Contingent Capital: The Case of COERCs

by

George Pennacchi,¹ Theo Vermaelen,² and Christian C.P. Wolff³

April 2014

We are grateful for valuable comments from Sanjiv Das, Douglas Diamond, Bernard Dumas, Denis Gromb, Mark Flannery, Pekka Hietala, George Hübner, Diny de Jong, Pascal Maenhout, Paul Malatesta (the Editor), Stephan Loesh, Hamid Mehran, Yves Nosbusch, Julian Presber, Suresh Sundaresan, Josef Zechner, anonymous referees, and participants of the 2012 European Financial Management Association Meetings, the 2012 Chulalongkorn Accounting and Finance Symposium, the 2013 American Finance Association Meetings, and seminars at the University of Chicago Booth School of Business, the Federal Deposit Insurance Corporation, the University of Lancaster, the Brussels Finance Club, the Tinbergen Institute, and INSEAD. We thank Dennis Ding and Fanou Rasmouki for research assistance.

¹University of Illinois at Urbana-Champaign, Email: gpennacc@illinois.edu, Phone: 1-217-244-0952 ² INSEAD, Email: theo.vermaelen@insead.edu, Phone: 33 (0)1 60 72 42 63 ³Luxembourg School of Finance, University of Luxembourg, and CEPR, Email: christian.wolff@uni.lu, Phone: +352 4666446800
Contingent Capital: The Case of COERCs

Abstract
This paper introduces and analyzes a new form of contingent convertible: a Call Option Enhanced Reverse Convertible (COERC). If an issuing bank’s market value of capital breaches a trigger, COERCs convert to many new equity shares that would heavily dilute existing shareholders, except that shareholders have the option to purchase these shares at the bond’s par value. COERCs have low risk: they are almost always fully repaid in cash. Yet, they reduce government bailouts by replenishing a bank’s capital. COERCs’ design also avoids problems with market-based triggers, such as manipulation or panic, while reducing moral hazard and debt overhang.
I. Introduction

The capital levels of many large banks were inadequate to withstand the 2007 to 2009 financial crisis. Threatened by negative externalities from these banks’ failures, several governments intervened to provide guarantees and inject capital. With the goal of avoiding such bailouts in the future, regulators have raised banks’ capital requirements and reconsidered what debt-like instruments should qualify as capital. Supported by academic proposals such as Flannery (2005), some countries have permitted their banks to partially meet higher capital standards by issuing contingent capital, also called contingent convertibles or CoCos. In general terms, CoCos are bonds that convert to equity, or are written off, after some triggering event such as a decline in a bank’s capital below a threshold. CoCos are growing in popularity, with issuance in 2014 projected to be in the range of $75 to $100 billion.¹ However, there is significant variation in the types of CoCos that banks have issued, as well as the types of CoCos that academics and policymakers have proposed that they issue. No consensus has yet emerged on the particular form that CoCos should take.

Our paper contributes to a growing literature on CoCos by a developing, valuing, and analyzing a new form of CoCo that addresses many of the criticisms that practitioners and academics have lodged against standard CoCos. We term this new security a Call Option Enhanced Reverse Convertible or COERC. As we detail below, COERCs differ from other CoCos in three main ways. First, COERCs convert when a bank’s market value of total capital to deposits ratio falls below a pre-specified trigger level. This trigger level is set high enough so that it is highly likely that the bank remains

an adequately-capitalized, going concern following conversion. Second, at conversion a large proportion of new shares are issued to COERC investors which has the potential to heavily dilute the bank’s initial shareholders. Third, this dilution can be avoided because COERCs also provide the initial shareholders with an option to repurchase the newly-issued shares at a price equal to the COERC bonds’ par value.

As we will demonstrate, the combination of these three features allows a bank that suffers losses to extinguish debt and replenish common equity when it is likely to still be financially stable. When this conversion occurs, COERC investors almost always receive their bond’s par value in cash. The threat of heavy dilution coerces the bank’s shareholders to exercise their option to repurchase the newly issued equity shares from the COERC investors. Consequently, COERC bonds are nearly default-free, which makes them easy to value and aligns the incentives of bank shareholders to maximize firm value, rather than shareholder value. Furthermore, the COERC’s embedded option protects shareholders from loss due to premature conversion that might result from stock price manipulation or panic.

We use a model calibrated to market and accounting data from the three largest U.S. banks to price COERCs, standard CoCos, and non-convertible subordinated debt. The model shows that credit spreads for COERCs are relatively low, and as a consequence the shareholders’ equity of banks that issue COERCs has risk characteristics similar to that of a bank with unlimited liability. Moreover, our analysis confirms that a bank that issues COERCs has a smaller moral hazard incentive to increase its asset risk (Jensen and Meckling (1976)) compared to a similar bank that issues CoCos or
nonconvertible subordinated debt. In addition, the “debt overhang” problem (Myers (1977)) is mitigated when the bank issues COERCs.

We also extend the model to consider the possibility that a bank’s stock price, and hence its observed market value of capital, could deviate from the fundamental value of the bank’s underlying assets. Such a deviation might be caused by the manipulation of a speculator or a more widespread “irrational” panic. Critics of CoCos with market value triggers argue that these deviations from fundamentals can lead to a premature triggering of conversion that would impose losses on the bank’s initial shareholders. Our results from this extended model show that COERCs better protect a bank’s shareholders from losses compared to a standard CoCo with a market value trigger.

The next section surveys regulatory issues and related research on CoCos. Section III introduces COERCs with a simplified numerical example. In Section IV a model for valuing COERCs, standard CoCos, and nonconvertible subordinated debt is described, while Section V provides this basic model’s results. Section VI compares COERCs and CoCos in an extension of the model that allows market values to deviate from fundamentals. Section VII discusses further considerations while Section VIII concludes.

II. Background on CoCos and Related Literature

This section briefly discusses the regulatory treatment of CoCos and research most closely related to our paper. See Flannery (2014) for an in-depth review of CoCos.

Flannery (2005) first proposed that banks issue CoCos, and interest in them grew rapidly during the 2007 to 2009 crisis because other debt-like capital, such as nonconvertible subordinated debt, failed to stabilize banks. Traditional subordinated debt was designed to absorb losses ahead of senior creditors, such as insured depositors, but
only after a bank had failed. During the financial crisis, however, regulators were loath to close large banks when many banks became distressed simultaneously. The response of many governments was to inject new capital into banks, with subordinated debtholders obtaining de facto protection. Consequently, post-crisis reforms to capital standards disqualified many traditional subordinated debt instruments as counting toward regulatory capital.

There are two general contract features that define CoCos. The first is the specific set of events that triggers when a CoCo is converted. The second is the specific terms of the payoff received by the CoCo investors at the time of conversion. We first discuss the triggering events and follow that with a review of conversion terms.

The new Basel III international capital standards restrict the triggering event for a CoCo to qualify as capital. The Basel Committee on Banking Supervision (BCBS) (2011) requires that for any non-common equity capital to qualify as Tier 1 or 2 regulatory capital, it must convert to common equity or be written off when supervisors determine that the bank has reached the “point of non-viability” (PONV). The PONV is defined as the time when a bank is unable to support itself and a government resolution or public capital injection is imminent; that is, the bank is a “gone-concern.” In addition, for a CoCo to count as Additional Tier 1 capital, such “going-concern” CoCos must convert when the ratio of Tier 1 common equity to risk-weighted assets (CE/RWA) falls to a triggering level not less than 5.125%. However, CoCos can count for no more than 1.5% of Additional Tier 1 capital, and they cannot fulfil the additional capital required of the

---

2 “In the recent crisis existing subordinated debt and hybrid capital largely failed in its original objective of bearing losses” (HM (UK) Treasury (2009)).
largest, “globally systemically important” banks (G-SIBs). The BCBS continues to review CoCos and does support their use to meet higher national capital requirements.³ At the national level, Switzerland has taken the lead by requiring its two major banks, UBS and Credit Suisse, to raise their capital ratios to 19%, with up to 9% being met with CoCos. To date, most banks issuing going-concern CoCos, including Credit Suisse, UBS, and Barclays, set triggers equal to CE/RWA in the range of 5.125% to 7%.

In contrast to the regulatory capital triggers specified in Basel III and used in CoCos issued to date, most academic proposals for CoCos envision a market-based conversion trigger, such as the bank’s stock price or its market value of capital ratio.⁴ The choice between a regulatory capital ratio trigger versus a market-based capital ratio trigger is important for whether conversion is triggered during a period of stress. Haldane (2011) documents that regulatory capital ratios, in contrast to market capital ratios, failed to forecast the financial crisis. Figures 1.A and 1.B (Charts 5 and 7 from Haldane (2011)) graph the average Tier 1 capital ratios and the average market capital to book value of debt ratios, respectively, for two groups of major financial institutions: 15 “crisis” banks that in the autumn of 2008 failed (or required government support or were taken over in distressed circumstances) and 18 “no crisis” banks. Figure 1.A shows that the Tier 1

---

³ A primary reason why the BCBS failed to strongly endorse going-concern CoCos was its expressed uncertainty on how they would perform and how they should be best designed (BCBS (2011, p.18-19)).

capital ratios for both groups remained stable and indistinguishable from May 2002 through 2007, but then actually rose in 2008 for the crisis banks. The picture is very different in Figure 1.B which clearly indicates that market capital ratios of crisis banks anticipated their financial distress, becoming much lower than that of no crisis banks. Hence, if the crisis banks had issued going-concern CoCos with regulatory capital triggers, conversion would have failed when it was needed most. Market value-based triggers, however, would have led to conversions.

The opposite signals provided by regulatory and market capital ratios are unsurprising since banks can more easily control the former: recognizing capital gains by selling appreciated assets, while holding on to depreciated assets, increases the regulatory ratio’s numerator; and portfolio reallocations can reduce the ratio’s risk-weighted asset denominator. Merrouche and Mariathasan (2014) document that banks manipulate regulatory capital ratios as they near financial distress, and banks more likely to receive public aid manipulate risk weights more severely. Consequently, CoCos that are classified as “going-concern” based on regulatory triggers may, in practice, fail to convert until regulators determine the PONV has been reached.

Market-based triggers are not without criticisms. Concerns relate to possible manipulation by speculators, “death spirals,” or equilibrium pricing problems. As we discuss below, a market-based trigger may create incentives for speculators to short the

---

5 A more recent case is Dexia, the French-Belgian bank that, before being rescued by the government in October 2010, reported a Tier1 ratio above 10% and was ranked 12th out of 90 banks that were subject to stress tests in the Spring of 2010 (De Groen (2011)).

bank’s stock and force an economically “unjustified” conversion that dilutes the bank’s shareholders. But even without short-sellers, shareholders may suffer losses from conversions following a market panic in which stock prices fall below their fundamental values. Another concern, pointed out by Sundaresan and Wang (2011) and analysed by Glasserman and Nouri (2012), is that market-based triggers may lead to “no-equilibrium” or “multiple equilibria” problems such that the bank’s stock value is ill-defined and, hence, may be an unreliable trigger. Problems with market value equilibria depend, in part, on the CoCo’s conversion terms.

Turning to the second main feature of CoCos, their conversion terms, one type of conversion is a principal writedown where CoCo investors receive no shares and have their bond’s principal reduced. For example, a 2010 Rabobank CoCo has its principal reduced by 75% when a regulatory capital trigger is breached. More recently, banks such as Barclays and KBC have issued CoCos with a 100% write-down. Another 2011 Rabobank CoCo has a partial writedown sufficient to increase the bank’s capital ratio above the regulatory capital ratio trigger.\(^7\) As noted by Flannery (2014), CoCos with principal write-downs are similar to “catastrophe bonds” issued by insurance companies. Any conversion creates a loss relative to the unconverted bond’s promised payments.

An alternative type of conversion pays new equity shares to CoCo investors. Flannery (2005) proposed that CoCos convert to shares worth the bond’s par value based on the stock’s closing price on the day that conversion is triggered. However, there are

\(^7\) This payoff is equivalent to converting only a part of the CoCo sufficient to replenish the bank’s capital ratio above a regulatory trigger level. Glasserman and Nouri (2012a) propose a similar partial conversion mechanism, though they specify that the CoCo not be written down but converted into new equity shares.
potential problems when the number of shares issued depends on the stock price. First, if more shares are issued the lower is the price, short sellers have a strong incentive depress the bank’s stock price, creating the possibility of a “death spiral” (Hillion and Vermaelen (2003)) that heavily dilutes the initial shareholders. Second, as discussed in Flannery (2009a) and Pennacchi (2011), if bank asset values suffer a large, discrete loss so that the bank’s total capital is worth less than the CoCo bond’s par value, no amount of new shares can be worth the CoCo’s par value, making the conversion infeasible.

