Abstract

Consider a firm facing two consumer segments with differing valuations for quality. The demand is stationary and known, and consumers make repeat purchase. However, once a premium product is introduced, the valuations of the consumers change in the next period. The firm derives a cost savings due to learning effect in the second period, the magnitude of which depends on the volume in the first period. Under such a situation, should a firm introduce a premium product before a basic product or vice-versa? Should it introduce two products simultaneously? What should it then do in the second period? Should it introduce a single product into the market first and expand its product offering later? Using a two-period stylized model we seek to answer these questions. We characterize the optimal product introduction strategy for the firm. We show how the firm’s choice is influenced by cost savings and valuation changes. The managerial implications of the model are also discussed.

Keywords: Marketing, Product Design; Segmentation; Valuation Change; Marketing/Manufacturing Interface
1. INTRODUCTION

Firms periodically introduce new products or line extensions of existing products. While introducing such products, firms often segment the market and offer differentiated products for the premium consumer segment and for the basic consumer segment. The timing of such product introductions is an important strategic decision that companies face. Many different product introduction strategies are observed in practice, which often differ within an industry. For example, on June 26, 1990, IBM released its PS/1 home computer, several years after its high-end PS/2 business-oriented microcomputer (Moorthy & Png, 1992). Intel Corporation followed the opposite strategy with its Pentium 4 line of microprocessors. To quote from *The New York Times* (Gaither, 2002):

> *Intel first began selling its Pentium 4 chips in personal computers about 15 months ago and is gradually working the new chip design into systems for servers, which are high-powered computers that run Web sites and databases. Intel has historically introduced new chip technologies into its products for servers first.*

The book publishers routinely introduce the hardbound books before the paperback editions (McDowell, 1989). Palm Inc. followed a similar strategy: the premium version, Palm Vx, was introduced in October 1999. Subsequently, it expanded the product line in August 2000 by introducing a basic product, the Palm m100 (Palm, 2000). On the contrary, Levi Strauss & Co introduced the Dockers brand trousers before introducing the premium brand of Slates trousers. In fact, while launching the Slates brand, a “basic” version of the brand was introduced in 1996. Subsequently, Levi upgraded the brand in 1999, withdrawing the basic version completely (Levi Strauss, 1999).

Why do some manufacturers introduce high-end products before the low-end products while others do the opposite? When technological feasibility exists, how should a firm decide whether to follow the strategy of introducing the high-end product followed by a low-end product or the reverse strategy of low-end product followed by the high-end product? Should it introduce two products simultaneously? Should it introduce a single product into the market first and expand its product offering later? When is it optimal to have only a single product instead of two differentiated products? How will the decisions
for the firm change if there is a valuation change for the consumers? We propose a formal model in this paper to answer these questions.

Wilson and Norton (1989) consider the problem of deciding the optimal timing of introducing new products. Cohen, Eliashberg, and Ho (1996) look at the tradeoff between time to market and product performance. Moorthy and Png (1992) address the problem of timing of introduction of two differentiated durable products serving two segments with the product introduction time periods fixed exogenously. They show the conditions under which the seller would commit to sell high-end product in the first period followed by a low-end product in the next period. We differ from Moorthy and Png (1992) in three important ways. First, we allow repeat purchase in multiple periods. Two additional features distinguish our paper from theirs: one from the customer side and the other from the cost side. When a new high-end feature is introduced, it usually commands a premium, which tends to disappear over time. With time, customer’s expectations change; what was considered a luxury in the past could become a necessity now. In authors own experience, a 3 mega-pixel digital camera deemed luxury feature in 2000 became a must have attribute a year later. Thus once a premium attribute is introduced in the market, the customers discount it in the future as its “value” diminishes for them. Our model incorporates this valuation change.

On the production side, we allow the cost in the second period to be lower than that of the first period because of the learning/experience curve (Hall and Howell, 1985). A function of the sales volume in the first period, this cost reduction permits the manufacturer to upgrade the quality of even the low end product and/or decrease prices, a phenomenon so often observed in the computer and semiconductor industry (Noyes, 1977).

Temporal product introduction strategy may involve one or multiple products in any time period. The different product introduction strategies considered are: single product strategies, multiple products strategies, product line expansion strategies, and product line consolidation strategies. We provide insights about the best product introduction strategy for a firm and how it is affected as cannibalization between products increases. We show that if the degree of cannibalization is very high between the basic and the premium product, it is optimal for the firm to offer a single product in each period. However, as
the degree of cannibalization goes down, multiple products strategies become attractive to the firm. Depending upon the degree of cannibalization, either an expansion strategy or a consolidation strategy can be optimal. We also discuss the best single product strategy for the firm and benchmark our results against the no valuation change, no cost savings case. Our work adds to the growing body of literature on product development decisions using an operations/marketing framework. Krishnan and Ulrich (2001) provide an excellent review of this literature.

The remainder of this paper is organized as follows. Our model is presented in Section 2. Section 3 describes our results for the single product strategies while Section 4 describes our results for multiple products strategies. We compare and contrast all strategies in Section 5 and describe the best product strategy for the firm. Section 6 summarizes the results and gives the implications for the practitioners.

2. THE MODEL

Consider a product that may be differentiated on some dimension on which consumers agree in their preference ordering, for example, focal lengths for cameras, style/color/fabric for garments etc. For brevity we call this dimension “quality”. We model repeat purchases by consumers over two time periods. Consider a market consisting of a high segment and a low segment, denoted by $H$ and $L$ respectively. The size of segment $H$ is $n_H$ and the size of segment $L$ is $n_L$. We will denote by $n = \frac{n_H}{n_H + n_L}$ the fraction of high-end customers. A maximum of two products can be introduced in each period; one with high quality and the other with low quality. We will use the subscript to denote the quality level and the superscript to denote the time period. Thus, $q_{H1}^t$ and $q_{L1}^t$ ($q_{H2}^t$ and $q_{L2}^t$) will denote the quality levels for the high and low products, respectively, during the first (second) period. The high segment values quality level of product $j$ at $v_H q_j$ and the low segment values it at $v_L q_j$ (with $v_H > v_L$). The surplus to customer $i$ ($i = H, L$) from product $j$ ($j = H, L$) priced at $p_j^t$ ($t = t_1, t_2$) is $p_j^t - v_i' q_j^t$. The customer buys the product that gives her the maximum non-negative surplus.
On the supply side, there is a single monopolistic supplier whose marginal cost of supplying the product increases with quality. Following the literature, we assume that the cost is quadratic in the quality level with cost coefficient $c$; for quality level $q$, the cost is $cq^2$. The customers are in the market in both periods; buying a product in each period if it provides them with non-zero surplus. If a high product is introduced in the first period, customers’ valuation goes down in the second period. Let $\psi_H$ and $\theta_L$, with $0 < \psi \leq 1$ and $0 < \theta \leq 1$, be the new valuations for the high and low segment respectively in the second period, if a high product in introduced in the first period. We assume that $\psi_H > \theta_L$. The manufacturer realizes cost savings in the second period due to learning effect if any product is sold in the first period. The magnitude of cost savings depends on the sales volume in the first period. Let $\alpha (0 \leq \alpha < 1)$ denote the maximum fractional cost savings per unit, which is achieved when both the segments buy the product in the first period. Under such a scenario, $c'=(1-\alpha)c$ is the cost coefficient in the second period. If only the high segment buys the product in the first period then the cost coefficient in the second period is $c''=(1-\alpha n)c$.

In each of the two periods the manufacturer has three choices: offer low product, offer high product, or offer both products. Thus, there are total nine strategies that we organized into five categories based on the number of products offered in each period. We will use category and strategy interchangeably.

1. **Stable Single-Product.** In this strategy, the manufacturer offers one and the same product in both periods. There are two possibilities. In strategy SS1, the manufacturer offers the low products in both periods while strategy SS2 refers to offering high product in both periods.

2. **Varying Single-Product.** The manufacturer offers only one product in each period, but the products change from the first to the second period. Under strategy VS1 the low product is followed by the high product; while under strategy VS2 the high product is followed by the low product.

3. **Expanding Product Line.** The manufacturer begins with a single product in period 1 and expands the product line to two products in period 2. In strategy EP1, the
low product is offered in period 1 while the high product is offered in period 1 in strategy EP2.

4. *Consolidating Product Line*. In this strategy two products are offered in period 1 but only the low product is offered in option CP1 and only the high product is offered in option CP2.

5. *Stable Product Line*. In this strategy both products are offered in each period. This single option under this strategy is denoted by SP.

In all these strategies, when there is only one product designed for the low segment is offered, the high segment also buys it since it derives positive surplus from such a product. When only one product designed for the high product is offered, the low segment’s surplus from it is negative and thus only the high segment buys it. Table 1 summarizes the customers that are served under each of the above strategies. Table 1 also shows the cost savings and the valuation changes under each strategy.

