $P(t, \tau)$ be the date $t$ price of a default-free bond paying $1$ at maturity $T = t + \tau$.
- Interpreting $c(T) = P(T, 0) = 1$, equation (17) is
  \[ P(t, \tau) = \mathbb{E}_t \left[ e^{-\int_t^T r(u) du} 1 \right] \]  
  (61)
- We now rederive the Vasicek (1977) model using this equation where recall that the physical process for $r(t)$ is
  \[ dr(t) = \alpha [\bar{r} - r(t)] dt + \sigma_r dz_r \]  
  (62)
- Assuming, like before, that the market price of bond risk $q$ is a constant,
  \[ \mu_p(r, \tau) = r(t) + q \sigma_p(\tau) \]  
  (63)
  where $\sigma_p(\tau) = -P_r \sigma_r / P$. 
Thus, recall that the physical process for a bond’s price is

\[
dP(r, \tau) / P(r, \tau) = \mu_p(r, \tau) \, dt - \sigma_p(\tau) \, dz_r\tag{64}
\]

\[
= [r(t) + q\sigma_p(\tau)] \, dt - \sigma_p(\tau) \, dz_r
\]

Defining \(d\hat{Z}_r = dz_r - qdt\), equation (64) becomes

\[
dP(t, \tau) / P(t, \tau) = [r(t) + q\sigma_p(\tau)] \, dt - \sigma_p(\tau) [d\hat{Z}_r + qdt]
\]

\[
= r(t) \, dt - \sigma_p(\tau) \, d\hat{Z}_r \tag{65}
\]

which is the risk-neutral process for the bond price since all bonds have the expected rate of return \(r\) under the \(Q\) measure.
Therefore, the process for $r(t)$ under the $Q$ measure is found by also substituting $d\hat{z}_r = dz_r - q dt$:

$$dr(t) = \alpha \left[ \bar{r} - r(t) \right] dt + \sigma_r \left[ d\hat{z}_r + q dt \right]$$

$$= \alpha \left[ \left( \frac{q \sigma_r}{\alpha} \right) - r(t) \right] dt + \sigma_r d\hat{z}_r \quad (66)$$

which has the unconditional mean $\bar{r} + q \sigma_r / \alpha$.

Thus, when evaluating equation (61)

$$P(t, \tau) = \hat{E}_t \left[ \exp \left( - \int_t^\tau r(u) \, du \right) \right]$$

this expectation is computed assuming $r(t)$ follows the process in (66).

Doing so leads to the same solution given in the previous chapter, equation (9.41) in the text.
Summary

- Martingale pricing is a generalization of risk-neutral pricing that is applicable in complete markets.
- With dynamically complete markets, the continuous-time state price deflator has an expected growth rate equal to minus the risk-free rate and a standard deviation equal to the market price of risk.
- Contingent claims valuation often can be simplified by an appropriate normalization of asset prices, deflating either by the price of a riskless or risky asset.
- Martingale pricing can be applied to options written on assets paying continuous, proportional dividends, as well as default-free bonds.