Given these problems when the number of shares is variable, subsequent CoCo proposals and actual issuance of CoCos specify a pre-determined number of shares to be issued at the time of conversion. For example, Bulow and Klemperer (2013), Calomiris and Herring (2013), Flannery (2009b), Glasserman and Nouri (2012b), and Hilscher and Raviv (2011) propose issuance of a fixed number of shares or, equivalently, a fixed proportion of total shares, at conversion. Moreover, Berg and Kaserer (2012) note that CoCos issued by Lloyds in 2009 and Credit Suisse in 2011 specify conversion to shares worth a fixed 17% and 30%, respectively, of the banks’ total shares.

However, conversion to a fixed number of shares can make CoCo payoffs risky. If a CoCo has a regulatory capital trigger, the previously-discussed evidence suggests that the bank’s stock price may be quite low at conversion. Alternatively, if a CoCo has a market-value trigger, the bank’s stock price following conversion could be significantly

---

8 Berg and Kaserer (2012) show that CoCos issued by Lloyds and Credit Suisse that convert to equity would likely suffer significant losses at conversion, similar to CoCos with a principal writedown.
less than its trigger price if a sudden, discrete asset and stock price decline triggers conversion.9

Moreover, if the market value trigger is based on the bank’s stock price, potential stock price equilibrium problems exist if conversion promises shares that are likely to be worth less than or greater than the CoCo bond’s par value. Sundaresan and Wang (2011) present a discrete-time model that shows if conversion terms promise CoCo investors less than their bond’s par value, then the stock price may have no equilibrium value in the vicinity of the conversion threshold. The intuition is that since conversion transfers value from CoCo investors to the bank’s shareholders, the stock price would tend to rise as the conversion threshold is approached. However, this rise in the stock price could, itself, prevent conversion, leading to no equilibrium in the vicinity of the threshold. Sundaresan and Wang (2011) also consider the opposite case of conversion terms that promise CoCo investors an amount of shares whose value exceeds their bond’s par value, so that value is transferred from shareholders to CoCo investors at conversion. Their discrete-time model yields multiple stock price equilibria in the vicinity of the conversion threshold.

Glasserman and Nouri (2012b) study the same equilibria problems but using a model that permits continuous trading in the bank’s stock. They show that when conversion terms transfer value from CoCo investors to shareholders, the no equilibrium result continues to hold. However, with continuous trading and conversion terms that transfer value from shareholders to CoCo investors, the possibility of multiple equilibria

9 In other words, if the stock price is initially above the trigger threshold and then suffers a discrete decline (crash) that leaves it strictly below the threshold, the market value of bank equity at conversion is less than its value at the trigger price. The results in Section 5, derived from a model calibrated to bank data, indicate the presence of such risk.
no longer holds. Rather, they show that a unique stock price equilibrium exists in the vicinity of the conversion threshold. Hence, with a stock price trigger, conversion terms that are disadvantageous to CoCo investors can be problematic (no equilibrium) but terms that are advantageous to CoCo investors are not (a unique equilibrium).

A CoCo’s conversion terms also influence the incentives of banks that act in the interest of their shareholders. If conversion terms, either from a writedown or issuance of few shares, leave CoCo investors with a loss relative to their bond’s par value, then conversion transfers wealth to the bank’s shareholders. Consequently, prior to conversion banks have a heightened moral hazard incentive to make risky loans and investments since shareholders benefit from risk’s greater upside gains but face relatively little downside costs when losses result in conversion. By the same logic, when losses push a bank close to the conversion point, shareholders have little incentive to replenish equity via a new share issue. This “debt overhang” is particularly problematic since greater equity would reduce the likelihood of conversion and greatly benefit CoCo investors at the expense of initial shareholders. Conversely, if conversion terms are likely to provide CoCo investors with a gain relative to their bond’s par value, moral hazard and debt overhang problems are mitigated or reversed. Empirical analysis by Berg and Kaserer (2012) finds that the conversion terms of actual CoCos issues significantly disadvantage CoCo investors, thereby likely worsening moral hazard and debt overhang.

Clearly the design of a CoCo’s conversion trigger and conversion terms influence the CoCo’s timeliness in replenishing equity, the CoCo’s risk, and the issuing bank’s incentives. Since regulatory capital may trigger conversion too late or not at all, most
academic proposals for CoCos specify a type of market trigger but often differ on conversion terms. We briefly describe proposals that are most related to ours.

Calomiris and Herring (2013) focus on the incentive effects of conversion terms. Under their design, conversion is triggered when a 90-day moving average of the bank’s market value of equity to debt falls below a threshold. At conversion, CoCos convert to a large number of new shares that are intended to give CoCo investors a gain relative to their bond’s par value. For example, CoCos might receive shares worth 1.05 times the par value of their bonds. Since these conversion terms heavily dilute the initial shareholders, debt overhang is reversed. Consequently, the bank has incentives to voluntary replenish equity in order to avoid conversion and shareholder dilution losses.

Bolton and Samama (2012) propose that banks issue a convertible bond that they name a Capital Access Bond (CAB). The bond’s proceeds are invested in cash collateral, making it default-free if conversion does not occur. However, opposite to Calomiris and Herring (2013), the CAB converts to a fixed number of equity shares that would likely transfer value from CAB investors to the bank’s initial shareholders.\textsuperscript{10} This is because, unlike standard CoCos, the bank’s Board of Directors would decide when the CAB

\textsuperscript{10} A related proposal by Kashyap, Rajan and Stein (2008) requires that banks purchase contingent capital insurance that would allow them to issue new equity during a crisis at a predetermined price. In essence, they would buy put options on their own stock. This proposal requires the existence of default-free insurers. As Duffie (2010) points out, if the source of distress is a general financial crisis, the put seller may itself be distressed and unable to honour its commitments. A CAB addresses this default problem by acting as a credit-linked note (pure bond invested in cash plus a short put option position).
converts, presumably when the stock price has fallen to a point where the converted shares are worth significantly less than the unconverted CAB’s promised payments.\textsuperscript{11}

Bulow and Klemperer (2013) would require that banks issue CoCo-like bonds named Equity Recourse Notes (ERNs). Any promised coupon or principal payment would be paid in new equity shares whenever the bank’s contemporaneous stock price is below a fixed proportion of its price at the ERN’s issue date. For example, if the proportion was $p = 25\%$ and the date 0 stock price when the ERN was issued was $S_0 = \$100$, then a bond payment at date $t > 0$ is paid in shares, not cash, whenever $S_t < pS_0 = \$25$. The equity shares issued per $\$1$ of promised payment is $1/(pS_0)$, which are worth less than $\$1$ since $S_t < pS_0$. If another promised payment occurs at date $\tau > t$ when $S_\tau \geq pS_0$, then the ERN resumes payment in cash. Since ERN investors sustain a loss when the stock price is low enough for payment in shares, potential moral hazard exists. Moreover, Bulow and Klemperer envision that a bank will issue multiple ERNs, so that if a second ERN is issued at date $T > 0$ when $S_T < S_0$, then this second ERN is paid in shares only when $S_t < pS_T < pS_0$, making the second ERN effectively senior to the first. Consequently, if the bank’s stock price falls, issuing more senior ERNs transfers value from prior ERN investors to shareholders, providing an incentive to issue more ERNs and avoid debt overhang but potentially worsening risk-shifting incentives.

Several proposals envision that CoCos should convert only during a generalized financial crisis, not when just an individual bank becomes distressed. Kashyap, Rajan and

\textsuperscript{11} Bolton and Samama note that the likelihood of conversion will tend to raise the bank’s stock price, but they do not explicitly solve for the price as an endogenous function of the bank’s assets and conversion terms. They also note that the conversion terms that favor the bank’s initial shareholders could induce moral hazard regarding bank risk-taking.
Stein (2008) link a CoCo’s conversion to a date when aggregate banking industry losses exceed a trigger level. McDonald (2013) recommends a dual price trigger whereby a CoCo converts only if the bank’s stock price breaches a trigger and an aggregate financial institutions stock index falls below another trigger. A similar dual trigger mechanism by the Squam Lake Working Group (2009) ties conversion to a situation where the bank violates covenants and its regulator declares a systemic crisis. The aim of these proposals is to permit individual banks to fail when distress is firm-specific and likely a result of mismanagement at the individual bank. Doing so would reduce managerial agency costs. However, as pointed out by Flannery (2014), if a bank is too-big-to fail, it is not clear that regulators will allow even a single bank’s failure, or that one large bank’s failure would not cause a systemic crisis at other banks.

Our COERC proposal is closest to Calomiris and Herring (2013) in that conversion terms potentially lead to dilution losses for the existing shareholders, rather than losses to CoCo bond investors. However, we differ in the design of the conversion trigger, making it equal to the ratio of the market value of total capital, not just the bank’s equity capital. We also differ by providing the bank’s shareholders the right to repurchase the newly-issued converted shares at the bond’s par value, so that the threat of heavy dilution coerces shareholders to repay the COERC investors in cash. Consequently, conversion under a COERC is more neutral in not transferring value from shareholders to bondholders (as in Calomiris and Herring) or vice-versa (as in CABs or ERNs). As will be demonstrated, the conversion trigger, conversion terms, and repurchase option of COERCs mitigate moral hazard incentives, debt overhang, and unintended value transfers due to stock price manipulation or a generalized stock price panic.
The next section provides intuition for the current paper’s COERC proposal via a highly simplified, static numerical example. A more formal dynamic valuation model is introduced in Section IV.

III. COERCs: A Simplified Example

This section provides intuition for the basic features of a COERC with a simplified numerical example that makes several strong assumptions. These assumptions are relaxed in the following section which uses a more explicit valuation model.

Assume at an initial date 0 that the market value of a bank’s assets, $A_0$, equals $1,100. The bank’s liabilities consist of senior deposits, COERCs, and shareholders’ equity with market values $D_0$, $V_0$, and $E_0$, respectively, so that $A_0 = D_0 + V_0 + E_0$. It is assumed that $D_0$ equals the deposits’ par value of $1,000 and that $V_0$ equals the COERCs’ par value of $B = 30. Finally, the initial market value of shareholders equity is $E_0 = S_0 \times n_0$, where $S_0 = 10$ is the initial stock price and $n_0 = 7$ is the initial number of shares outstanding. Note that the sum of the market value of the COERC and the market value of equity is equal to $E_0 + V_0 = A_0 - D_0 = 100$, which we refer to as the market value of the bank’s total capital. We now describe the three main features of COERCs.

First is the COERC’s conversion trigger. Conversion occurs on the first date $t$ prior to its maturity when the market value of capital, $E_t + V_t = A_t - D_t$ falls to $65 or less. Equivalently, if we subtract off the COERC’s par value of $B = 30$, conversion is triggered when initial shareholders’ equity is $65 - 30 = 35$ or the equity-to-deposit ratio is 3.5%. However, we specifically make the trigger a function of the bank’s total capital (equity plus COERCs) for reasons that are detailed later: it discourages stock price manipulation and it avoids potential equilibrium problems.
Second is the COERC’s conversion terms. At conversion COERC investors receive a large number of new shares, say \( n_1 \), that have the potential to heavily dilute the bank’s initial shareholders. We assume \( n_1 = 30 \), and since the COERC’s par value is \( B = \$30 \), the “conversion price” is \( \$1 \). Thus COERC investors’ share of total capital equals 
\[
\alpha = \frac{n_1}{n_0 + n_1} = \frac{30}{37}.
\]
If, at conversion, the total capital of the bank is \( \$65 \), then COERC investors would receive \( \alpha \times \$65 = \$52.70 \), a significant gain relative to their bond’s par value. Since the new per share stock price would be \( \frac{\$65}{37} = \$1.76 \), the initial shareholders’ value equals \( 7 \times \$1.76 = \$12.30 \).