We now describe the models corresponding to the strategies mentioned above. For the reasons of brevity we describe the model formulation for only two of the above strategies; the rest have similar formulation.

First consider strategy SS1, offering low product in both periods. Since high product is not introduced in the first period, there is no valuation change in the second period. Cost of quality in period 2 is $c'$ and the seller will discount the profit obtained from the second period by a factor $\delta (0<\delta \leq 1)$. The firm’s problem is described below.

**Model SS1**

$$\text{Max } (n_H + n_L)[p^{\text{L1}} - c(q^{\text{L1}})^2] + \delta(n_H + n_L)[p^{\text{L2}} - c'(q^{\text{L2}})^2] \quad (1)$$

Subject to:

$$v_L q^{\text{L2}} - p^{\text{L2}} \geq 0 \quad (2)$$

$$v_L q^{\text{L1}} - p^{\text{L1}} \geq 0 \quad (3)$$

$$q^{\text{L1}}, q^{\text{L2}} \geq 0 \quad (4)$$

In this formulation, (1) is the total discounted profit for the manufacturer from the two periods. The constraints (2) and (3) represent the participation constraints for the
consumers in periods 1 and 2 respectively. With our assumption of \( v_H > v_L \), serving only the low segment is impossible because when the low segment buys (i.e., \( p \leq v_L q_L \)), so does the high segment. Thus, the participation constraints for the high segment are redundant.

Once the first period’s customers are known, the cost coefficient and valuations for the second period are determined. This enables us to analyze each period’s problem independently. Consider the second period problem only. Extracting the entire surplus from the high segment is not possible. However, nothing prevents the manufacturer from extracting the entire surplus from the low segment. Hence, the participation constraint for the low segment given by equation (2) must satisfy to equality. This implies we need to set \( p_L^{i2} = v_L q_L^{i2} \). Similarly, no surplus for the low segment during the first period means that \( p_L^{i1} = v_L q_L^{i1} \). Substituting these in the objective function and using the first order conditions we obtain the optimal quality and profit as

\[
q_L^{i2*} = \frac{v_L}{2c'} = \frac{v_L}{2c(1-\alpha)} ; \quad q_L^{i1*} = \frac{v_L}{2c} ,
\]

\[
\pi^*(SSI) = (n_H + n_L) \frac{v_L^2}{4c} \left(1 + \frac{\delta}{1-\alpha}\right).
\]

We next describe the model formulation for EP2. Under this strategy, the firm offers a high product in the first period and two products, one high and one low, in the second period. Recall that offering high product in first period causes valuation changes in the second period for both segments. Cost per unit quality in the second period is given by \( c'' = (1-\alpha)c \). The manufacturer’s problem is described below.

**Model EP2**

\[
\text{Max } n_H [p_H^{i1} - c(q_H^{i1})^2] + \delta \{n_L [p_L^{i2} - c''(q_L^{i2})^2] + n_H [p_H^{i2} - c''(q_H^{i2})^2]\} \quad (7)
\]

Subject to:

\[
\theta v_L q_L^{i2} - p_L^{i2} \geq 0 \quad (8)
\]

\[
\psi v_H q_H^{i2} - p_H^{i2} \geq 0 \quad (9)
\]

\[
v_H q_H^{i1} - p_H^{i1} \geq 0 \quad (10)
\]
The objective function (7) maximizes the total discounted profit for the manufacturer. The constraints (8), (9), and (10) are the participation constraints and equations (11) and (12) are the self selection constraints for the low and high segment respectively in the second period. Since the manufacturer cannot identify the customer’s type, she must design the product line so that each segment voluntarily chooses the product-price combination meant for it. Note that such a situation does not arise in the first period as the manufacturer offers only one product targeted to the high segment.

As described in the earlier model, once the first period’s customers are known, the cost coefficient and valuations are determined. This enables us to analyze each period’s problem independently. Consider the second period problem only. If the manufacturer tries to extract the high segment’s entire consumer surplus by setting \( p_H^{i_2} = \psi v_H q_H^{i_2} \), then these customers will switch to the lower quality product and will get a positive surplus. The manufacturer will extract the lower segment’s entire surplus and set \( p_L^{i_2} = \theta v_L q_L^{i_2} \).

Given our assumption \( \psi v_H > \theta v_L \), only one of the two self-selection constraints, (11) and (12), will be binding. Since the high segment customers are more willing to pay for higher quality, the manufacturer should direct the higher quality product to them, i.e., set price \( p_H^{i_2} \) such that the high segment will be indifferent between the two products. Thus (12) will be binding to give

\[
p_H^{i_2} = p_L^{i_2} + \psi v_H (q_H^{i_2} - q_L^{i_2}) = \psi v_H q_H^{i_2} - (\psi v_H - \theta v_L) q_L^{i_2}.
\]

Now consider the first period problem. The manufacturer offers only a high product in the first period. As a result, she will set the price such that she is able to extract the entire surplus from the high segment. Thus,

\[
p_H^{i_1} = v_H q_H^{i_1}.
\]
Substituting (14), (15), and (16) into the objective function (7) and carrying out the optimization we get the following results for the EP2 model.

\[
q_{1H}^* = \frac{v_H}{2c}, \quad q_{1L}^* = \frac{\theta v_L (1 - \bar{R})}{2c} = \frac{v_L}{2c} \frac{\theta}{1 - \alpha n} (1 - \bar{R}); \quad q_{2H}^* = \frac{\psi v_H}{2c}, \quad q_{2L}^* = \frac{\psi}{2c}\frac{v_L}{1 - \alpha n};
\]

\[
\pi^* (EP2) = \frac{n_H v_H^2}{4c} (1 + \frac{\delta \psi^2}{1 - \alpha n}) + \frac{n_L v_L^2}{4c} \frac{\delta \theta^2}{1 - \alpha n} (1 - \bar{R})^2,
\]

where, \( \bar{R} = \frac{n_H}{n_L} \left( \frac{v_H}{v_L} - 1 \right) \).

In the similar manner, the optimal quality and profits for each strategy can be obtained by solving the profit maximization problem with appropriate participation and self-selection constraints. Table 2 describes the optimal quality and profit level under each strategy. We have used the notation \( R = \frac{n_H}{n_L} \left( \frac{v_H}{v_L} - 1 \right) \) in Table 2. As we will see in the next section, \( \bar{R} \) is measure of cannibalization between the high product and the low product when the valuations for the two segments are \( \psi v_H \) and \( \theta v_L \) respectively; while \( R \) is a measure of cannibalization between the high product and the low product when the valuations for the two segments are \( v_H \) and \( v_L \) respectively. Note that the solutions for the EP1, CP1, and CP2 are feasible if and only if we have a non-negative quality, i.e. \( R \leq 1 \). Similarly, the EP2 strategy is feasible if and only if \( \bar{R} \leq 1 \). The feasibility of the SP strategy requires both \( R \leq 1 \) and \( \bar{R} \leq 1 \).

The next three sections describe our results. The single product strategies are discussed in Section 3. Section 4 describes our results for the multiple products strategies. We compare all strategies in Section 5 and characterize the best product strategy for the firm.

### 3. SINGLE PRODUCT STRATEGIES

Under a single product strategy, the firm can introduce at most one product in each period. The relevant strategies are (refer to Table 1): SS1, SS2, VS1, and VS2. We begin with a brief discussion about the single period problem. The firm has only three
options in a single period: (1) offer a low product, so that both segments will buy, (2) offer a high product for the high segment only, and (3) offer two products, one for the low segment and other for the high segment. Under Case (1), it is easy to see that the firm will offer a quality \( \frac{v_L}{2c} \), which is the efficient quality for the low segment, and the profit for the firm is \((n_H + n_L) \frac{v_L^2}{4c}\). When the firm offers only a single high product (Case 2), it offers a quality \( \frac{v_H}{2c} \) to extract the entire surplus from a high customer. The profit for the firm is \( n_H \frac{v_H^2}{4c} \). Under Case 3, the firm lets the consumers self-select the products. The optimal quality levels are \( q_L = \frac{v_L}{2c} (1 - R) \) and \( q_H = \frac{v_H}{2c} \); and the profit for the firm is \( n_H \frac{v_H^2}{4c} + n_L \frac{v_L^2}{4c} \). Note that these results are feasible only for \( R < 1 \). As noted by Moorthy and Png (1992), \( R \) is a measure of cannibalization between the low and the high product. It increases with the increasing values of the ratios \( n_H / n_L \) or \( v_H / v_L \). When \( R > 1 \), the cannibalization between the two products is so high that it is no longer optimal for the firm to offer two products. In other words, an optimal two-product solution does not exist for \( R > 1 \). Reasoning in a similar manner, \( \frac{n_H}{n_L} \left( \frac{qv_H}{q_H} - 1 \right) \) is measure of cannibalization between the two products when the valuations for the two segments are \( qv_H \) and \( q_H \). Thus, \( \frac{n_H}{n_L} \) is a measure of cannibalization in the second period, if a change in valuation has taken place (because of the introduction of the high product in the first period).

