
Chapter 2 - Measuring Portfolio Return

2-1:

$$A. R = \frac{\$2,124,770.}{\$2,000,000.} = 1.062385 \quad r = +6.2385\%$$

$$B. R = \frac{\$1,843,748.}{\$2,000,000.} = 0.921874 \quad r = -7.8126\%$$

$$C. R = \frac{\$2,000,000.}{\$2,000,000.} = 1.0000 \quad r = 0.0000\%$$

2-2:

- A. The pension payments and contribution are not part of the decrease or increase in the value of the portfolio that we need to measure, so we subtract them out. Dividends and bond income are received as a result of the way in which the portfolio was invested; they stay in.

$$\begin{aligned}
 R &= \frac{\$34,993,897.09 - \frac{15}{31} (\$3,098.10) - \frac{21}{31} (-\$9,879.20) - \frac{31}{31} (\$3,098.10)}{\$34,978,567.03 + \frac{16}{31} (\$3,098.10) + \frac{10}{31} (-\$9,879.20) + \frac{0}{31} (\$3,098.10)} \\
 &= \frac{\$34,995,992.27}{\$34,976,979.21} \\
 &= 1.000543588
 \end{aligned}$$

$$r = 0.05436\%$$

- B. The total cash flow to be taken out is -\$3,683.00 so, using the mid-month cash flow adjustment we get

$$\begin{aligned}
 R &= \frac{\$34,993,897.09 - \frac{1}{2} (-\$3,683.00)}{\$34,978,567.03 + \frac{1}{2} (-\$3,683.00)} \\
 &= \frac{\$34,995,738.59}{\$34,976,725.53} \\
 &= 1.000543592
 \end{aligned}$$

$$r = 0.05436\%$$

The rate of return is identical to the seventh decimal place - fifth decimal place in percentage points. When the cash flows are distributed through the month in this way then the mid-month method is a pretty good estimate.

2-3:

- A. Dividends are not cash flow in the rate of return calculation because the dividend is the result of the investment made not a flow out of the investor's control.
- B. The portfolio begins with \$57,000. (1,000 shares * \$57.00) and ends with \$57,050. (\$56,750. in shares and \$300 in cash (1,000 shares * 30¢ per share dividend). Thus the rate of return is 0.0877%

$$\frac{\$56,750. + \$300.}{\$57,000.} = 1.000877 = 1 + 0.0877\%$$

$$\text{or } \frac{\$56,750. + \$0.30}{\$57.} = 1.000877 = 1 + 0.0877\%$$

2-4:

- A. *January:* $R = \frac{\$28}{\$25} = 1.120 = 1 + 12.0\%$
- February:* $R = \frac{\$30 + \$0.25}{\$28} = 1.080357143 = 1 + 8.036\%$
- March:* $R = \frac{\$29.25}{\$30.25} = 0.966942149 = 1 - 3.305\%$

Note that for the month of March we take a portfolio approach and measure the rate of return on the portfolio value of \$29.25. The 25¢ dividend we have sitting in idle cash thus increases the portfolio return to -3.305%: the \$29 earns a return of -3.333% and the 25¢ earns a return of 0%.

- B. $R = (1.120)(1.080357143)(0.966942149) = 1.170 = 1 + 17.0\%$
- C. $R = \frac{\$29.25}{\$25} = 1.170 = 1 + 17.0\%$

If there are no cash flows that require an adjustment the total time weighted rate of return equals the holding period rate of return.

If we were calculating the expected rate of return $E[R]$ we would calculate the rate of return for March independently from anything that happened in February; we assume we purchased a new share at \$30 at the close February 28 and watch it decline to \$29 by the close March 31. We calculate the expected return as 5.56746%

$$E[R] = \frac{12.0\% + 8.0357143\% - 3.3\%}{3} = \frac{16.70238095\%}{3} = 5.56746\%$$

2-5:

- A. Total rate of return: $- 50\% + 50\% - 50\% + 50\% - 50\% + 50\% \Rightarrow + 0.0000\%$
 Average rate of return: $0.000\% / 6 \Rightarrow + 0.0000\%$

If The Quark Investment Fund claims that it "on average" broke even, it may be statistically correct but the poor client who put in \$1,000,000. and now has \$421,875. might feel that "broke even" doesn't quite describe his investing experience.