A third critical feature of the COERC is that when conversion is triggered, a rights issue is announced inviting the initial shareholders to buy the 30 new shares at the \( \$1 \) conversion price, with their new funds used to repay the COERC debt at its par value. Rational investors will exercise this option since the fully-diluted stock price of \( \$1.76 \) is higher than the rights issue price of \( \$1 \). If the initial shareholders are financially constrained, they can sell their rights to other non-constrained investors. Consequently, the COERC investors are paid their par value of \( \$30 \) in cash, rather than receive shares, and suffer no loss on their initial investment. The bank’s total capital of \( \$65 \) is now in the form of shareholders’ equity, with the initial shareholders’ claim of \( \$70 \) reduced to 
\[
S_i \times (n_0 + n_1) - B = \$1.76 \times 37 - \$30 = \$35.
\]
Hence, they bear the entire decline in bank capital.

The above calculations presume that the bank’s market value of capital at conversion equals the trigger level of \( \$65 \). Suppose, instead, that the triggering event coincided with a sudden, severe plunge in the bank’s asset value, as might occur during a financial crisis, and capital crashed from somewhere above the \( \$65 \) trigger to much below it, say \( A_t - D_t = \$50 \). Would initial shareholders still choose to exercise their option to...
repurchase the COERC investors’ shares? They will as long as the value of the new shares $\alpha \times (A_t - D_t) = (30/37) \times 50 = 40.54$ is above the COERC par value of $B = 30$.

Only if capital plunged below $B/\alpha = 37$, so that the fully-diluted stock price is below $1$, would COERC investors not be repaid in full and receive equity worth less than par.

However, in this very severe case COERCs would still perform as a standard CoCo and re-capitalize the bank by converting debt to new equity. Of course, it also is possible that capital could suddenly decline even more severely into negative territory where $A_t - D_t < 0$ and both COERC investors and shareholders would be wiped out, likely requiring regulators to resolve the failed bank. No capital instruments, short of ones with unlimited liability, could protect against such catastrophic failure. In summary, if $t_r$ is the rights issue date when initial shareholders must decide whether to exercise their option, the payoff to COERC investors can be written as

$$V_{t_r} = \begin{cases} 
B & \text{if } B \leq \alpha \left( A_{t_r} - D_{t_r} \right) \\
\alpha \left( A_{t_r} - D_{t_r} \right) & \text{if } 0 < \alpha \left( A_{t_r} - D_{t_r} \right) < B \\
0 & \text{if } A_{t_r} - D_{t_r} \leq 0 
\end{cases}$$

We next present a structural model of the bank that examines how the likelihood of these different payoffs, along with the triggering event, affects the risk borne by COERC investors. The model is used to analyze how different parameters of the COERC contract affect its risk, and also allows a comparison of a COERC’s risk to that of other capital instruments. The model also provides insights on the risk-taking incentives of the bank’s shareholders. Finally, it is used to study how robust different capital instruments are to deviations of market prices from fundamentals.
IV. Valuation Technique

Our model is aimed at analyzing COERCs and other capital instruments in a setting that allows for the possibility of sudden declines in bank asset values, as typically occurs during a financial crisis. Permitting the possibility of crises is critical for comparing the performance of different types of bank capital.

We employ a modified version of the credit risk model of Pennacchi (2011) which is especially suited for capturing potentially severe asset value declines. It assumes that bank asset values follow a mixed jump-diffusion process and derives the equilibrium credit spreads on bank deposits and capital instruments. Incorporating jumps is essential for deriving different credit spreads on convertible bank debt. If asset values follow only a standard diffusion process and convert to par when capital hits the triggering threshold, then all floating-rate debt would have zero credit spreads since bond investors would always receive their par value at conversion.

A. Basic Model Assumptions

Initially, our model follows standard structural credit risk models, such as Merton (1974), by assuming a Modigliani-Miller setting where the market value of a bank’s total liabilities, including shareholders’ equity and other capital instruments, equals the market value of its underlying assets. Later, we loosen this assumption to permit the market prices of bank shareholders’ equity and capital instruments to depart from the underlying asset “fundamentals.” Such a departure allows us to study the robustness of different capital instruments to manipulation or panic that could change the conversion triggering date from that based purely on fundamentals.
A bank is assumed to issue short-maturity deposits (senior debt), shareholders’ equity, and longer-maturity bonds in the form of COERCs, standard CoCos, or non-convertible subordinated debt. The funds raised by these liabilities are invested in assets whose date $t$ value is denoted $A_t$. The change in the bank’s assets equals the asset’s return plus changes due to the bank’s cash inflows less cash outflows. Using the superscript * to distinguish asset changes solely due to their rate of return, these assets’ risk-neutral rate of return, $dA^*_t / A^*_t$, satisfies the jump – diffusion process:

$$
\frac{dA^*_t}{A^*_t} = (r_t - \lambda k) dt + \sigma dz + (Y_{q_t} - 1) dq_t,
$$

where $dz$ is a Brownian motion process, $q_t$ is a Poisson counting process that increases by 1 with probability $\lambda dt$, and $\ln(Y_{q_t}) \sim N(\mu_y, \sigma^2_y)$ where $k \equiv E^Q[Y_{q_t} - 1]$ =

$$
\exp[\mu_y + \frac{1}{2} \sigma^2_y] - 1 \text{ is the risk-neutral expected value of a jump. In equation (2), } \sigma \text{ is the standard deviation of the continuous diffusion movements in the bank’s assets while } \lambda \text{ is the per unit time probability of a jump in the assets’ value. The jump size is lognormally distributed, where } \mu_y \text{ controls the mean jump size and } \sigma_y \text{ is its standard deviation.}
$$

We permit the short-term default-free interest rate (Treasury bill rate), $r_t$, to vary such that its risk-neutral process follows the Cox, Ingersoll, and Ross (1985) model:

$$
\frac{dr_t}{\sqrt{r_t}} = \kappa(\bar{r} - r_t) dt + \sigma_r \sqrt{r_t} dz,
$$

where $dz dz_t = \rho dt$.

Our model assumes bank deposits have a very short (instantaneous) maturity, but are default-risky and pay a fair, competitive interest rate. This assumption fits many large
“money-center” banks that rely on short-term, wholesale sources of funds, such as large-denomination deposits paying LIBOR. Assuming deposits have a short maturity also simplifies our analysis because the conversion of CoCos or COERCs does not affect the current value or yield on deposits. Conversion does not change the total amount of claims that are junior to deposits. It only changes the composition.\footnote{Conversion may change the bank’s future cash outflows, due to a reduction in coupon payments and a rise in dividends, thereby affecting future yields on deposits. Our model accounts for this fact.}

Thus, let $D_t$ be the date $t$ quantity of bank deposits which are assumed to have an instantaneous (e.g., overnight) maturity and to pay an interest rate of $r_t + h_t$, where $h_t$ is the deposits’ fair credit spread. We also assume that the bank targets a capital ratio or asset-to-deposit ratio, so that its leverage tends to be mean-reverting. Much empirical evidence, including Flannery and Rangan (2008), Adrian and Shin (2010), and Memmel and Raupach (2010), finds that deposit growth expands (contracts) when banks have an excess (a shortage) of capital.\footnote{Another structural model with mean-reverting leverage is Collin-Dufresne and Goldstein (2001). They show that allowing leverage to mean-revert is necessary for matching the term structure of corporate bond credit spreads. Since Adrian and Shin (2010) show that bank leverage has stronger mean-reversion than that of non-financial corporations, modeling this phenomenon is particularly important for valuing bank bonds.} Defining $x_t = A_t / D_t$, as the date $t$ asset-to-deposit ratio, the bank targets this ratio by adjusting deposit growth according to:

\begin{equation}
\frac{dD_t}{D_t} = g (x_t - \hat{x}) dt
\end{equation}

where the positive constant $g$ measures the strength of mean-reversion and $\hat{x} > 1$ is the bank’s target asset-to-deposit ratio.
The bank is assumed to fail (be closed by regulators) when assets fall to, or below, the par value of deposits (plus any non-convertible bonds). If failure occurs, total losses to depositors are $D_t - A_t$. Though deposits are default-risky, prior to failure their value always equals their par value $D_t$ since their short maturity allows their credit spread $h_t$ to continually adjust to its fair value. This assumption simplifies the valuation of the bank’s other liabilities since they always sum to equal the bank’s total capital of $A_t - D_t$.

Moreover, the Appendix shows that this fair deposit credit spread equals

\[ h_t = \lambda \left[ N(-d_1) - x_t \exp \left( \mu_y + \frac{1}{2} \sigma_y^2 \right) N(-d_2) \right] \]

where $d_1 = \left[ \ln \left( x_t + \mu_y \right) / \sigma_y \right]$ and $d_2 = d_1 + \sigma_y$.\(^\text{14}\) Note that $h_t$ is a strictly decreasing, convex function of the bank’s asset to deposit ratio, $x_t$.

Besides deposits, at date 0 the bank issues a subordinated bond having par value of $B$ and maturity date of $T > 0$. Prior to maturity or conversion, it pays a continuous coupon per unit time, $c_t dt$. Since we focus on credit spreads, the bond is assumed to pay floating-rate coupons so that $c_t = r_t + s$ where $s$ is a fixed spread over the short-term default-free rate.\(^\text{15}\) At the issue date 0, $s$ is set so that the bond initially sells for its par value, $B$. The method of solving for this equilibrium coupon spread is discussed shortly.

We now specify a CoCo or COERC bond’s conversion trigger. Conversion occurs when the bank’s total capital-to-deposit ratio breaches the fixed threshold $\chi$:

\[ \chi = \frac{\bar{A}_t - D_t}{D_t} \]

\(^\text{14}\) The credit spread depends only on the bank’s current asset-to-deposit ratio and the parameters of the asset jump process since only jumps that exceed the bank’s capital can cause losses to depositors.

\(^\text{15}\) Our results are qualitatively and quantitatively similar if we assume that bonds pay fixed coupons.
where, given the current level of deposits, $\bar{A}_t$ is the corresponding threshold value of assets at which conversion is triggered. Recall for the example of Section III, $D_0 = $1,000, $\bar{A}_0 = $1,065, and $\chi = 6.5\%$. The only difference is that equation (6) allows for the realistic possibility that the quantity of deposits can change over time.

The trigger based on asset values in equation (6) is well-defined, but a bank’s market value of assets is not directly observable. However, the fundamental relation that the market values of assets and liabilities must be equal implies $A_t = S_t \times n_0 + V_t + D_t$, where recall that $S_t$ is the bank’s date $t$ stock price, $n_0$ is the number of shares outstanding, and $V_t$ is the market value of the convertible bond. Since the deposits’ short maturity and continuously-adjusting fair credit spread makes their market value equal their par value, $D_t$, the market value of assets (and total capital) can be observed from the sum of the market values of shareholders equity plus the convertible bond: $A_t - D_t = S_t \times n_0 + V_t$. Thus, equation (6) can be implemented so that conversion is triggered whenever

$$ \frac{S_t \times n_0 + V_t}{D_t} \leq \chi. $$

Alternatively, a trigger might be based on only the equity/deposit ratio, say $S_t \times n_0 / D_t$. However, Glasserman and Nouri (2012b) show that a trigger using the stock price, and not the bond price, can be ill-defined in that there is no equilibrium when the conversion terms transfer value from bondholders to shareholders. The intuition is that if the event of conversion makes the stock more valuable, investor recognition of this value transfer maintains the stock price above the trigger, leading to no equilibrium.

---

16 We are grateful to Stewart Myers for first suggesting this trigger.
The Appendix proves that the trigger in (7) is immune to this problem because any capital value transfer causes the bond price to fall and offset the rise in the stock price, thereby leading to a unique equilibrium. Consequently, their sum continues to reflect the bank’s total capital and the underlying asset value.\textsuperscript{17} Although the current paper assumes that the conversion terms of CoCos and COERCs do not transfer value from bondholders to shareholders, we propose this total capital ratio trigger because it is robust to alternative conversion terms.\textsuperscript{18} Note also that our trigger based on the total capital ratio, \((A_t - D_t)/D_t = x_t - 1\), is analytically convenient since \(x_t\) can be considered the “state” variable that triggers conversion whenever \(x_t \leq 1 + \chi\).