Let \( X = \frac{v_H^2}{v_L^2} \) be the ratio of profit under case (2) to profit under case (1) above. It follows that for a single period problem restricted to introducing a single product, the firm prefers a high product to a low product if \( X \geq 1 \); and prefers a low product to a high product if \( X < 1 \). Thus we call \( X \) high product preference index or \( h-index \) for short. The \( h-index \) is a measure of relative profitability of the two products for the manufacturer. Note that the condition \( X < 1 \) implies \( R < 1 \); and \( R > 1 \) implies \( X > 1 \).

We are now ready to look at our original multi-period problem where we allow repeat purchase. We begin our analyses with the stable single product strategy. Proofs of
all results are included in the Appendix. The following proposition describes our first result.

**Proposition 1**: SS1 is the best stable single product strategy if and only if

\[
X \leq \frac{1 + \delta / (1 - \alpha)}{1 + \delta \psi^2 / (1 - \alpha)}.
\]  

The relative profitability of the SS1 and SS2 strategies depends on the cost side effects, which favor the former, and the demand side effects, which favor the latter. The manufacturer obtains more customers, greater cost savings, and there is no valuation change under strategy SS1. However, this strategy does not allow the manufacturer to fully extract the surplus from the high segment. In fact, the high segment receives a strictly positive surplus under SS1. On the other hand, the SS2 strategy allows the firm to extract all surpluses from the high segment. But in period two the firm gets only the high segment customers and that too with lower surplus to extract because of valuation change. The condition in Proposition 2 represents the net of these effects. The proposition allows us to divide the entire parameter space into two zones depending upon the value of a single parameter, \(h\)-index.

The critical value of \(X\), depends on the parameters like \(n_H, n_L, \alpha, \delta, \text{ and } \psi\). When \(n_H \gg n_L\), SS1 will be a preferred strategy for a relatively smaller zone, while for \(n_H \ll n_L\), SS2 will be a preferred strategy for relatively larger zone. It is interesting to note the following anecdotal evidence in this context. The popular Operations Management textbook by Chase, Acquilano, and Jacobs (McGraw-Hill) is available in the US in the hardcover form only (SS2 strategy). While, in other parts of the world (Poland, for example), the same textbook is available in soft-cover version only (SS1 strategy). Presumably, the fraction of high-end customers is higher in the US market then in some other markets around the world.

Note that the right hand side of (18) is greater than one. This lets us conclude the following result immediately.

**Corollary 1**: If \(X < 1\), then SS1 is the best stable single product strategy.
The proof of Corollary 1 follows directly from Proposition 1. Cost savings, discount factor and valuation change factor has no effect on the optimality of SS1 when the h-index is less than one. For the first period problem, h-index less than one implies low product is the best single product strategy. Valuation change for the high segment makes it more attractive to offer the low product in the second period. Thus Corollary 1 is easy to deduce from two independent single period analyses. However, Proposition 1 shows how the region in which SS1 is optimal is expanded beyond the value of 1 when two periods are considered.

**Corollary 2**: The price of the product increases and the quality improves in the second period under strategy SS1. However, under strategy SS2 there can be a situation where the quality improves but the price goes down in the second period.

This corollary captures the effect of valuation change in strategy SS2. Under strategy SS1, there is no valuation change in the second period. However, the firm derives cost savings. This helps the firm to improve the quality in the second period, which, in turn, extracts a higher price from the consumer. Under strategy SS2, the firm derives a lesser cost savings compared to SS1. In addition, there is a valuation change. If $\psi > 1 - \alpha n$, i.e., the magnitude of valuation change is higher than the cost savings, the manufacturer is forced to improve the quality in order to appeal to the consumers in the second period. The price in the second period is higher if $\psi^2 > 1 - \alpha n$. Since $\psi^2 < \psi$, depending upon the relative magnitude of $\psi$ and $\alpha$, there can be a situation where $\psi^2 < 1 - \alpha n < \psi$. Under this scenario, the manufacturer is forced to improve quality and reduce price in order to make a sale in the second period. If $\psi^2 > 1 - \alpha n$, then both the quality and the prices are higher in the second period. We next look at the results for varying single product strategies. The following proposition describes our first result.

**Proposition 2**: VS1 is the best varying single product strategy if and only if
\[ X[1 - \delta/(1 - \alpha)] \leq 1 - \delta \theta^2 / (1 - \alpha n) \] \hspace{1cm} (19)

The proof of proposition 2 is straightforward and, hence, is omitted. We have compared the two varying single product strategies in Proposition 2. As in Proposition 1, the profitability of the firm is determined by trading off demand-side revenue benefits against supply-side cost benefits. Under the VS1 strategy, the firm is unable to extract the surplus from the high segment in the first period. However, it sells to both low and high customers in the first period and derives a higher cost savings. The firm can extract all surpluses from the high segment in the second period. Under the VS2 strategy, the firm sells only to the high customers in the first period and extracts all surpluses. However, it derives a lower cost savings. The firm serves both the segments in the second period. As a result, it will not be able to extract all surpluses from the high segment. In addition, there is a valuation change in the second period. The firm provides the efficient quality (adjusted for valuation changes and cost savings) in each period under a varying single product strategy. Proposition 2, again, divides the parameter space into two regions, depending upon a critical value of h-index. The region for optimality of VS1 expands with the cost savings parameter \( \alpha \).

It is worth comparing our results with those of Moorthy and Png (1992). Moorthy and Png (1992) do not consider repeat purchase. They conclude that under sequential product introduction strategy, a firm should introduce the high product first followed by the low product. Their conclusion hinges on the fact that a customer will, at most, buy one unit of the product. In contrast, in our analysis with the possibility of repeat purchase there are circumstances where it might be more profitable for the firm to introduce the low product first, followed by a high product. In addition, and unlike Moorthy and Png, we consider valuation change and cost savings. Introducing the low product in the first period helps the firm to achieve a higher cost savings in the second period. Simultaneously, it helps the firm to avoid valuation change.
**Best Single Product Strategy**

We have so far discussed four single product strategies. We next compare these strategies. The following proposition characterizes the best single product strategy for a firm.

**Proposition 3:** *If there is no valuation change and no cost savings, then SS1 is the best single product strategy for \( X \leq 1 \), and SS2 is the best single product strategy for \( X > 1 \). Even in presence of valuation change and cost savings, SS1 is the best single strategy for \( X \leq 1 \). However, there is no unique best single product strategy for \( X > 1 \).*

Proposition 3 describes how the choice of best single product strategy is affected in presence of valuation change and cost savings. In absence of these effects, we have a simple recommendation for the firm: choose SS1 strategy if \( X \leq 1 \), otherwise choose SS2 strategy. Thus, in absence of any valuation change or cost savings, a firm will never choose a varying single product strategy. In presence of valuation change and cost savings, SS1 is still the best strategy for the firm for \( 1 \leq X \leq 1 \).

Next, consider the case \( X > 1 \). Table 3 gives the pair-wise comparison of each of the four single product strategies. It is easy to see from Table 3 that \( \lambda_1, \lambda_2, \) and \( \lambda_4 \) are positive (with \( \lambda_1 \) and \( \lambda_2 \) greater than one). However, \( \lambda_3 \) and \( \lambda_5 \) can take any arbitrary value depending upon the problem parameters. We make an additional simplifying assumption to characterize the optimal single product strategy for the firm analytically. Let \( \theta = \psi \), so that \( \lambda_4 = 1 \). It is now possible to characterize the optimal strategy of the firm. Four scenarios can arise depending upon whether \( \lambda_3 \) is positive or negative; and whether \( \lambda_5 \) is greater than or less than one. Table 4 shows an ordered arrangement of the \( \lambda \)s under each of these scenarios, and the optimal product introduction strategy for the firm under each scenario. The rank order for the \( \lambda \)s can be derived easily from the expressions in Table 3. Once this rank order is known, the optimal strategy for the firm can be determined by using the pair-wise comparison from Table 3. Note that the optimal strategy for the firm for \( X \leq 1 \) is determined by Proposition 3.
Consider Case 1 in Table 4, for example. When $\lambda_3 > 0$ and $\lambda_5 > 1$, simple but tedious calculations show that: $1 < \lambda_1 < \lambda_2 < \lambda_3 < \lambda_5$. The optimal product strategy for the firm can be characterized by using the pair-wise comparison from Table 3. Figure 1 schematically describes the strategy. The entire parameter space can be divided into three zones, each with distinct product strategies, depending upon the value of the h-index. For low values of the h-index only the low product is introduced. As the h-index increases, the manufacturer’s optimal strategy calls for dropping the low segment in period 2 and eventually not offering any product to the low segment in both periods. The other three cases in Table 4 will have similar interpretations. We next look at the multiple products strategies.

4. MULTIPLE PRODUCTS STRATEGIES

Under a multiple products strategy, the firm introduces two products in at least one of the two periods. The possible strategies are: expanding product line (EP1 and EP2), consolidating product line (CP1 and CP2), and stable product line (SP). We look at each of these strategies in turn.

**Proposition 4**: If $\psi \geq \theta$, then strategy EP1 dominates strategy EP2 for $X \leq 1$.

In other words, if profit from a single low product is better than the profit from a single high product in one period, higher contrast in the valuation of the two customers (after valuation change) will make EP1 strategy more desirable than EP2 strategy. Unlike the single product strategies, it is very difficult to compare the expanding product line strategies of the firm analytically. The condition $\psi \geq \theta$ ensures that $R \geq R$, which is required for the proof of the proposition. For $X > 1$, there is no unique dominant expanding product line strategy for the firm. It is easy to see (from Table 2) that $\pi^*(EP1) \geq \pi^*(EP2)$ if and only if
Thus, we need to evaluate the expression in (20) to compare EP1 and EP2 strategies for arbitrary values of the problem parameters. Suppose the above relationship holds for a given values of valuation changes. As \( \psi \) increases, EP2 may become more profitable unless \( \theta \) also increases. An increase in cost savings parameter makes EP1 more profitable. Finally, it is worth mentioning that the EP1 strategy is feasible for \( R \leq 1 \), while the EP2 strategy is feasible for \( \bar{R} \leq 1 \). When any of these relationships does not hold, the corresponding strategy reduces to one of the single product strategies discussed earlier. For example, if \( \bar{R} > 1 \), the cannibalization is so severe in the second period that it is no longer optimal for the firm to offer two products. Thus, the EP2 strategy reduces to either to SS2 strategy or to VS2 strategy.

We next look at our results for consolidating product line and the stable product line. Recall that under the consolidation strategy, the firm offers two products in the first period, but only one product in the second period. Specifically, in the second period, under CP1 low product is offered while high product is offered in CP2. Under the stable product strategy, the firm offers the two products (low and high) in both the periods. The following proposition describes our first result.

**Proposition 5:** The profit from strategy CP1 dominates that from strategy CP2 if and only if \( X \leq \theta^2 / \psi^2 \).

This proposition captures the tradeoff between the cost-side and the demand-side effects under product consolidation strategies. There is no difference in the first period offerings under the two consolidation strategies. Similar cost savings is realized in the second period under both the strategies. Only the valuation change in the second period is different under the two strategies. As a result, the condition for Proposition 5 involves
only the valuation change parameters $\psi$ and $\theta$. The range of values of $h$-index for which the CP1 strategy dominates the CP2 strategy is greater if the percentage change in valuation is lower for the low segment compared to the high segment ($\theta > \psi$). With this condition, the new valuations of the customers in period two become closer to each other. In particular, if $\theta = \psi$, CP1 strategy dominate the CP2 strategy for $X \leq 1$, while the CP2 strategy dominates the CP2 strategy for $X > 1$. Note that the two consolidation strategies for the firm are feasible for $R \leq 1$. For $R > 1$, the consolidation strategies will reduce to one of the four single product strategies discussed earlier.

We next compare the two consolidation strategies with the stable product strategy. Recall that under a stable product strategy, the firm offers two products (the high and the low product) in each of the two periods under consideration. The following proposition describes our result.

**Proposition 6**: The profit from stable product strategy (SP) dominates those from the consolidation strategies CP1 and CP2.

Proposition 6 demonstrates that the firm will never choose the CP1 or CP2 strategy over the SP strategy. The first-period product offerings and the first-period profits are identical under the three strategies. Under each of these three strategies, the firm is able to extract the entire surplus from the low segment in the first periods. The self-selection constraint for the high segment will be binding in the first period under each of these strategies. However, the second period scenarios differ. Under the CP1 (CP2) strategy, the firm extracts the entire surplus from the low (high) segment in the second period. The high segment receives a strictly positive surplus in the second period under CP1. Under the SP strategy, the self-selection constraint for the high segment is binding in the second period. Thus, from an intuitive point, we can expect the SP strategy to give higher profit to the firm compared to the CP1 and the CP2 strategy. Proposition 6 validates this intuition. Finally, in order for this comparison to be valid, the feasibility conditions $R \leq 1$ and $\bar{R} \leq 1$ must hold. The other values of $R$ and $\bar{R}$ are discussed in the next section where we consider all possible strategies.
5. BEST PRODUCT STRATEGY FOR THE FIRM

We have discussed several single and multiple products introduction strategies for the firm in Sections 3 and 4. We now seek to find the best product strategy for the firm by considering all strategies. We begin with the case of no valuation change and no cost savings.

**Proposition 7:** If there is no valuation change (i.e. \( \theta = \psi = 1 \)), then the SP strategy is the best strategy for the firm for \( R < 1 \).

Proposition 7 compares all nine strategies for the firm under the assumption of \( \theta = \psi = 1 \). Note that the result is independent of the cost savings parameter \( \alpha \). What happens when \( R \geq 1 \)? The cannibalization is severe and it is no longer optimal for the firm to offer two products in the same period. Therefore, the only feasible strategies for the firm are the four single product strategies (refer to Table 2 and note that \( \overline{R} = R \) when \( \theta = \psi \)) with optimal choice governed by Proposition 3.

We next look at the firm’s choice in presence of valuation change and cost savings. Depending upon the magnitude of \( R \) and \( \overline{R} \) we will get several different scenarios here. The following proposition describes our first result.

**Proposition 8:** In presence of cost savings and valuation changes, either SP or EP1 is the best product strategy for the firm for \( R < 1 \) and \( \overline{R} < 1 \). Furthermore, EP1 strategy is the best product strategy for the firm if and only if

\[
\frac{\delta n_H v_H^2}{1-\alpha} (1-\psi^2) + \frac{\delta n_L v_L^2}{1-\alpha} [(1-R)^2 - \theta^2 (1-\overline{R})^2] - n_L v_L^2 (R^2 - 2R) - n_H (v_H^2 - v_L^2) \geq 0.
\]

(21)

The above proposition states that when the cannibalization is low, the firm can restrict itself to only two product introduction strategies, EP1 or SP, out of all possible product strategies. It is interesting to note that the consolidation strategies as well as single
product strategies do not figure in the potential choice for optimal product introduction strategies. It is easy to see that the term $n_i v_i^2 (R^2 - 2R)$ in (21) is negative for $R < 1$. Thus, (21) is more likely to be satisfied for low values of $\psi$ and $\theta$. The result is quite intuitive. The cost savings are similar under the EP1 and SP strategies. However, unlike the SP strategy, there is no valuation change under EP1 strategy. Therefore, the EP1 strategy is likely to be more profitable than the SP strategy when there is a drastic change in consumer valuations (low values of $\psi$ and $\theta$). Similarly, the EP1 strategy is likely to be more profitable as $n_H$ decreases.

What happens when $R \geq 1$ and $\bar{R} \geq 1$? As explained in Proposition 7, under such a scenario, the cannibalization is so severe that it is no longer profitable for the firm to introduce two products simultaneously in any period. Therefore, the only feasible strategies for the firm are the four single product strategies; and the optimal choice of the firm is governed by Proposition 3. Recall from Section 3 that $R \geq 1$ implies $X \geq 1$. Therefore, from Table 4, the optimal choice for the firm is either VS1 or SS2.

The following two propositions describe the other two possible scenarios involving $R$ and $\bar{R}$.

**Proposition 9:** When $\bar{R} < 1$ and $R \geq 1$, the best product strategy for the firm is either VS1 or EP2. Furthermore, the EP2 strategy is the best product strategy for the firm if and only if

$$X(1 + \frac{\delta \psi^2}{1 - \alpha n} - \frac{\delta}{1 - \alpha}) + \frac{n \delta \theta^2}{1 - \alpha n} (1 - \bar{R})^2 \geq 1. \quad (22)$$

This is a situation with significant cannibalization between the high and the low products in period 1 but with product valuation changes, cannibalization is not significant in the second period. Under such a situation only five strategies are feasible: the four single product strategies, and the EP2 strategy. Proposition 9 states that out of these five strategies, the firm can restrict itself to only two strategies. Equation (22) is more likely to be satisfied for smaller values of the ratio $\delta/(1 - \alpha)$. Therefore, if the cost savings in the second period is very high (high value of $\alpha$) compared to the discount factor $\delta$, the
firm will choose VS1 strategy over the EP2 strategy. This makes intuitive sense as the firm derives a higher cost savings in the VS1 strategy compared to the EP2 strategy. The firm favors the EP2 strategy when the cannibalization is less significant in the second period. This will happen when $\theta$ is much larger than $\psi$, i.e., when the valuation change for the high segment is more drastic compared to the low segment. Note that the condition $\bar{R} < 1$ and $R \geq 1$ also implies that $\theta > \psi$.