We next turn to specifying conversion terms. For standard CoCos we follow Flannery (2010) and assume CoCo investors receive a fixed number of shares that would be worth their bond’s par value if the bank’s asset value equals the asset trigger threshold. If \(n_1\) is the number of shares issued to CoCo investors, then it satisfies \(n_1 \times S_t = B\) where

\textsuperscript{17} The Appendix presents evidence that large U.S. banks’ subordinated bonds are sufficiently liquid to make frequent observation of their prices feasible. Glasserman and Nouri (2012b) Section 4.6 also give another example of a trigger based on the sum of two security prices that avoids the no equilibrium problem. More generally, they and Sundaresan and Wang (2011) show that if trading occurs discretely, rather than continuously, a trigger based only on a stock price can lead to multiple equilibria. Such multiple equilibria due to discrete trading are also avoided by a total capital trigger in (7) because, once again, any value transfers between shareholders and bondholders do not affect the sum of the two securities.

\textsuperscript{18} As in Bond, Goldstein, and Prescott (2010), if a government intervention at the time of conversion changes the total value of the bank’s assets, no equilibrium or multiple equilibria may reappear. However, COERCs are specifically designed to avoid the need for government intervention.
\( \bar{S}_t \) is the post-conversion stock price when the bank’s capital equals the trigger threshold.

Since total equity in this case equals \((n_0 + n_1) \times \bar{S}_t = n_0 \bar{S}_t + B = \bar{A}_t - D_t = \chi D_t\), we have

\[
n_1 = \frac{n_0 B}{\bar{A}_t - B - D_t} = \frac{n_0 B}{\chi D_t - B}.
\]

Consistent with (8), if \( \alpha = n_1/(n_0 + n_1) \) denotes the share of total bank capital that is owned by the CoCo investors at the time of conversion, then \( \alpha = B / (\bar{A}_t - D_t) \).

The COERC’s conversion terms differ from this standard CoCo because the new shares issued at the time of conversion are specified to be greater than that in equation (8). Equivalently, the share of total bank capital that COERC investors are entitled to receive is assumed to exceed \( B / (\bar{A}_t - D_t) \). For the example given in Section III, standard CoCos would receive the proportion of capital \( \alpha = B / (\bar{A}_t - D_t) = 30/65 = 6/13 \approx 0.46 \) while the corresponding proportion for COERCs was assumed to be \( \alpha = 30/37 \approx 0.81 \).

The COERC’s conversion terms also differ in another crucial way because conversion triggers a rights offering that gives the bank’s shareholders the option to repurchase the newly issued shares at a price equal to the COERC bond’s par value. If we let \( t_c \) be the conversion trigger date when inequality (7) is first satisfied, then in practice a rights offering period would follow that gives the initial shareholders the time to decide whether to exercise their option. Let \( t_r \) be the end of the rights offering period where, for example, \( t_r = t_c + 20 \) trading days if a rights offering is typically completed in one month.

Based on these conversion terms, the payoffs to standard CoCo and COERC investors following conversion can both be represented by equation (1) but where this
payoff occurs at date $t_c$ for standard CoCos and the slightly later date $t_r$ for COERCs. 19 Moreover, these bonds’ values of $\alpha$ in equation (1) differ, with the capital proportion of $\alpha = B / (\bar{A}_t - D_t)$ for standard CoCos and a specifically higher capital proportion, $\alpha$, for COERCs. Equation (1) shows the effect of this higher $\alpha$ that threatens dilution and coerces the initial shareholders to repurchase the newly issued shares from COERC investors: there are more states of the world where COERC investors receive their bond’s par value (in cash) compared to the states where CoCo investors receive theirs (in equity), making COERCs less default-risky than standard CoCos.

An additional bond that we analyze is non-convertible subordinated debt. As with CoCos and COERCs, subordinated debt investors receive a floating-rate coupon and the par value of their bond, $B$, at maturity as long as the bank’s asset value exceeds $D_t + B$, the total par value of debt. If, prior to maturity, the bank’s asset value no longer exceeds total debt, the subordinated debt holders receive a final payoff of $\max[A_t - D_t, 0]$.

The last component of capital is the bank’s shareholders’ equity. It is assumed to be paid a continuous dividend equal to a constant proportion of the bank’s net worth, $\delta(A_t - B - D_t)dt$, where $A_t - (B + D_t)$ is the difference between the bank’s asset value and the par value of its debt, and $\delta$ is approximately the dividend yield of bank equity. At the date of bond conversion or bond maturity, the value of the initial shareholders’ equity equals the bank’s residual capital after payoffs are made to bondholders as was specified above.

19 Since the asset value relevant for COERC investors’ payoff, $A_t$, is for a date following the asset value triggering conversion, $A_{t_c}$, our model and calculations account for the possibility that COERC investors are exposed to further asset value declines during the rights offering period.
While a closed-form solution exists for the equilibrium credit spread paid on the bank’s short-term deposits (equation (5)), it is necessary to numerically calculate the equilibrium credit spread on the bank’s bonds, $s$. Similar to Boyle (1977), we use Monte Carlo valuation by simulating the risk-neutral processes for the bank’s asset-to-deposit ratio, $x_t$, and the instantaneous-maturity interest rate, $r_t$. The Appendix shows how the process for $x_t$ is derived from the bank’s asset process, $A_t$, where the change in assets equals the return earned on assets (equation (2)), less interest payments to depositors, coupon payments to bondholders, and dividend payments to shareholders, plus deposit changes that target capital (equation (4)). The value of bonds reflects their coupon payments prior to conversion or maturity plus their payoffs at conversion or maturity, where conversion is determined by whether the value of assets breaches the capital ratio threshold given in equation (6). This Monte Carlo valuation leads to a date 0 bond value, $V_0$, for a given spread, $s$. Then, the bond’s fair initial credit spread, $s^*$, is determined by varying $s$ until $V_0 = B$; that is, until the bond initially sells for its par value.

B. Model Parameter Estimates

By recapitalizing banks prior to severe financial distress and avoiding a government bailout, CoCos (and COERCs in particular) are most valuable when issued by “too-big-to-fail” banks. Therefore, we calibrate the model’s parameters using data on the three largest U.S. banks: Bank of America, Citigroup, and JPMorgan Chase. For each bank, we estimate a daily asset return process over the period January 2, 2003 to December 30, 2011 by calculating market returns on the bank’s total liabilities, based on the assumption that they equal total asset returns. From quarterly Federal Reserve Y9-C (Bank Holding Company) Reports, we obtained daily estimates of each bank’s amounts
of short-term senior debt (mainly deposits), senior bonds, subordinated bonds, and preferred stock. Along with the bank’s daily market value of shareholders’ equity obtained from the Center for Research in Security Prices (CRSP), we calculated daily proportions of the bank’s liabilities in these five different liability classes. For example, the average liability proportions for these three banks over the sample period was 71.6%, 15.5%, 2.5%, 0.5%, and 9.8% for short-term debt, senior bonds, subordinated bonds, preferred stock, and common shareholders’ equity, respectively.

By calculating the product of a bank’s daily liability proportions and the market rates of return earned by each liability class, we obtained the daily return on the bank’s total liabilities. Consistent with our model, we proxied the daily return on the bank’s short-term debt by the overnight LIBOR. The daily return on senior bonds was estimated from daily changes in each bank’s 5-year credit default swap (CDS) spread. Subordinated bond returns were computed using daily TRACE transaction prices of a representative subordinated debt issue of each bank. Daily preferred stock returns were obtained from Bloomberg while daily common stock returns were obtained from CRSP.

Calculations for the nine year period led to 2,270 daily total liability (and asset) returns for each bank. A “jump” event was assumed to occur on a given day if the return differed from the mean daily return by more than three standard deviations, an event that averaged approximately 5 times per year per bank, providing a jump frequency estimate

---

20 To estimate daily amounts from quarterly data, a cubic spline was fit for each trading day between the quarterly observations.

21 The subordinated debt issues that were used are those given in Table 1.
of $\lambda = 5$. The sample standard deviation of the size of these jump returns provided an estimate of $\sigma_y = 0.0125$; that is a 1.25% daily asset change. After eliminating these jump events from the daily return series, the standard deviation of the diffusion-generated returns was approximately $\sigma = 0.03$, or an annual asset return standard deviation of 3%.

Calibrating the risk-neutral jump frequency and jump volatility parameters, $\lambda$ and $\sigma_y$, from the time series of returns assumes that they equal their physical process counterparts. Pan (2002) makes this same assumption when estimating a similar jump-diffusion process for the S&P500 index return. However, she assumes that the risk-neutral expected jump size differs from the physical (actual) expected jump size by a risk premium. In our sample, the average jump size for the three banks over the nine-year period, call it $k^P$, is close to zero, equal to 0.00025 (or 2.5 basis points). We follow Pan (2002) and assume that the risk premium from jumps is reflected solely in the mean jump size and set $\mu_y = -0.0025$ so that the risk-neutral expected jump size is $k = E^Q\left[ Y_{q,t} - 1 \right] = \exp\left[ \mu_y + \frac{1}{2} \sigma_y^2 \right] - 1 = -0.0024$ and the implied jump risk premium is $\lambda (k^P - k) = 1.3\%$.

This excess rate of return on bank assets is slightly greater than the 1% excess rate of return that others have estimated for a large sample of banks, but it is consistent with the evidence finding greater systematic risk for the largest of banks.23

22 In the absence of jumps where returns are generated by only a (normal) diffusion process, the expected number of returns exceeding three standard deviations would be less than one per year.

23 Pennacchi (2000) estimates a 1% excess asset return from all commercial banks listed on CRSP during 1926 to 1996. Demsetz and Strahan (1997) and De Jonghe (2010) find that the largest commercial banks have greater systematic risk, particularly if they undertake investment banking activities.
A dividend yield of $\delta = 2\%$ is assumed, which is slightly lower than the average dividend yields for Bank of American, Citigroup, and JPMorgan over our sample period, but significantly higher than their current dividend yields of less than 1%. The target total capital-to-deposit ratio for banks is assumed to be 14% ($\hat{x} = 1.14$), which is approximately the sample average ratio of total capital to short-term and senior debt for these three banks. Consistent with Adrian and Shin (2010) and Memmel and Raupach (2010), we also assume a capital mean reversion speed of $g = \frac{1}{2}$, so that about one-half of a bank’s capital deviation from its target is expected to be reduced over the next year.

The remaining parameters relate to the Cox, Ingersoll, and Ross (1985) default-free term structure. We chose estimates similar to Duan and Simonato (1999) with $\kappa = 0.114$, $\bar{r} = 6.45\%$, $\sigma_r = 0.07$, and $\rho = -0.2$. Assuming an initial short-rate of $r_0 = 2\%$, these parameters lead to a five-year, fixed-coupon default-free bond having a par yield of 3%.

V. Basic Model Results

A bank’s bonds (CoCos, COERCs, or non-convertible subordinated debt) are assumed to have a five-year maturity and an initial par value equal to 3% of deposits; that is, $B/D_0 = 3\%$. This is the assumption of our previous example and is comparable to the amounts of non-common equity capital that our three sample banks have typically issued. For CoCos and COERCs, the conversion trigger is assumed to be when the total capital to deposit ratio, $\chi$, breaches 6.5%. Given an initial bond par value to deposit ratio of 3%, this total capital threshold implies a common equity trigger of about 3.5%. As in the example, COERC investors are entitled to receive a total capital share of $\alpha = 30/37$.

---

24 From 2003 to 2011, the average ratio of subordinated debt plus preferred stock to short-term and senior debt was 4.1%, 2.7%, and 3.4% for Bank of America, Citigroup, and JPMorgan Chase, respectively.
Figure 2 graphs the new issue credit spreads for COERCs, CoCos, and non-convertible subordinated debt based on the previous section’s valuation method and parameter values. The horizontal axis gives the percent of total bank capital per deposits, \((A_0 - D_0)/D_0\), at the time of the bonds are issued. The vertical axis is the new issue credit spreads, \(s\), in basis points. As expected, as a bank’s initial total capital declines, all three bonds’ new issue credit spreads rise. For non-convertible subordinated debt, lower initial bank capital increases the likelihood that the bonds’ investors would suffer losses if the bank failed. That would occur if, over the bond’s five-year life, there was a sudden decline in capital strictly below the bond’s par value, equal initially to 3% of deposits.