We next look at another potential situation where $\bar{R} > 1$ and $R < 1$. The following proposition summarizes our result.

**Proposition 10:** When $\bar{R} \geq 1$ and $R < 1$, the best product strategy for the firm is either EP1 or CP2. Furthermore, EP1 strategy is the best product strategy for the firm if and only if

$$n_h v_h^2 \left[ \frac{\delta}{1 - \alpha} - 1 \right] - \frac{\delta \psi^2}{1 - \alpha} + n_L v_L^2 (1 - R)^2 \left[ \frac{\delta}{1 - \alpha} - 1 \right] + (n_h + n_L) v_L^2 \geq 0. \tag{23}$$

When $\bar{R} \geq 1$ and $R < 1$, the feasible strategies for the firm are: four single product strategies, the EP1 strategy, and the two consolidation strategies. Under this scenario, the interplay of $\psi$ and $\theta$ gives rise to significant cannibalization in the second period that makes the EP2 and the SP strategies infeasible. The cannibalization in the first period is less significant compared to that in the second period. Note that the cost savings in the second period are identical under EP1 and CP2 strategies. However there is no valuation change under EP1 strategy. Therefore, EP1 is likely to be the preferable strategy for a firm over the CP2 strategy when there is a drastic valuation change. Our calculations validate this intuition as (23) is more likely to be satisfied for smaller values of $\psi$. However, $\theta$ has to be smaller than $\psi$ for $\bar{R} > 1$. As (23) demonstrates, the optimal strategy choice for the firm also depends on the relative magnitudes of $\delta$ and $\alpha$.

Figure 2 schematically describes the optimal strategy for the firm. The values of $R$ and $\bar{R}$ partition the parameter space into four regions. In the figure, optimal choice of the firm is indicated in each of the regions.
Single Vs. Multiple Time Periods

We explore the effect of considering a two-period model vs. a single period model in this Section. Considering multiple time periods allows us to capture the effect of cost savings explicitly in our model. The realized cost savings in the second period is proportional to the demand in the first period. Similarly, considering two periods in our model lets us capture the effect of valuation change in a straightforward manner: there is a valuation change in the second period for both the segments whenever a high product is offered in the first period. Considering multiple periods in our model allows us to prove the following interesting result.

Proposition 11: In a single period context, offering one low product is never an optimal strategy for a firm unless the firm follows a single product strategy. However, in a two-period context, offering one low product, either in first period or in second period, can be a part of an optimal strategy for the firm.

This proposition captures the effect of multiple time periods in our model. In a single period context it is not optimal for the firm to offer one low product (unless it is restricted to a single product strategy) as offering two products (for $R \leq 1$) or a single high product (for $R > 1$) yields higher profit. However, valuation change and cost savings come into play in a two-period model. There is no valuation change if the low product is offered in the first period. In addition, cost savings are higher in the second period if the low product is offered in the first period. Interplay of these factors may result in the low product being offered in a period. For example, under EP1 strategy only the low product is offered in the first period, which can be an optimal strategy for the firm by Proposition 8. Similarly, under CP1 strategy only the low product is offered in the second period, which can be an optimal strategy for the firm by Proposition 10. In addition to Proposition 11, our model allows us to prove the following result.

Corollary 3: For $1 < X \leq \frac{1}{\delta \psi^2 + \delta} < \frac{1}{\delta} \quad \text{the following statements hold:}$

$$1 + \frac{\delta \psi^2}{1-\alpha n} - \frac{\delta}{1-\alpha}$$
(a) In a single-period context, offering a low product is less profitable for the firm than offering a high product; however
(b) In a two-period context, offering a low product in first period followed by a high product in second period (VS1 strategy) is more profitable for the firm than offering one high product in each of the two periods (SS2 strategy).

6. SUMMARY AND MANAGERIAL IMPLICATIONS

We considered different product introduction strategies for a firm over two periods in the presence of repeat purchase, valuation change for the customer, and cost savings for the manufacturer. Our analyses included single as well as multiple products introduction in each period, including product line expansion and consolidation. We characterized the optimal product introduction strategy for a firm and have shown that the optimal choice for the firm changes with the degree of cannibalization. We also analyzed the single product strategies for the firm and characterized the best single product strategy. To compare profit from different product introduction strategies, we developed an index called $h$-index. We characterized the optimal single product strategies for different values of $h$-index. If low product in both periods (SS1 strategy) is optimal with no valuation change and no cost savings then it will continue to be optimal even with valuation changes and cost savings. Contrary to Moorthy and Png (1992), we have shown that introducing a high product followed by a low product (VS2) cannot be optimal but a low product before a high product (VS1) can be an optimal strategy for a firm under a repeat purchase scenario.

When multiple product introduction is possible, SS1 strategy is no longer optimal for any parameter value. We showed that stable product line (SP) is the best product introduction strategy when there is no valuation change. However, a firm needs to consider several parameters to find the best product introduction strategy in presence of valuation changes. With high cannibalization in both periods, only high product is introduced in the second period. When cannibalization is low in both periods, EP1 strategy (low product in periods 1 and both products in period 2) can be optimal. When the possibility of cannibalization is high in the second period, the optimal strategies are
SS2, EP1, VS1, or CP2. With the former three, the valuation change is avoided by not introducing the high product in the first period. With the latter strategy, high valuation change is accepted and only high segment is served in period 2. High cannibalization in period 1 with high cost savings favor VS1 strategy where both segments are served in period 1 over EP2 where only high segment is served in period 2. When cannibalization is low, both segments are served in the first period on all optimal strategies but the cost saving factor does affects the exact strategy choice in both periods. Our qualitative insights on the trade-offs involved in valuation change and cost savings provide useful guidance to decision makers in design, operations, and marketing functions.

We observe different product introduction strategies in practice. For example, Palm Inc. (described in Section 1) followed a strategy similar to the EP2 strategy. While purely a conjecture, Proposition 10 might explain this. Palm \textit{m100} would have cannibalized the sales of Palm \textit{Vx} if both were introduced simultaneously. However, valuation change of consumers might have helped Palm to reduce cannibalization in the second period, resulting in introduction of the \textit{m100} model. We have mentioned several other examples in Section 1. Levi Strauss & Co followed a strategy equivalent to the VS1 strategy in our model while introducing the \textit{Slates} brand.

Finally, our model underscores the fact that optimal product introduction strategies change in presence of valuation change of the consumers. What does it mean from the perspective of the product introduction strategies for a firm? Our model provides guidelines to the decision makers on how to handle such concerns. It also underscores the importance of the degree of cannibalization in product introduction strategies.

Our approach allows us to derive interesting qualitative insights; it also has limitations inherent in any analytical model. Our assumptions about costs and utility functions are consistent with the existing literature. We assume a 0-1 market share with known size of each segment. Similar assumption has been used by Moorthy and Png (1992) and Kim and Chhajed (1999). When the customer choice process is probabilistic, the computation of cost savings will be considerably complicated. However, examining other types of market share model, such as diffusion model or logit model, will be an useful extension.
We have exogenously assumed the existence of the magnitude of valuation changes in our model. Examining the existence of these parameters, and determining the factors influencing the magnitude of these parameters will be an interesting empirical extension. Our model considers a single attribute, denoted by quality. Considering product introduction strategy under multiple attributes will be another extension of the current work. Finally, like most other papers (Moorthy and Png (1992), Desai et al. (2001)) in this field, our paper does not consider the effect of competition.
APPENDIX: PROOFS OF PROPOSITIONS

Proof of Proposition 1

SS1 is the best stable single product strategy if \( \pi^*(SS1) \geq \pi^*(SS2) \). The above relationship will hold if and only if (refer to Table 2)

\[
(n_H + n_L) \frac{v_L^2}{4c} (1 + \frac{\delta}{1-\alpha}) \geq n_H \frac{v_H^2}{4c} (1 + \frac{\delta\psi^2}{1-\alpha}) .
\]

Rearranging terms in the above inequality, we get

\[
\frac{v_H^2}{v_L^2} n \leq \frac{1 + \delta/(1-\alpha)}{1 + \delta\psi^2/(1-\alpha)} .
\]

Proof of Corollary 2

Using the expressions for quality for Strategy SS1 from Table 2, we get \( p_L^{1*} = \frac{v_L^2}{2c} \), and \( p_H^{1*} = \frac{v_H^2}{2c(1-\alpha)} \). Given \( 0 \leq \alpha < 1 \), it is easy to see that both price and quality increases in the second period under strategy SS1. Now consider strategy SS2. Using the expressions of quality from Table 2, and solving for prices we get, \( p_L^{2*} = \frac{v_L^2}{2c} \) and \( p_H^{2*} = \frac{v_H^2}{2c} + \frac{\psi^2}{1-\alpha} \).

Recall that \( 0 < \psi \leq 1 \). Therefore, there can be case where \( \psi > 1-\alpha \), but \( \psi^2 < 1-\alpha \). This means that the quality improves but the price goes down in the second period.

Proof of Proposition 3

First, consider no valuation change (\( \theta = \psi = 1 \)), and no cost savings (\( \alpha = 0 \)). Using Proposition 1 with \( \theta = \psi = 1 \) and \( \alpha = 0 \) we get \( \pi^*(SS1) \geq \pi^*(SS2) \) if and only if \( X \leq 1 \). Similarly, Using Proposition 2 with \( \theta = \psi = 1 \) and \( \alpha = 0 \) we get \( \pi^*(VS1) \geq \pi^*(VS2) \) if and only if \( X \leq 1 \). Thus, the candidates for best single product strategy are SS1 and VS1 for \( X \leq 1 \), and SS2 and VS2 for \( X > 1 \). Using the results from Table 2 (with \( \theta = \psi = 1 \) and \( \alpha = 0 \)), we see that \( \pi^*(SS1) \geq \pi^*(VS1) \) if and only if

\[
(n_H + n_L) \frac{v_L^2}{4c} (1 + \delta) \geq (n_H + n_L) \frac{v_L^2}{4c} + \delta \frac{n_H v_H^2}{4c} .
\]
Simplification of the above inequality yields $X \leq 1$. Thus, SS1 is the best single product strategy for $X \leq 1$. Now consider $X > 1$. Using the results from Table 2 (with $\theta = \psi = 1$ and $\alpha = 0$), we see that $\pi^*(SS2) \geq \pi^*(VS2)$ iff $(1 + \delta) \frac{n_H v_H^2}{4c} + \frac{n_h v_h^2}{4c} \geq (n_H + n_L) \frac{v_L^2}{4c} + \frac{n_h v_h^2}{4c}$.

Simplification of the above inequality yields $X > 1$. Thus, SS1 is the best single product strategy for $X > 1$.

Next, consider the problem with valuation change and cost savings. Using Corollary 1, the candidates for best single product strategy are only SS1, VS1, and VS2 for $X \leq 1$. Using the expressions from Table 2, $\pi^*(SS1) \geq \pi^*(VS1)$ if and only if

$$(n_H + n_L) \frac{v_L^2}{4c} (1 + \delta \frac{1}{1 - \alpha}) \geq (n_H + n_L) \frac{v_L^2}{4c} + \delta \frac{n_H v_H^2}{4c}.$$

Canceling terms and simplifying, we see that the above inequality is equivalent to $X \leq 1$.

Next, compare SS1 and VS2. Again, from Table 2, $\pi^*(SS1) \geq \pi^*(VS2)$ if and only if

$$(n_H + n_L) \frac{v_L^2}{4c} (1 + \delta \frac{1}{1 - \alpha}) \geq n_H v_H^2 + (n_H + n_L) \frac{v_L^2}{4c} \frac{\delta \theta^2}{1 - \alpha n}, \text{ or, } X \leq 1 + \delta \frac{1}{1 - \alpha} - \frac{\delta \theta^2}{1 - \alpha n}.$$

Note that the right hand side of the last inequality is strictly greater than one. Thus, the inequality is satisfied for $X \leq 1$. However the above inequality is not equivalent to $X \leq 1$.

Therefore, SS1 is the best strategy for $X \leq 1$.

**Proof of Proposition 4**

Note that when $\psi \geq \theta$ , $R \geq \bar{R}$. Thus, $(1 - R)^2 \geq (1 - \bar{R})^2$ for $R \leq 1$. With this assumption, compare the second period profits for EP1 and EP2 (refer to Table 2). Clearly,

$$\frac{n_H v_H^2}{4c(1 - \alpha)} + \frac{n_L v_L^2}{4c(1 - \alpha)} (1 - R)^2 > \frac{n_H v_H^2}{4c(1 - \alpha n)} \frac{\psi^2}{4c} + \frac{n_L v_L^2}{4c(1 - \alpha n)} (1 - \bar{R})^2,$$

since $\frac{1}{1 - \alpha} > \frac{1}{(1 - \alpha n)}$. This means that the profit in the second period in strategy EP1 is strictly greater than that of EP2 in the second period for $R \leq 1$. When $X \leq 1$, we have,

$$(n_H + n_L) \frac{v_L^2}{4c} \geq \frac{n_H v_H^2}{4c}.$$ Also, $X \leq 1$ implies $R \leq 1$. Hence the condition $R \leq 1$ is redundant when $X \leq 1$. Therefore, when $\psi \geq \theta$ and $X \leq 1$, EP1 dominates EP2.
Proof of Proposition 5

Note that the first period profits are identical under the strategies CP1 and CP2. Thus, it is sufficient to compare the second period profits. \( \pi^*(CP1) \geq \pi^*(CP2) \) if and only if
\[
(n_H + n_L) \frac{v_H^2}{4c} \frac{\delta \theta^2}{1 - \alpha} \geq \frac{n_H v_H^2}{4c} \frac{\delta \psi^2}{1 - \alpha}.
\]
Simplifying and rearranging, we get \( X \leq \theta^2 / \psi^2 \).

Proof of Proposition 6

Refer to Table 2. We want to show \( \pi^*(SS) \geq \pi^*(CP2) \), i.e.
\[
\frac{n_H v_H^2}{4c} (1 + \frac{\delta \psi^2}{1 - \alpha}) + \frac{n_L v_L^2}{4c} \{(1 - R)^2 + \frac{\delta \theta^2}{1 - \alpha} (1 - R)^2 \} \geq \frac{n_H v_H^2}{4c} (1 + \frac{\delta \psi^2}{1 - \alpha}) + \frac{n_L v_L^2}{4c} (1 - R)^2.
\]
Simplifying the above expression we get
\[
\frac{n_L v_L^2}{4c} \frac{\delta \theta^2}{1 - \alpha} (1 - R)^2 \geq 0, \text{ which is trivially true. Thus, } \pi^*(SS) \geq \pi^*(CP2).
\]

Refer to Table 2 again. \( \pi^*(SS) \geq \pi^*(CP1) \) if and only if
\[
\frac{n_H v_H^2}{4c} (1 + \frac{\delta \psi^2}{1 - \alpha}) + \frac{n_L v_L^2}{4c} \{(1 - R)^2 + \frac{\delta \theta^2}{1 - \alpha} (1 - R)^2 \} \geq \frac{n_H v_H^2}{4c} (1 + \frac{\delta \psi^2}{1 - \alpha}) + \frac{n_L v_L^2}{4c} (1 - R)^2 + \frac{n_H v_H^2}{4c} + \frac{n_L v_L^2}{4c} \frac{\delta \theta^2}{1 - \alpha}.
\]
Simplifying, we get,
\[
n_H v_H^2 \psi^2 + n_L v_L^2 \theta^2 (1 - R)^2 \geq (n_H + n_L) v_L^2 \theta^2.
\]
Substituting \( R = \frac{n_H}{n_L} (\frac{\psi v_H}{\theta v_L} - 1) \) in the above expression and simplifying yield the following inequality: \( (\psi v_H - \theta v_L)^2 (n_H + n_L) \geq 0 \), which is trivially true. Thus, \( \pi^*(SS) \geq \pi^*(CP1) \). Therefore, the SP strategy dominates both CP1, and CP2 strategies.

Proof of Proposition 7

A closer look at the profit expressions in Table 2, and some simplifying calculations yield the following results.
\[
\pi^*(SS) \geq \{ \pi^*(CP1), \pi^*(CP2) \} \text{ (from Proposition 6)} \quad \text{(A1)}
\]
\( \pi^*(SS) \geq \pi^*(EP2) \) (always true, irrespective of \( \theta = \psi = 1 \)) \hspace{1cm} (A2)

\( \pi^*(EP2) \geq \pi^*(VS2) \) (always true, irrespective of \( \theta = \psi = 1 \)) \hspace{1cm} (A3)

\( \pi^*(CP2) \geq \pi^*(SS2) \) (always true, irrespective of \( \theta = \psi = 1 \)) \hspace{1cm} (A4)

\( \pi^*(EI1) \geq \pi^*(VS1) \) (always true, irrespective of \( \theta = \psi = 1 \)) \hspace{1cm} (A5)

\( \pi^*(EI1) \geq \pi^*(SS1) \) (always true, irrespective of \( \theta = \psi = 1 \)) \hspace{1cm} (A6)

Combining (A1)-(A4), we get

\[ \pi^*(SS) \geq \{ \pi^*(VS2), \pi^*(CP1), \pi^*(CP2), \pi^*(SS2), \pi^*(EP2) \}. \] \hspace{1cm} (A7)

Combining (A5) and (A6) we get

\[ \pi^*(EI1) \geq \{ \pi^*(VS1), \pi^*(SS1) \}. \] \hspace{1cm} (A8)

Using (A7) and (A8), SS is the best product strategy for the firm if \( \pi^*(SS) \geq \pi^*(EI1) \).