For CoCos, lower initial bank capital increases the likelihood of a conversion where CoCo investors could suffer losses. Losses would occur if, over the bond’s five-year life, there was a sudden decline in capital strictly below the \(\chi = 6.5\%\) capital to deposit threshold. Only if there is a gradual (continuous) decline in capital that led to conversion exactly at the 6.5% capital ratio trigger would CoCo investors receive shares that are worth their bond’s par value and, thereby, avoid a loss.

Similar to CoCos, COERCs can also sustain losses at conversion, and the likelihood is greater when the bank’s initial capital is less. However, COERC losses require a much greater decline in bank capital around the time of conversion. As discussed earlier, when \(\alpha = 30/37\) the capital/deposit ratio would need to fall from above the 6.5% trigger to below 3.7% before the bank’s shareholders would lack the incentive to repurchase the shares issued to COERC investors at the bond’s $30 par value.\(^{25}\) Since

\(^{25}\) Recall from the COERC payoff in equation (1), capital below 3.7% implies a payoff indicated by the right-hand side’s second or third lines, rather than the first which returns the bond’s par value.
the likelihood of this event is relatively small, COERCs are better protected from losses compared to CoCos and subordinated debt. Thus, for any level of initial bank capital, COERC credit spreads are significantly lower than those for the other two bonds.\textsuperscript{26}

It should be noted that if bank assets followed a pure Brownian motion diffusion process, so that $\lambda = 0$ or $\sigma_y = \mu_y = 0$, the bank’s asset value would have a continuous sample path and the credit spreads for the three bonds in Figure 2 would equal zero for every initial level of capital.\textsuperscript{27} For subordinated debt, regulators could always close the bank at the point that assets exactly equal the par value of total debt, $D_t + B$, allowing full recovery by debt holders. Also, CoCos and COERCs would always convert when the bank’s capital value exactly equals the trigger level, also ensuring that investors receive the par value of their bonds. Thus, the realistic possibility of sudden asset value losses is what generates differences in the three bonds’ risks and credit spreads.

To illustrate how the threat of dilution protects COERC investors, Figure 3 reports new issue credit spreads for COERCs that differ by the number of shares that investors are entitled to receive at conversion. Specifically we consider $n_1 = 40, 30, 20, \text{ or } 10$ shares, so that the dilution ratio is $\alpha = 40/47, 30/37, 20/27, \text{ or } 10/17$. Clearly, for a given

\textsuperscript{26} Given the same conversion threshold, $\chi$, COERC credit spreads are lower than comparable CoCo credit spreads as long as the share of capital that COERC investors are entitled to receive, $\alpha$, exceeds that for CoCo investors. However, COERC (and CoCo) credit spreads will not, in general, be less than subordinated debt credit spreads. As will be illustrated next, reducing the dilution ratio, $\alpha$, and the conversion threshold, $\chi$, can raise COERC credit spreads above those of subordinated debt.

\textsuperscript{27} Moreover, as can be seen from equation (5), credit spreads on deposits, $h_t$, would also equal zero. An example where asset returns follow a pure diffusion process is Albul, Jaffee and Tchistyi (2010), and CoCos in their model have zero credit spreads (are default-free).
level of initial bank capital, new issue credit spreads are lower when COERC investors are entitled to greater proportion of the bank’s capital. The intuition is the same as discussed earlier. COERC investors would be subject to losses only when there is a sudden decline from the 6.5% capital ratio trigger to a capital ratio below \((B/\alpha)/1000\), which are capital/deposit ratios of 3.53%, 3.70%, 4.05%, and 5.10%, respectively.

Another contract feature that affects the risk of COERCs (as well as CoCos) is the trigger level of capital. As was just mentioned, when \(\alpha = 30/37\) capital would need to fall suddenly from the trigger level to below 3.70% in order for COERC investors to suffer a loss. The likelihood of this happening is less the higher is the trigger capital ratio. Previous graphs assumed a trigger capital/deposit threshold of \(\chi = 6.5\). Figure 4 graphs new issue credit spreads for COERCs where the trigger capital ratio threshold equals either \(\chi = 5\), 5.5\%, 6.0\%, or 6.5\%. Clearly for any level of initial bank capital, new issue COERC credit spreads are lower when the trigger threshold, \(\chi\), is higher. While conversion is less likely the lower is the threshold, if conversion does occur at the lower threshold COERC investors are more likely to sustain losses. For example, starting from a capital value just above the thresholds, when \(\chi = 5.0\) an asset value decline of over 1.3\% (5.0\% – 3.7\%) would lead to COERC losses, whereas when \(\chi = 6.5\) an asset value decline of over 2.8\% (6.5\% – 3.7\%) would be required for COERC losses.

The design features that reduce the default risk of COERCs have implications for a bank’s risk-shifting incentives. Merton (1974) noted that the shareholders’ equity of a levered, limited-liability firm is comparable to a call option written on the firm’s assets with a strike price equal to the promised payment on the firm’s debt. By raising the risk (volatility) of the firm’s assets, shareholders can increase the value of their call option at
the expense of the firm’s debt value. This moral hazard incentive to transfer value from debt holders to shareholders tends to rise as the firm becomes more levered.

The risk-shifting incentives of banks that issue COERCs, CoCos, and subordinated debt can be compared in the context of our model. We calculate the change in the value of shareholders’ equity following a rise in the volatility of jump risk, $\frac{\partial E}{\partial \sigma_y}$. Note that this comparative static exercise does not change the bank’s asset value, though from equation (5) it raises the credit spread on short-term deposits, with the effect that the deposits’ market value continues to equal their par value. Consequently, the rise in jump risk transfers value to the bank’s shareholders at the expense of its bondholders; that is, $\frac{\partial E}{\partial \sigma_y} = -\frac{\partial V}{\partial \sigma_y}$. Figure 5 reports numerical estimates of the derivative $\frac{\partial E}{\partial \sigma_y}$ for a bank that issues subordinated debt, CoCos, or COERCs.28 The calculation is done for current capital levels ranging from 7% to 20% of deposits. The benchmark parameters and contract features are assumed for CoCos and COERCs ($\chi = 6.5\%, \alpha = 30/37$).

Figure 5 shows that for any level of capital, $\frac{\partial E}{\partial \sigma_y}$ is lowest when the bank issues COERCs, second lowest for CoCos, and highest for subordinated debt. Moral hazard tends to be greater as the bank’s capital declines, except for convertible bonds at capital near the conversion threshold.29 The most important finding is that a bank that issues

---

28 The bank is assumed to have issued each bond at its fair credit spread when the bank’s total capital equaled 10% of deposits. For each bond, the derivative is calculated numerically by the discrete approximation $\frac{\Delta E}{\Delta \sigma_y}$ where $\Delta \sigma_y = (0.0150 - 0.0125) = 0.0025$. In other words, for each current level of bank capital, we valued bank equity by 200,000 Monte Carlo simulations when $\sigma_y = 0.0125$ (the benchmark case) and then repeated the equity valuation but with $\sigma_y = 0.0150$.

29 For convertible bonds near the conversion threshold, it can be relatively more likely that the threshold will be hit exactly (due to diffusion movements in asset values) which would result in repayment at par.
COERCs has a smaller incentive to choose activities or investments that increase jump volatility. The relatively high number of shares that COERC investors can receive at conversion better protects the par value of their bond compared to investors in CoCos. Furthermore, since COERC investors are likely to be fully repaid in cash at conversion, they benefit from exiting the bank earlier than non-convertible bond investors.

While not reported here, similar results occur for the derivative $\partial E/\partial \lambda$, which captures a bank’s moral hazard to choose assets that increase the frequency of jumps. When a bank issues COERCs, it has the least incentive to raise jump frequency, followed by when it issues CoCos, then when it issues subordinated debt. The same ordering occurs if one considers a bank’s moral hazard incentive to reduce the mean jump size, $\mu_y$. A COERC’s greater protection against jump risk reduces moral hazard.\(^{30}\)

Subordinated debt, CoCos, and COERCs also affect another bank incentive, namely, debt overhang (Myers (1977)). When debt is subject to possible default losses, issuing new equity makes these losses less likely and increases the debt’s value. Given that investors pay a fair price for the new equity issue, the increase in the debt’s value

Furthermore, at low levels of capital, the market value of equity is also low, so that its absolute increase from greater risk tends to be smaller, though it may be greater as a proportion of equity.

\(^{30}\)Unreported calculations also show that a bank’s incentive to raise assets’ diffusion volatility, $\partial E/\partial \sigma$, is smaller for COERCs, except when capital is very low. With low capital, greater (continuous) Brownian motion risk makes it more likely that assets exactly equal the trigger threshold at conversion or assets exactly equal total debt at the time the bank is closed. In such scenarios, CoCo and subordinated debt investors suffer no losses. This contrasts to higher jump risk that makes it more likely that conversions and bank closures occur following downward jumps where these investors would suffer losses.
comes at the expense of the bank’s initial shareholders’ equity. This loss in shareholder value is a disincentive to replenishing bank equity following a decline in bank’s capital.

We quantify debt overhang by calculating the change in the value of the bank’s shareholders’ equity, \( \partial E \), following a new equity issue that increases the bank’s assets by \( \partial A \). Since new equity is fairly priced, the change in the value of the pre-existing shareholders’ equity is \( \partial E/\partial A - 1 \). A negative value for this quantity indicates debt overhang. Similar to previous figures that analyzed risk-shifting incentives, Figure 6 shows calculations of \( \partial E/\partial A - 1 \) for a bank that issued either subordinated debt, CoCos, or a COERC. As before, benchmark parameters and contract features are assumed.31

Relative to subordinated debt, Figure 6 shows that COERCs reduce debt overhang for any level of bank capital from 7% to 14% of deposits. In addition, for most capital levels the debt overhang is smaller for a bank that issues COERCs relative to one that issues CoCos. The only exception occurs at low capital when the two bonds are close to their conversion thresholds and \( \partial E/\partial A - 1 \) actually turns positive. The intuition for this result is that conversion due to a diffusion movement in asset value becomes more likely when capital is close to the threshold, an event that pays the bondholders’ their par values and that shareholders wish to avoid. However, taken as a whole, this analysis indicates that COERCs mitigate debt overhang and could improve financial stability by removing much of the bank’s disincentive to replenish capital.

---

31 The bank is assumed to have issued each bond at its fair credit spread when the bank’s total capital equaled 10% of deposits. For each bond, the derivative \( \partial E/\partial A \) is calculated numerically by the discrete approximation \( \Delta E/\Delta A \) where \( \Delta A = 0.125\% \) of deposits. The different values of equity for each asset (capital) level were calculated by 1 million Monte Carlo simulations.
VI. Extending the Model to Incorporate Deviations from Fundamentals

CoCos with market price triggers have been criticized because prices of bank stocks and CoCos, on which triggers are based, may not reflect the bank’s underlying fundamental asset value. Deviations of market prices from fundamentals can harm the bank’s shareholders if premature conversions provide CoCo investors with undervalued shares that heavily dilute the initial shareholders. Indeed, when a bank issues CoCos, speculators have the incentive to buy them and then short sell the bank’s stock to force an economically “unjustified” conversion that benefits CoCo investors at the expense of the diluted initial shareholders. Even without short-sellers, bank shareholders may be concerned that an irrational market panic or a “death spiral” could lead to unjustified conversions and dilutions of their ownership stake. A related concern is shareholders’ loss of control if CoCo investors hold a significant share of equity after conversion.

To illustrate the potential harm to shareholders, recall the simple example where a bank issues $1,000 in deposits, CoCos with a par value of $30, and 7 shares of stock initially valued at $10 per share, so that total capital is $100. Conversion is triggered, and CoCo investors are issued 6 new shares, when the market value of total capital falls to $65. Now suppose that a speculator buys the CoCos for $30 and, via short-selling or an unjustified rumour that generates a panic, manipulates the bank’s stock price down to $5. Conversion is triggered and CoCos convert to 6 new shares since the market value of capital now equals $65, the sum of the $30 CoCos plus the 7×$5 = $35 value of equity. After the short-selling ends or the panic subsides, suppose that the bank’s total capital is again recognized to equal its fundamental value of $100, so that the price per share is

32 Calomiris and Herring (2013) propose to outlaw short-selling of bank stocks to prevent death spirals.
$100/13 = $7.69. CoCo investors’ claim is now worth 6×$7.69 = $46.15 while the initial shareholders’ claim is worth 7×$7.69 = $53.85. As a result, this temporary manipulation or panic transfers $70 - $53.85 = $16.15 from shareholders to CoCo investors.