Using the profit expressions from Table 2 along with \( \theta = \psi = 1 \) (note that \( \bar{R} = R \) for \( \theta = \psi \)), we get \( \pi^*(SS) \geq \pi^*(EI1) \) if

\[ \frac{n_H v_H^2}{4c} (1 + \frac{\delta}{1 - \alpha}) + \frac{n_L v_L^2}{4c} \{ (1 - R)^2 + \frac{\delta}{1 - \alpha} (1 - R)^2 \} \geq \]

\[ (n_H + n_L) \frac{v_L^2}{4c} + \frac{\delta}{1 - \alpha} \{ \frac{n_H v_H^2}{4c} + \frac{n_L v_L^2}{4c} (1 - R)^2 \} \]

Simplifying the above inequality, we get,

\[ \frac{n_H v_H^2}{4c} + \frac{n_L v_L^2}{4c} (1 - R)^2 \geq (n_H + n_L) \frac{v_L^2}{4c} , \] \hspace{1cm} which is true (Moorthy & Png, 1992). Thus, SS1 is the best product strategy for the firm if \( \theta = \psi = 1 \).

**Proof of Proposition 8**

Comparing (A7) and (A8), we get, \( EI1 \) is the best product strategy for the firm if \( \pi^*(EI1) \geq \pi^*(SP) \). Otherwise, \( SP \) is the best product strategy for the firm. The condition for strategy \( EI1 \) to dominate strategy \( SP \) in the proposition follows directly from Table 2.

**Proof of Proposition 9**

When \( \overline{R} < 1 \) and \( R \geq 1 \), the feasible strategies are: SS1, SS2, VS1, VS2 and EP2.
From Table 2 we get, \( \pi^* (EP2) \geq \pi^* (SS2) \). 

(A9)

From equation (A3), we get, 
\( \pi^* (EP2) \geq \pi^* (VS2) \) 

(A10)

From the discussion at the beginning of Section 3, we get, \( R \geq 1 \Rightarrow X \geq 1 \). Therefore, from the proof of Proposition 3, we get, 
\( \pi^* (VS1) \geq \pi^* (SS1) \) for \( X > 1 \).

(A11)

Combining (A9), (A10), and (A11), the potential candidates for the best strategy are VS1 and EP2. From the profit equations from Table 2 we get, if and only if:

\[
\frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1 - \alpha n} \right) + \frac{n_L v_L^2}{4c} \left(1 - \frac{\delta \theta^2}{1 - \alpha n} \right) \geq \left(n_H + n_L\right) \frac{v_L^2}{4c} + \frac{\delta}{1 - \alpha} n_H v_H^2.
\]

Simplification the above inequality yields the condition for Proposition 9.

**Proof of Proposition 10**

When \( \overline{R} \geq 1 \) and \( R < 1 \), the feasible strategies for the firm are: SS1, SS2, VS1, VS2, EP1, CP1, and CP2. From (A8), we get \( \pi^* (EP1) \geq \{ \pi^* (SS1), \pi^* (VS1) \} \). Similarly, from (A4) we get, \( \pi^* (CP2) \geq \pi^* (SS2) \). From Table 2 we get, \( \pi^* (CP1) \geq \pi^* (VS2) \). Therefore, the potential candidates for the best strategy are EP1, CP1, and CP2. Next we show that \( \pi^* (CP2) \geq \pi^* (CP1) \) for \( \overline{R} \geq 1 \) and \( R < 1 \). From the definition of \( R \) and \( \overline{R} \), we must have \( \psi > \theta \) (or \( \theta / \psi < 1 \)) for \( \overline{R} \geq 1 \) and \( R < 1 \). Now, \( \overline{R} \geq 1 \) implies \((n_H / n_L)(v_H / v_L - 1) \geq 1\).

Simplification of this inequality yields \( n(v_H / v_L) \geq \theta / \psi \). Given \( v_H > v_L \), the last inequality implies \( X \geq \theta / \psi \). Given \( \theta / \psi < 1 \), \( X \geq \theta / \psi \) implies \( X \geq \theta^2 / \psi^2 \). Therefore, by Proposition 5, \( \pi^* (CP2) \geq \pi^* (CP1) \). Hence, the optimal choice for the firm is either EP1 or CP2. The condition for strategy EP1 to dominate strategy CP2 in the proposition follows directly from Table 2.

**Proof of Proposition 11**

When the firm offers a single low product in a single period scenario, the profit for the firm is given by \( (n_H + n_L)v_L^2 / 4c \). If the firm offers two products, one for the H and the
other for the L segment, then the profit for the firm is \( n_H v_H^2 / 4c + (1-R)^2 n_L v_L^2 / 4c \). A simple but tedious calculation shows that the later expression is larger than the former. However, the later equation is feasible for \( R \leq 1 \). When \( R > 1 \), the firm offers only high product, resulting in a profit of \( n_H v_H^2 / 4c \). Thus, it is never optimal for the firm to offer one low product unless it is restricted to a single product strategy. However, as seen in propositions 8 and 10, offering one low product can be an optimal strategy in a two-period model.

**Proof of Corollary 3**

Using the expressions from Table 2, \( \pi^*(VS1) \geq \pi^*(SS2) \) if and only if

\[
\frac{(n_H + n_L) v_L^2}{4c} + \frac{\delta}{1-\alpha} \cdot \frac{n_H v_H^2}{4c} \geq \frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1-\alpha n}\right).
\]

Rearranging terms, we can write the above inequality as

\[
\frac{n_H v_H^2}{(n_H + n_L) v_L^2} \left[1 + \frac{\delta \psi^2}{1-\alpha n} - \frac{\delta}{1-\alpha}\right] \leq 1, \quad \text{or}, \quad X \leq \frac{1}{1 + \frac{\delta \psi^2}{1-\alpha n} - \frac{\delta}{1-\alpha}}.
\]

Note that for any \( \delta, \psi, \text{ and } \alpha \), \( \delta / (1-\alpha) > \delta \psi^2 / (1-\alpha n) \). Thus the right hand side of the above inequality is always greater than one as long as \( 1 + \delta \psi^2 / (1-\alpha n) - \delta / (1-\alpha) > 0 \), or,

\[
\frac{(n_H + n_L) \delta \psi^2}{(1-\alpha)n_H + n_L} > \frac{\delta}{1-\alpha} - 1. \quad \text{In a single period context, the profit from the high product is more than that for the low product if } X > 1. \quad \text{Thus, Proposition 9 will be valid for } 1 < X \leq \frac{1}{1 + \frac{\delta \psi^2}{1-\alpha n} - \frac{\delta}{1-\alpha}}, \quad \text{and is feasible for } \frac{(n_H + n_L) \delta \psi^2}{(1-\alpha)n_H + n_L} > \frac{\delta}{1-\alpha} - 1.
### Table 1. Customers Served Under Various Product Strategies