We now extend the basic model of Section IV to formally analyze how such deviations from market price fundamentals affects the values of CoCos and COERCs relative to the bank’s initial shareholders’ equity. As before, the “true” of “fundamental” value of the bank’s assets equals $A_t$ and follows the same risk-neutral rate of return process assumed in equation (2). However, the market value of capital no longer needs to satisfy the fundamental value relation $S_t \times n_0 + V_t = A_t - D_t$. Instead, the market value of the bank’s total liabilities can differ from the fundamental value of its assets:

$$S_t \times n_0 + V_t + D_t = A_t e^\eta_t$$

where $\eta_t$ is a “noise” term reflecting a deviation of the market value of liabilities from the fundamental value of assets, $A_t$. When $\eta_t$ is positive (negative), the bank’s liabilities are over- (under-) valued relative to the bank’s fundamental asset value.

It is natural to think that $\eta_t$ is non-zero because the bank’s stock price, $S_t$, fails to reflect the bank’s fundamental underlying assets, $A_t$. Asset values may be unobserved by outside investors because the bank’s portfolio holdings are not always known and some assets (e.g., most loans) are not traded. Moreover, manipulation by better-informed speculators could force stock prices from fundamental values due to limited arbitrage by lesser-informed investors. Overly optimistic or pessimistic beliefs regarding the banks’ prospects (“bubbles” or “panics”) may cause non-fundamental stock (and bond) prices.

As do Jurek and Yang (2007), we assume that the deviation from fundamentals, which we simply call “noise,” follows the mean-reverting Ornstein-Uhlenbeck process
where $\kappa_\eta > 0$ measures the speed at which noise is expected to revert to its unconditional mean of zero, and $\sigma_\eta$ is the volatility of changes in the level of noise.

As before, conversion occurs when inequality (7) holds, but the market value of capital, $S_{t} \times n_{0} + V_{t}$, now equals $A_{t} e^{\eta_{t}} - D_{t}$ rather than $A_{t} - D_{t}$. For standard CoCos, we assume that after conversion the noise, $\eta_{t}$, returns to zero, so CoCo investors receive max $[\alpha(A_{t} - D_{t}), 0]$ where, for example, $\alpha = 6/13$. For COERCs, we assume that during the rights offering period, shareholders make their decision whether to repurchase shares based on capital observed with noise, $A_{t} e^{\eta_{t}} - D_{t}$, but following that decision COERCs are again valued at the fundamental payoff (1). Thus, the payoff to COERCs is

\[
V_{t} = \begin{cases} 
B & \text{if } B \leq \alpha \left( A_{t} e^{\eta_{t}} - D_{t} \right) \\
\alpha \left( A_{t} - D_{t} \right) & \text{if } 0 < \alpha \left( A_{t} e^{\eta_{t}} - D_{t} \right) < B \\
0 & \text{if } A_{t} - D_{t} \leq 0 
\end{cases}
\]

where, for example, $\alpha = 30/37$.

Our analysis incorporating noise maintains the earlier parameter and contract assumptions. In addition, estimates for $\kappa_\eta$ and $\sigma_\eta$ of the noise process (10) are needed. One gauge of size and persistence of stock price deviations from fundamentals comes from “Siamese Twins,” which are two classes of shares that own fixed proportions of a firm’s dividends and assets. Rosenthal and Young (1990) note that the prices of these two share classes should always trade at a fixed ratio, equal to the ratio of their cash flow rights. But empirical evidence from two firms with such dual share classes, Royal
Dutch/Shell and Unilever NV/Unilever PLC, finds that the ratio of dual share prices persistently deviates from the fixed ratio of their fundamental cash flow rights. Jurek and Yang (2007) use daily stock price data of Royal Dutch/Shell and Unilever NV/PLC from 1970 to 2006 to estimate the noise process for $\eta$ in (10) and obtain average annualized values of $\sigma_\eta = 6.4\%$ and $\kappa_\eta = 3.56\%$, the latter estimate implying a mean reversion half-life of 49 trading days. We use their estimates assuming that bank total capital has the same deviations from fundamentals as these firms’ stocks. Since we model deviations at the asset level, we adjust for an average bank capital/asset ratio of roughly 14%, so that our estimate of $\sigma_\eta$ is $0.14 \times 6.4\% = 0.896\%$; that is, observed bank assets deviate from fundamental value with an annual standard deviation of slightly less than 1%. This noise deviation is about one-quarter of our model’s fundamental bank asset return standard deviation of slightly less than 4%).

As detailed in the Appendix, with noise affecting the observed asset value and capital, $S_t \times n_0 + V_t = A_t e^{\eta_t} - D_t$, the previously described risk-neutral valuation method is used to solve for the fair new issue credit spreads for CoCos and COERCs. Figure 7 graphs these credit spreads when banks have starting capital from 9% to 20% and when the fundamental value of assets is observed both without and with noise. As seen in the figure, noise has a much greater impact on the value of CoCos. For example, if a bank issues CoCos when its total capital/deposits is 9%, the fair credit spread without noise is

---

33 As discussed earlier, a 3.96% fundamental asset return standard deviation was the average calibrated from Bank of America, Citigroup, and JPMorgan Chase, and includes both diffusion and jump risks.

34 Our fair value calculations with noise assume that at the initial date 0, $\eta_0 = 0$. The calculations without noise are the same as those in Figure 1.

39
243 basis points (bps) but only 202 bps if total capital is observed with noise, a difference of 41 bps. In contrast, if the same bank issues COERCs, the fair credit spreads without and with noise would be 38 and 36 bps, respectively, a difference of only 2 bps.\(^{35}\)

The intuition for these results comes from the contractual payoffs of CoCos and COERCs. When \(\eta_t > 0\) so that the market value of capital is above its fundamentals (e.g., a “bubble”), conversion is delayed relative to fundamentals if 
\[
\frac{(A_t e^{\eta_t} - D_t)}{D_t} > \chi > \frac{(A_t - D_t)}{D_t}.
\]
This case does not transfer much value between bond investors and initial shareholders since the unconverted bonds continue to receive coupon payments.\(^{36}\) Alternatively \(\eta_t < 0\) where the market value of capital is below fundamentals (e.g., manipulation or panic). Here if 
\[
\frac{(S_t n_0 + V_t)}{D_t} = \left(\frac{A_t e^{\eta_t} - D_t}{D_t} \right) < \chi < \frac{(A_t - D_t)}{D_t},
\]
conversion gives CoCo investors a share of fundamental bank capital that exceeds their bond’s par value, benefiting them at the expense of the initial shareholders.\(^{37}\) Thus, for a given coupon spread, CoCos are worth more, and initial shareholders’ equity are worth

\(^{35}\) For better visual clarity, Figure 7 graphs credit spreads starting with a bank capital/deposit ratio of 9%. However, the same qualitative differences occur when the bank’s initial capital is as low as 7%: the CoCo credit spreads without and with noise are 788bp versus 705bp, a difference of 83bp. The COERC credit spreads without and with noise are 120bp and 121bp, a difference of less than 1 bp.

\(^{36}\) Delayed conversion may harm bondholders, but the effect is relatively small. If conversion is delayed but eventually occurs despite \(\eta_t > 0\), convertible bonds are more likely to suffer a loss of par value. This case is similar to that with no noise but a lower conversion threshold, \(\chi\), as illustrated in Figure 4. With smaller fundamental capital, a jump below the threshold makes it less likely that bonds would convert at par.

\(^{37}\) Since from (8) CoCo investors’ share of capital is \(\alpha = B/(\chi D_t)\), then \(B = \alpha \chi D_t = \alpha(A_t - D_t)\).
less, in the presence of noise. Hence, when this ex-post transfer of value is recognized initially, the fair credit spread of CoCos is significantly lower in the presence of noise.

In contrast, when $\eta_t < 0$ the same premature triggering of conversion is unlikely to transfer value from shareholders to COERC investors. Since shareholders repurchase the new shares issued to COERC investors whenever the shares’ observed market value exceeds the bonds’ par value, $B < \alpha \left(A_t e^{\eta_t} - D_t\right)$, that event is likely to continue. Only if $\alpha \left(A_t e^{\eta_t} - D_t\right) < B < \alpha \chi D_t < \alpha (A_t - D_t)$ is there a transfer from shareholders to COERC investors, which is unlikely because, as discussed in Section III, $\alpha$ for COERCs is significantly greater than $B/(\chi D_t)$, the proportion of shares given to CoCo investors. The noise at the end of the rights offering period, $\eta_t$, must be very negative to persuade shareholders to not exercise their repurchase option.

Since ex-post transfers from shareholders to COERC investors are much less likely, COERC and equity values are insensitive to noise from manipulation or panic. Recognizing this fact, speculators have little incentive to short sell the bank’s stock to prematurely trigger conversion. While noise is an exogenous process in our model, a more general model with incentive-based manipulation suggests that manipulation is less, and bank stock prices are more transparent, with COERCs compared to CoCos.

**VII. Further Considerations**

While not directly analysed by our model, several practical considerations relate to the adoption of CoCos and COERCs. Standard CoCos, especially ones with regulatory capital triggers whose conversion is likely to be decided by regulators, are often criticized
as hard to value. Credit rating agencies have been reluctant to rate them. COERCs’ relatively low default risk should make them much easier to value. Even if the timing of conversion is hard to predict, that COERCs are almost always repaid their par value in cash should qualify them for a high quality credit rating.

Under current U.S. tax law, the deductibility of interest paid on CoCos is in question. A CoCo bond might not be treated as debt for tax purposes if there is a “high” likelihood that it will be converted to equity (IRS Code Section 163). However, since COERCs are almost always repaid in cash, and only repaid in equity at a loss, a strong argument can be made that their tax treatment should be the same as standard bonds.

A bank that wishes to issue CoCos might find that their shareholders are reluctant to authorise a large share issue, not only due to possible dilution but also due to loss of control to new shareholders. However, COERCs allow existing shareholders to preserve their pre-emptive rights by purchasing the new share issue. A December 2013 Barclays CoCo was the first to have such a COERC-like feature. Its prospectus states that Barclays intends to “give shareholders the opportunity to purchase the ordinary shares created on conversion.” Moreover, when COERCs convert, the bank’s shareholders need not fear

---

38 For example, Credit Suisse’s CoCo, issued in February 2011, converts to equity if the bank’s core Tier I capital ratio falls below 7%. However, the Swiss regulator, FINMA, can also force conversion if it sees that Credit Suisse needs public funds to avoid insolvency. Fitch rated this CoCo BBB, but Moody’s and Standard & Poor’s did not rate it, citing uncertainty regarding potential losses. See “Credit Suisse CoCo Investors Uncertain How to Value Notes,” The Financial Times April 15, 2011.

39 In Europe, CoCo interest is tax deductible, which may explain why only European banks have issued CoCos thus far. European bank CoCos have often been marketed to U.S. investors.

40 However, unlike the COERC, the prospectus did not pre-specify the purchase price.
being liquidity constrained since they can sell their repurchase rights to nonconstrained investors. In many countries brokers automatically sell unexercised rights if investors fail to inform them of a decision, thereby guaranteeing the success of a profitable rights issue.

VIII. Conclusions

This paper introduces and analyses a new security, the COERC, whose design modifies the original CoCo proposal of Flannery (2005, 2009a). COERCs deal with several fundamental concerns regarding CoCos, especially ones with market value triggers. First, COERCs protect the bank’s shareholders from dilution losses due to manipulation or a panic-driven “death spiral.” COERCs avoid these losses by providing shareholders the option to buy back the shares from the COERC investors at the bond’s par value. Second, fixed-income investors may be reluctant to buy CoCos if they are exposed to high risks. COERCs’ relatively low credit risk alleviates this concern since at conversion the threat of dilution coerces shareholders to fully repay COERCs in cash. Third, the security is designed to rule out the problems of multiple equilibria or no equilibrium associated with market value triggers. COERCs are immune to such problems because their conversion is triggered by the market value of total capital.

Finally, relative to CoCos or non-convertible bonds, COERCs’ lower credit risk mitigates the excessive risk-taking incentives that are typically present in a levered firm. By reducing the possibility of wealth transfers between bond investors and shareholders, COERCs also help solve the debt overhang problem of high leverage.
Appendix

Derivation of the Deposit Credit Spread

The following is a derivation of the formula for $h_t$ in equation (5). Note that the risk-neutral rate of return on deposits equals

\[(A-1) \quad dD_t^* / D_t^* = (r_t + h_t) dt - \max \left( \frac{D_t - Y \cdot A_r}{D_t}, 0 \right) dq_t \]

In words, deposits earn the instantaneous risk-free return of $r_t + h_t$ per unit time but suffer a sudden loss equal to the bank’s negative net worth per deposit if the bank fails. Now define $H_t$ as the value of this loss per deposit conditional on a jump occurring. Then

\[(A-2) \quad H_t \equiv E_t^Q \left[ \max \left( \frac{D_t - Y \cdot A_r}{D_t}, 0 \right) \right] = E_t^Q \left[ \max \left( 1 - Y_t x_t, 0 \right) \right]
= \int_0^{U \times (1 - Y_t x_t)} \exp \left[ -\frac{\left( \ln Y - \mu_y \right)^2}{2 \sigma_y^2} \right] \frac{1}{Y \sigma_y \sqrt{2 \pi}} dy

Make the change in variable $y \equiv (\ln Y - \mu_y) / \sigma_y$. Then $y|_{y=0} = -\infty, y|_{y=1/x} = -(\ln x + \mu_y) / \sigma_y, Y = \exp[\mu_y + y \sigma_y], \text{ and } dy = dY/(Y \sigma_y). \text{ Defining } d_1 \equiv [\ln x + \mu_y] / \sigma_y, \text{ then}

\[(A-3) \quad H_t = \int_{-d_1}^{-d_2} \left( 1 - \exp \left[ \mu_y + y \sigma_y \right] \right) x e^{-y^2/2} \sqrt{2 \pi} dy = N(-d_1) - x e^{d_1} \int_{-d_1}^{-d_2} \exp \left[ y^2 / 2 \right] \frac{1}{\sqrt{2 \pi}} dy

Completing the square in the exponent, one obtains

\[(A-4) \quad \int_{-d_1}^{-d_2} \exp \left[ y \sigma_y - \frac{y^2}{2} \right] dy = e^{d_1^2/2} \int_{-d_1}^{-d_2} e^{-y^2/2} \frac{1}{\sqrt{2 \pi}} dy = e^{d_1^2/2} \int_{-d_1}^{-d_2} \frac{1}{\sqrt{2 \pi}} dy

where $d_2 = d_1 + \sigma_y \equiv [\ln x + \mu_y] / \sigma_y + \sigma_y$. Collecting terms together, one finds

\[(A-5) \quad H_t = N(-d_1) - x \exp \left[ \mu_y + \frac{1}{2} \sigma_y^2 \right] N(-d_2) = N(-d_2) - \exp \left[ \ln x + \mu_y + \frac{1}{2} \sigma_y^2 \right] N(-d_2)\]
For deposits to earn an instantaneous risk-neutral expected rate of return equal to the risk-free rate \( r_t \), it must be that \( h_t dt - \lambda dt H_t = 0 \), or \( h_t = \lambda H_t \). Multiplying \( H_t \) by the risk-neutral probability of a jump, \( \lambda \), gives equation (5) in the text.

**Uniqueness of Equilibrium and Feasibility of a Total Capital Trigger**

Define date \( \tau_f \) as the bank's failure date. Specifically, if \( A_t \) is the date \( t \) market value of the bank’s assets and \( D_t > 0 \) is the date \( t \) par value of the bank’s deposits, then

(A-6) \[ \tau_f = \inf \{ t \geq 0 : A_t \leq D_t \} \]

Now from Appendix equations (A-1) to (A-5), for any date \( t < \tau_f \) the instantaneous fair credit spread on deposits is derived such that the market value of deposits equals the par value of deposits, \( D_t \).\(^{41}\) Consequently, at any date \( t \) prior to bank failure at date \( \tau_f \), \( D_t \) denotes both the market value and par value of deposits.

Next define \( \tau_c \) as

(A-7) \[ \tau_c = \inf \{ t \geq 0 : A_t \leq D_t (1 + \chi) \} \]

Clearly, a comparison of (A-6) and (A-7) indicates that if \( \chi > 0 \), then \( \tau_c \leq \tau_f \). The date \( \tau_c \) can be considered a unique conversion date if a conversion trigger is based on the bank’s market value of assets. We now show that this same conversion date is equivalently defined by our paper’s proposed conversion trigger in equation (7) of the text.

As in the text, let \( S_t \) be the bank’s date \( t \) stock price, \( n_0 \) be the number of shares outstanding, and \( V_t \) be the market value of the convertible bond. Then in the absence of

\(^{41}\) Moreover, this credit spread \( h_t = \lambda H_t \) given in (A-5) depends only on the asset-to-deposit ratio, \( x_t = A_t / D_t \). It does not depend on how total capital, \( A_t - D_n \), is allocated between the market value of bank equity and the market value of convertible bonds. Thus, the credit spread is unaffected by the conversion event.
arbtrage, the market value of the bank’s assets must equal the market value of its total liabilities. At a date \( t \leq \tau_c \), this conservation of value – no arbitrage relation is

\[(A-8) \quad A_t = S_t \times n_0 + V_t + D_t \]

or rearranging

\[(A-9) \quad S_t \times n_0 + V_t = A_t - D_t \]

Our proposed conversion trigger (7) can be written as

\[(A-10) \quad S_t \times n_0 + V_t \leq D_t \chi \]

and substituting (A-9) into the left-hand side, it is equivalent to

\[(A-11) \quad A_t - D_t \leq D_t \chi \]

Adding \( D_t \) to both sides of inequality (A-11), one obtains

\[(A-12) \quad A_t \leq D_t \left(1 + \chi \right) \]

which is the same event defining date \( \tau_c \) in (A-7). Hence, since (A-10) and (A-12) describe equivalent events, the conversion date based on the market value of total capital

\[(A-13) \quad \tau_c = \inf \left\{ t \geq 0 : \left(S_t \times n_0 + V_t \right) \leq D_t \chi \right\} \]

describes exactly the same conversion date as (A-7) which is based on bank assets.

The feasibility of a total capital to senior debt (deposits) trigger depends on the availability of information on senior debt and the market prices of COERCs. Large U.S. banks already report their senior debt on a weekly basis.\(^{42}\) Whether COERCS will be sufficiently liquid to observe their market prices on a frequent basis cannot be known with certainty before they exist. However, unlike some other corporate bonds, our

evidence finds that subordinated debt issues of large banks are liquid.

To investigate the liquidity of large banks’ subordinated debt (for which COERCs will be a special case), we collected TRACE transactions from January 2007 to December 2011 for three different subordinated notes issued by Bank of America, Citigroup, and JPMorgan Chase. Summary statistics in Table 1 show that the average daily trades in these three bonds exceeded 32. Moreover, trading increased during 2008 and 2009 at the height of the crisis, so it was not the case that liquidity dried up during the crisis. For these three bonds, there were 20 instances when one of the bonds did not trade on a given day. However, five of these cases were on a November 11, which is the Veterans Day holiday when bond markets were open but overall trading is expected to be light. If we excluded these instances, there would be only 15 bond-days with zero trades, or an average of only one day per year with no trading. Since COERCs are designed to have relatively low credit risk and Bühler and Trapp (2009) find that lower risk increases bond liquidity, this evidence suggests that COERCs should be relatively liquid.

Monte Carlo Simulation Method

The following describes the risk-neutral valuation method for the case where bank assets are observed with noise. Valuation for the basic model without noise is a special case with the noise term, $\eta_t$, set to zero.

The risk-neutral process for the fundamental value of the bank’s assets, $A_t$, equals the assets’ risk-neutral rate of return plus deposit growth less the payouts of interest to depositors and dividends to shareholders. In addition, as long as bonds remain unconverted, payouts include bond coupons. Thus, if $A_t e^{\eta_t}$ equals the observed (with noise) bank asset value, the actual (fundamental) asset value follows the process
\[
\begin{align*}
DA_t &= \left(\frac{DA^*_t}{A^*_t}\right) A_t + dD - (r_i + h_i) D_t dt - c_i B - \delta (A_t e^{\eta} - B - D_t) dt \\
&= \left[\left(r_i - \lambda k - \delta e^{\eta}\right) A_t + \left(g \left(x_t e^{\eta} - \tilde{x}\right) - r_i - h_i + \delta \left(c_i - \delta\right) b_t\right) \right] dt \\
&\quad + \sigma A_t dz + \left(Y_{q^r} - 1\right) A_t dq
\end{align*}
\]

where we have substituted in equations (2) and (4). Equation (A-14) can be rewritten as

\[
\begin{align*}
\frac{dA_t}{A_t} &= \left[\left(r_i - \lambda k - \delta e^{\eta}\right) + \frac{g \left(x_t e^{\eta} - \tilde{x}\right) - r_i - h_i + \delta \left(c_i - \delta\right) b_t}{x_t} \right] dt + \sigma dz + \left(Y_{q^r} - 1\right) dq
\end{align*}
\]

where \(b_t \equiv B/D_t\). Thus, the risk-neutral process for the asset/deposit ratio is

\[
\begin{align*}
\frac{dA_t}{A_t} &= \left[\left(r_i - \lambda k - \delta e^{\eta}\right) + \frac{g \left(x_t e^{\eta} - \tilde{x}\right) - r_i - h_i + \delta \left(c_i - \delta\right) b_t}{x_t} \right] dt \\
&\quad + \sigma dz + \left(Y_{q^r} - 1\right) dq
\end{align*}
\]

A simple application of Itô’s lemma for jump-diffusion processes implies

\[
\begin{align*}
d\ln x_t &= \left[\left(r_i - \lambda k - \delta e^{\eta}\right) + \frac{g \left(x_t e^{\eta} - \tilde{x}\right) - r_i - h_i + \delta \left(c_i - \delta\right) b_t}{x_t} \right] dt \\
&\quad + \sigma dz + \ln Y_{q^r} dq
\end{align*}
\]

When bonds are assumed to pay floating coupons, \(c_t = r_i + s\) in equation (A-17). Also, when bank assets are observed with noise, the deposit credit spread, \(h_t\), is given by equation (5) in the text but with \(x_t\) replaced by \(x_t e^{\eta}\). Furthermore, a trivial application of Itô’s lemma shows that \(b_t = B/D_t\) evolves as

\[
\begin{align*}
\frac{db_t}{b_t} &= g \left(\tilde{x} - x_t e^{\eta}\right) dt
\end{align*}
\]

To value subordinated debt or a CoCo or COERC bond, we compute the expression

\[
\begin{align*}
V_0 &= E^Q_0 \left[\int_0^T e^{-\int_0^t r_s ds} V(t) dt\right]
\end{align*}
\]
where \( \nu(t) \) is the bond’s cash flow per unit time paid at date \( t \). \( \nu(t) = c_tB = (r_t+s)B \) as long as the bond is not converted or the bank has not failed. If date \( T \) is reached without the bond converting or the bank failing, there is a final cash flow of \( B \). Given inequality (7), conversion of a CoCo or COERC is triggered the first time that the observed capital to deposit ratio falls below the trigger threshold:

\[
(A-20) \quad x_t e^{\eta t} - 1 \leq \chi
\]

In this case, there is a final cash flow given by equation (1). For the case of subordinated debt, bank failure occurs the first time that the par value of total debt exceeds observed assets, or \( x_t e^{\eta t} \leq 1 + b_t \). In that case, subordinated debt holders receive a final cash flow of \( \min\{ B, \max\{A_t-D_t,0\}\} \).

The right-hand side of equation (A-19) is calculated using a technique similar to Zhou (2001), which is a discretization method for Monte Carlo valuation of a mixed jump-diffusion process. We generalize his approach for the case of our four state variables: the default-free short rate, \( r_t \), which follows the process (3); the asset/deposit ratio, \( x_t \), which satisfies the jump-diffusion process (A-17); the bond par value/deposit ratio, \( b_t \), which follows the process (A-18); and noise, \( n_t \), which follows the process (10).