<table>
<thead>
<tr>
<th>Product Strategy</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 2</th>
<th>Cost</th>
<th>Valuation change?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stable Single-Product</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS1</td>
<td>L,H</td>
<td>L,H</td>
<td></td>
<td>$c'$</td>
<td>No</td>
</tr>
<tr>
<td>SS2</td>
<td>H</td>
<td>H</td>
<td></td>
<td>$c''$</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Varying Single-Product</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS1</td>
<td>L,H</td>
<td></td>
<td>H</td>
<td>$c'$</td>
<td>No</td>
</tr>
<tr>
<td>VS2</td>
<td>H</td>
<td></td>
<td>L,H</td>
<td>$c''$</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Expanding Product-Line</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EP1</td>
<td>L,H</td>
<td>L</td>
<td>H</td>
<td>$c'$</td>
<td>No</td>
</tr>
<tr>
<td>EP2</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>$c''$</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Consolidating Product Line</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP1</td>
<td>L</td>
<td>L,H</td>
<td></td>
<td>$c'$</td>
<td>Yes</td>
</tr>
<tr>
<td>CP2</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>$c'$</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Stable Product</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP</td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>$c'$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 2: Optimal Quality and Profit Under Different Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Optimal Quality, Optimal Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS1</td>
<td>$q_{l1}^* = \frac{v_L}{2c}$; $q_{l2}^* = \frac{v_L}{2c(1-\alpha)}$; $\pi^* (SS1) = (n_H + n_L) \frac{v_L^2}{4c} \left(1 + \frac{\delta}{1-\alpha}\right)$</td>
</tr>
<tr>
<td>SS2</td>
<td>$q_{Hl}^* = \frac{v_H}{2c}$; $q_{H2}^* = \frac{v_H}{2c} \frac{\psi}{(1-\alpha)}$; $\pi^* (SS2) = \frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1-\alpha n}\right)$</td>
</tr>
<tr>
<td>VS1</td>
<td>$q_{l1}^* = \frac{v_L}{2c}$; $q_{H2}^* = \frac{v_L}{2c(1-\alpha)}$; $\pi^* (VS1) = (n_H + n_L) \frac{v_L^2}{4c} + \frac{\delta}{1-\alpha} \frac{n_H v_H^2}{4c}$</td>
</tr>
<tr>
<td>VS2</td>
<td>$q_{Hl}^* = \frac{v_H}{2c}$; $q_{l2}^* = \frac{v_H}{2c} \frac{\theta}{(1-\alpha)}$; $\pi^* (VS2) = \frac{n_H v_H^2}{4c} + (n_H + n_L) \frac{v_L^2}{4c} \frac{\delta \theta^2}{1-\alpha}$</td>
</tr>
<tr>
<td>EP1</td>
<td>$q_{l1}^* = \frac{v_L}{2c}$; $q_{l2}^* = \frac{v_L}{2c(1-\alpha)} (1-R)$; $q_{H2}^* = \frac{v_H}{2c}$; $\pi^* (EP1) = (n_H + n_L) \frac{v_L^2}{4c} + \frac{\delta}{1-\alpha} \left{ \frac{n_H v_H^2}{4c} + \frac{n_L v_h^2}{4c} (1-R) \right}$</td>
</tr>
<tr>
<td>EP2</td>
<td>$q_{l1}^* = \frac{v_L}{2c} \frac{\theta}{(1-\alpha)} (1-R)$; $q_{l2}^* = \frac{v_H}{2c} \frac{\psi}{(1-\alpha)}$; $q_{H2}^* = \frac{v_H}{2c}$; $\pi^* (EP2) = \frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1-\alpha n}\right) + \frac{n_L v_h^2}{4c} \left(1-R\right)^2 \left(1-R\right)^2$</td>
</tr>
<tr>
<td>CP1</td>
<td>$q_{l1}^* = \frac{v_L}{2c} (1-R)$; $q_{H2}^* = \frac{v_H}{2c}$; $q_{l2}^* = \frac{v_L}{2c} \frac{\theta}{1-\alpha}$; $\pi^* (CP1) = \frac{n_H v_H^2}{4c} + \frac{n_L v_h^2}{4c} (1-R)^2 + (n_H + n_L) \frac{v_L^2}{4c} \frac{\delta \theta^2}{1-\alpha}$</td>
</tr>
<tr>
<td>CP2</td>
<td>$q_{l1}^* = \frac{v_L}{2c} (1-R)$; $q_{H2}^* = \frac{v_H}{2c}$; $q_{l2}^* = \frac{v_H}{2c} \frac{\psi}{1-\alpha}$; $\pi^* (CP2) = \frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1-\alpha}\right) + \frac{n_L v_h^2}{4c} (1-R)^2 \left(1-R\right)^2$</td>
</tr>
<tr>
<td>SP</td>
<td>$q_{l1}^* = \frac{v_L}{2c} (1-R)$; $q_{H2}^* = \frac{v_H}{2c}$; $q_{l2}^* = \frac{v_L}{2c} \frac{\theta}{1-\alpha} (1-R)$; $q_{H2}^* = \frac{v_H}{2c} \frac{\psi}{1-\alpha}$; $\pi^* (SP) = \frac{n_H v_H^2}{4c} \left(1 + \frac{\delta \psi^2}{1-\alpha}\right) + \frac{n_L v_h^2}{4c} \left(1-R\right)^2 + \frac{\delta \theta^2}{1-\alpha} \left(1-R\right)^2 \left(1-R\right)^2$</td>
</tr>
</tbody>
</table>
Table 3: Pair wise Comparison of Four Single Product Strategies

<table>
<thead>
<tr>
<th>Strategies Compared</th>
<th>Relationship</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS1, SS2</td>
<td>( \pi^<em>(SS1) \geq \pi^</em>(SS2) ) iff ( X \leq \lambda_1 )</td>
<td>( \lambda_1 = \frac{1+\delta/(1-\alpha)}{1+\delta\psi^2/(1-\alpha\eta)} )</td>
</tr>
<tr>
<td>SS1, VS1</td>
<td>( \pi^<em>(SS1) \geq \pi^</em>(VS1) ) iff ( X \leq 1 )</td>
<td>( \lambda_2 = 1 + \frac{\delta}{1-\alpha} - \frac{\delta\psi^2}{1-\alpha\eta} )</td>
</tr>
<tr>
<td>SS1, VS2</td>
<td>( \pi^<em>(SS1) \geq \pi^</em>(VS2) ) iff ( X \leq \lambda_2 )</td>
<td>( \lambda_3 = \frac{1}{1-\alpha} + \frac{\delta\psi^2}{1-\alpha\eta} )</td>
</tr>
<tr>
<td>SS2, VS1</td>
<td>( \pi^<em>(VS1) \geq \pi^</em>(SS2) ) iff ( X \leq \lambda_3 ), for ( \lambda_3 \geq 0 ) ( X \geq \lambda_3 ), for ( \lambda_3 &lt; 0 )</td>
<td>( \lambda_4 = \frac{\theta^2}{\psi^2} )</td>
</tr>
<tr>
<td>SS2, VS2</td>
<td>( \pi^<em>(VS2) \geq \pi^</em>(SS2) ) iff ( X \leq \lambda_4 )</td>
<td>( \lambda_5 = \frac{1-\delta\psi^2/(1-\alpha\eta)}{1-\delta/(1-\alpha)} )</td>
</tr>
<tr>
<td>VS1, VS2</td>
<td>( \pi^<em>(VS1) \geq \pi^</em>(VS2) ) iff ( X \leq \lambda_5 ), for ( \lambda_5 \geq 1 ) ( X \geq \lambda_5 ), for ( \lambda_5 &lt; 1 )</td>
<td>( \lambda_6 = \frac{1-\delta\psi^2}{1-\delta/(1-\alpha)} )</td>
</tr>
</tbody>
</table>

Table 4: Best Single Product Strategy For the Firm For Different X

<table>
<thead>
<tr>
<th>ID</th>
<th>( \lambda_3 )</th>
<th>( \lambda_5 )</th>
<th>( \lambda ) Relationship</th>
<th>Best Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Positive</td>
<td>( \lambda_5 &gt; 1 )</td>
<td>( 1 &lt; \lambda_1 &lt; \lambda_2 &lt; \lambda_3 &lt; \lambda_5 )</td>
<td>SS1 for ( X \leq 1 ) &lt;br&gt;VS1 for ( 1 &lt; X \leq \lambda_3 ) &lt;br&gt;SS2 for ( X &gt; \lambda_3 )</td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td>( \lambda_5 &lt; 1 )</td>
<td>( 1 &lt; \lambda_1 &lt; \lambda_2 &lt; \lambda_3 )</td>
<td>Same as Case 1</td>
</tr>
<tr>
<td>3</td>
<td>Negative</td>
<td>( \lambda_5 &gt; 1 )</td>
<td>( 1 &lt; \lambda_1 &lt; \lambda_2 &lt; \lambda_5 )</td>
<td>SS1 for ( X \leq 1 ) &lt;br&gt;VS1 for ( 1 &lt; X \leq \lambda_5 ) &lt;br&gt;SS2 for ( X &gt; \lambda_5 )</td>
</tr>
<tr>
<td>4</td>
<td>Negative</td>
<td>( \lambda_5 &lt; 1 )</td>
<td>( 1 &lt; \lambda_1 &lt; \lambda_2 )</td>
<td>SS1 for ( X \leq 1 ) &lt;br&gt;VS1 for ( X &gt; 1 )</td>
</tr>
</tbody>
</table>
Figure 1: Best Single Product Strategies for Different Values of X and for $\lambda_3 > 0, \lambda_5 > 1$

Figure 2: Best Product Strategy for the Firm
REFERENCES


Website, Levi Strauss & Co: www.levistrauss.com

Website, Palm Inc.: www.palm.com/about/corporate/timeline.html