Divide the time interval \([0,T]\) into \( n \) equal sub-periods, where \( \Delta t = T/n \) is the length of each period. \( n \) is chosen to be large so that a small \( \Delta t \) leads to an accurate discrete-time approximation of the model’s continuous-time processes. With time measured in years, our empirical work assumes \( \Delta t = 1/250 \), the length of one trading day.

Let \( t \) denote the end of trading day \( t-\Delta t \) and the beginning of trading day \( t \). Then the discrete-time process corresponding to (3) is
\[(A-21)\]
\[
r_{t+\Delta t} = r_t + \kappa_r (\bar{r} - r_t) \Delta t + \sigma_r \sqrt{\Delta t} \xi_{t+\Delta t} + \sigma_r \sqrt{\Delta t} \xi_{t+\Delta t} \]
\[
= \bar{r} \kappa_r \Delta t + r_t (1 - \kappa_r \Delta t) + \sigma_r \sqrt{\Delta t} \xi_{t+\Delta t} \]

where \(\xi_{t+\Delta t} \sim N(0,1)\) are serially independent shocks representing Brownian motion uncertainty. Similarly, the discrete-time process corresponding to \((A-17)\) is

\[(A-22)\]
\[
\ln x_{r+\Delta t} = \ln x_t + \left[ r_t - \lambda k - \delta e^\rho + \frac{g\left(x_t e^\rho - \hat{x}\right) - r_t - \eta_t - \delta - (c_t - \delta) b_t}{x_t} - g\left(x_t e^\rho - \hat{x}\right) - \frac{\sigma^2}{2} \right] \Delta t
\]
\[
+ \sigma \sqrt{\Delta t} \varepsilon_{t+\Delta t} + \ln Y_{t+\Delta t} \theta_{t+\Delta t}
\]

where \(\varepsilon_{t+\Delta t} \sim N(0,1)\) are serially independent shocks, \(E_t \left[\varepsilon_{t+\Delta t} \xi_{t+\Delta t}\right] = \rho, \ln Y_{t+\Delta t} \sim N\left(\mu_y, \sigma_y^2\right)\), and

\[(A-23)\]
\[
\theta_{t+\Delta t} = \begin{cases} 
1 & \text{with probability } \Delta t \lambda \\
0 & \text{with probability } 1 - \Delta t \lambda 
\end{cases}
\]

Finally, the discrete-time analogues of equations \((A-18)\) and \((10)\) are

\[(A-24)\]
\[
b_{t+\Delta t} = b_t \exp\left[-g\left(x_t e^\rho - \hat{x}\right) \Delta t\right]
\]

\[(A-25)\]
\[
\eta_{t+\Delta t} = \left(1 - \kappa_\eta \Delta t\right) \eta_t + \sigma_\eta \sqrt{\Delta t} \psi_{t+\Delta t}
\]

where \(\psi_{t+\Delta t} \sim N(0,1)\) are serially independent shocks uncorrelated with \(\xi_{t+\Delta t}\) and \(\varepsilon_{t+\Delta t}\).

Each Monte Carlo simulation of \((A-21)\) to \((A-25)\) calculates one realization of the right-hand side of \((A-19)\), and taking the average of at least 100,000 of them gives \(V_0\) for a given spread, \(s\). \(s\) is varied until the (fair) one is found such that \(V_0 = B\).
References


TABLE 1

Daily Trading Statistics for Subordinated Debt of Major US Banks

Table 1 reports trading statistics computed from daily TRACE data over the period January 2, 2007 to December 30, 2011. The statistics are for trades in three different bonds: 1) Bank of America’s 12-year subordinated note issued December 2003 with 5.25% semi-annual coupon, cusip 060505BG8; 2) Citigroup’s 10-year subordinated note issued August 2002 with 5.625% semi-annual coupon, cusip 172967BP5; and 3) JP Morgan Chase’s 10-year subordinated note issued October 2005 with 5.15% semi-annual coupon, cusip 46625HDF4. The statistics are for the number and dollar volume of trades each day. TRACE does not specify the dollar amount for trades exceeding $5 million, and in those cases a $5 million transaction size is assumed.

<table>
<thead>
<tr>
<th></th>
<th>Bank of America</th>
<th></th>
<th>Citigroup</th>
<th></th>
<th>JP Morgan Chase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trades</td>
<td>$ Volume</td>
<td>Trades</td>
<td>$ Volume</td>
<td>Trades</td>
<td>$ Volume</td>
</tr>
<tr>
<td>Average</td>
<td>33.4</td>
<td>2674066</td>
<td>33.3</td>
<td>4965545</td>
<td>32.9</td>
<td>3979899.8</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>27.8</td>
<td>4338850</td>
<td>23.5</td>
<td>7664606</td>
<td>19.4</td>
<td>5863343.9</td>
</tr>
<tr>
<td>1 Percentile</td>
<td>1</td>
<td>13880</td>
<td>3</td>
<td>55880</td>
<td>5</td>
<td>30000</td>
</tr>
<tr>
<td>5 Percentile</td>
<td>5</td>
<td>101350</td>
<td>11</td>
<td>299850</td>
<td>10</td>
<td>242000</td>
</tr>
<tr>
<td>25 Percentile</td>
<td>13</td>
<td>533750</td>
<td>20</td>
<td>1164750</td>
<td>20</td>
<td>756000</td>
</tr>
<tr>
<td>50 Percentile</td>
<td>26</td>
<td>1258000</td>
<td>28</td>
<td>2428500</td>
<td>30</td>
<td>1746000</td>
</tr>
<tr>
<td>75 Percentile</td>
<td>47</td>
<td>2963750</td>
<td>40</td>
<td>5853750</td>
<td>42</td>
<td>5002750</td>
</tr>
<tr>
<td>95 Percentile</td>
<td>85</td>
<td>9562050</td>
<td>75</td>
<td>16686150</td>
<td>66</td>
<td>14780900</td>
</tr>
<tr>
<td>99 Percentile</td>
<td>122</td>
<td>20980080</td>
<td>119</td>
<td>32975180</td>
<td>86</td>
<td>30608380</td>
</tr>
<tr>
<td>Median 2007</td>
<td>9</td>
<td>538000</td>
<td>22</td>
<td>1742000</td>
<td>15</td>
<td>2367000</td>
</tr>
<tr>
<td>Median 2008</td>
<td>14</td>
<td>708000</td>
<td>27</td>
<td>1666000</td>
<td>30</td>
<td>1556000</td>
</tr>
<tr>
<td>Median 2009</td>
<td>44</td>
<td>1916500</td>
<td>47</td>
<td>5032500</td>
<td>39</td>
<td>1597500</td>
</tr>
<tr>
<td>Median 2010</td>
<td>43</td>
<td>2050500</td>
<td>30</td>
<td>2428500</td>
<td>37</td>
<td>2344500</td>
</tr>
<tr>
<td>Median 2011</td>
<td>33</td>
<td>1571500</td>
<td>27</td>
<td>2331000</td>
<td>31.5</td>
<td>1503500</td>
</tr>
<tr>
<td>Total Days With Zero Trading</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 1

Tier 1 Capital Ratios versus Market Capital Ratios

Figure 1 Panel A is Chart 5 of Haldane (2011, page 14) and Figure 1 Panel B is Chart 7 of Haldane (2011, page 15). The following is from his descriptions. The data sources are Capital IQ and Bank of England calculations. “Crisis” banks are a set of major financial institutions which in autumn 2008 either failed, required government capital or were taken over in distressed circumstances. These are RBS, HBOS, Lloyds TSB, Bradford & Bingley, Alliance & Leicester, Citigroup, Washington Mutual, Wachovia, Merrill Lynch, Freddie Mac, Fannie Mae, Goldman Sachs, ING Group, Dexia and Commerzbank. The “no crisis” institutions are HSBC, Barclays, Wells Fargo, JP Morgan, Santander, BNP Paribas, Deutsche Bank, Crédit Agricole, Société Générale, BBVA, Banco Popular, Banco Sabadell, Unicredit, Banca Popolare di Milano, Royal Bank of Canada, National Australia Bank, Commonwealth Bank of Australia and ANZ Banking Group. Panels A and B show unweighted averages for these “crisis” and “no crisis” institutions in the sample for which data are available on the given day. Panel A reports the banks’ average Tier 1 capital ratios and Panel B reports the banks’ average 30-day moving average of the market value of equity to book-value of debt.

Panel A. Tier 1 Capital Ratios for “Crisis” and “No Crisis” Banks
Panel B. Market Capitalisation to Book-Value of Debt
FIGURE 2

Credit Spreads of Subordinated Debt, CoCos, and COERCs

Figure 2 shows credit spreads on 5-year maturity, floating-coupon bank bonds as a function of the bank’s capital to deposit ratio when the bonds are issued at their par values. The spreads, in basis points, represent a bond’s annual coupon rate in excess of the short-term default-free interest rate and are computed from the model and parameter values described in the text. Graphed are the spreads for a standard CoCo bond, a COERC bond, and a non-convertible subordinated bond. CoCos and COERCs convert when the ratio of the bank’s market value of total capital to deposits falls below 6.5%.
Figure 3 shows credit spreads on 5-year maturity, floating-coupon COERC bonds as a function of the bank’s capital to deposit ratio when the bonds are issued at their par values. The spreads, in basis points, represent a bond’s annual coupon rate in excess of the short-term default-free interest rate and are computed from the model and parameter values described in the text. COERCs convert when the ratio of the bank’s market value of total capital to deposits falls below 6.5%. Spreads are graphed for COERCs with different dilution ratios, $\alpha$, defined as the proportion of total bank shares issued to COERC investors at the time of conversion.
Figure 4 shows credit spreads on 5-year maturity, floating-coupon COERC bonds as a function of the bank’s capital to deposit ratio when the bonds are issued at their par values. The spreads, in basis points, represent a bond’s annual coupon rate in excess of the short-term default-free interest rate and are computed from the model and parameter values described in the text. Spreads are graphed for COERCs with different trigger thresholds, \( \chi \), defined as the bank’s market value of total capital to deposits ratio at which conversion occurs.
Figure 5 shows the increase in a bank’s market value of shareholders equity, $E$, when the bank increases the standard deviation of jumps in its assets’ value, $\sigma_y$. This risk-shifting incentive is computed from the model and parameter values described in the text and is shown for a bank that issues a standard CoCo bond, a COERC bond, or a non-convertible subordinated bond. In each case, the bond is assumed to be issued at its par value when the bank’s initial capital to deposit ratio was 10%. CoCos and COERCs convert when the ratio of the bank’s market value of total capital to deposits falls below 6.5%. The risk-shifting incentives, $\partial E/\partial \sigma_y$, are shown for current capital to deposit ratios between 7 and 20%.
Debt Overhang when Banks Issue Subordinated Debt, CoCos, and COERCs

Figure 6 shows the increase in a bank’s market value of initial shareholders equity when it issues new shareholders equity that increases its asset value by one unit, $\frac{\partial E}{\partial A} - 1$. A negative value for $\frac{\partial E}{\partial A} - 1$ represents “debt overhang.” This measure of debt overhang is computed from the model and parameter values described in the text and is shown for a bank that issues a standard CoCo bond, a COERC bond, or a non-convertible subordinated bond. In each case, the bond is assumed to be issued at its par value when the bank’s initial capital to deposit ratio was 10%. CoCos and COERCs convert when the ratio of the bank’s market value of total capital to deposits falls below 6.5%. The debt overhang measures, $\frac{\partial E}{\partial A} - 1$, are shown for current capital to deposit ratios between 7 and 14%.
Figure 7 shows credit spreads on 5-year maturity, floating-coupon bank bonds as a function of the bank’s capital to deposit ratio when the bonds are issued at par. The spreads, in basis points, represent a bond’s annual coupon rate in excess of the short-term default-free interest rate and are computed from the model and parameter values described in the text. Graphed are the spreads for a standard CoCo bond and a COERC bond where conversion occurs when the ratio of the bank’s market value of total capital to deposits falls below 6.5%. For each type of bond, in one case the bank’s market value of total capital is assumed to reflect the bank’s fundamental asset value without error. In the other case it is assumed to reflect the bank’s fundamental asset value with noise.