- B. Holding Period Rate of Return: $R = 421,875/1,000,000 = 0.421875 = 1 - 57.8125\%$
- C. Total Time Weighted rate of Return: $R = (0.5)(1.5)(0.5)(1.5)(0.5)(1.5) = 0.421875 = 1 - 57.8125\%$
 Monthly Time Weighted Rate of Return: $\sqrt[6]{0.421875} = 0.86602540 = 1 - 13.3975\%$

If there are no cash flows that require an adjustment the total time weighted rate of return equals the holding period rate of return.

2-6:

The holding period return for the five year period is 1,015%

$$R = \frac{\$27.875}{\$2.50} = 11.15 = 1 + 1015\%$$

Calculating for the average monthly rate of return over the 60 months and average annual rate of return over the 5 years we get 4.101% and 61.978% respectively.

A. $R = \sqrt[60]{\frac{\$27.875}{\$2.50}} = 1.041009232 = 1 + 4.101\%$

B. $R = \sqrt[5]{\frac{\$27.875}{\$2.50}} = 1.619776051 = 1 + 61.978\%$

If we had perversely taken the statistical measure in this instance we would have calculated average returns of 16.9% per month and 203% per year, which would be grossly misleading.

Remember that a holding period rate of return, by definition, assumes that the gains and losses in any one period are reinvested in the next. If we really expected an average of 200% per year returns then after five years our \$2.50 investment should have tripled five times for a total value of \$607.50 not \$27.875

2-7:

A. \$1,000,000

B. $R = \sqrt{(0.50)(2.00)} = 1.00 = 1 + 0.00\%$

C. $E[R] = \frac{-50\% + 100\%}{2} = 25\%$

2-8:

	Susan Q. Speculator		John Q. Investor	
	Hypothetical Resources (HR)	Discovery Café (DVC)	Hypothetical Resources (HR)	Discovery Café (DVC)
Dec 31	1,000,000.00	1,000,000.00	250,000.00	250,000.00
Jan 31	1,036,000.00	937,000.00	509,000.00	484,250.00
Feb 28	1,279,460.00	816,127.00	878,615.00	671,781.75
Mar 31	1,114,409.66	1,007,916.85	1,015,273.66	1,079,650.46
Apr 30	1,044,201.85	1,044,201.85	951,311.42	1,118,517.88
Holding Period return	4.4202%	4.4202%	-4.8689%	11.8518%

For Susan the time weighted rate of return for the four months is the same for each investment: $(1.036)(1.235)(0.871)(0.937)=1.044202$. It doesn't matter to Susan if gains or losses come first because all gains and losses are reinvested.

John is putting his money into HR and DVC at the rate of \$250,000 per month. When HR has positive returns there is less money in the stock than when HR is making negative returns. Conversely, when DVC has positive returns there is more money in the stock than when DVC is making negative returns. The statistical rate of return for the four months is 7.9% and the time weighted rate of return is 4.4202%.

The difference in the two investment patterns becomes obvious if we take the more accurate accounting approach and provide the market value of the portfolios in each case. This means we must add the uninvested cash to the market value of the portfolio.

Now we can see that in January JQ earns one quarter of the rate of return in the underlying stock because he has only one quarter of his portfolio invested. In February one half, in March three quarters, and in April the full portfolio. Now it makes a great deal of difference weather the gains or losses come at the beginning or at the end of the period.

	John Q. Investor			
	Hypothetical Resources (HR)		Discovery Café (DVC)	
Dec 31	1,000,000.00		1,000,000.00	
Jan 31	1,009,000.00	0.9000%	984,250.00	-1.575%
Feb 28	1,128,615.00	11.8548%	921,781.75	-6.3468%
Mar 31	1,015,273.66	-10.0425%	1,079,650.46	17.1264%
Apr 30	951,311.42	-6.3000%	1,118,517.88	3.6000%
Holding Period return	-4.8689%		11.8518%	

2-9:

In G., the monthly return for May is recalculated as
$$\frac{\$1,730,000 - \frac{1}{31} (500,000)}{\$1,074,750 + \frac{30}{31} (500,000)} = 1.099607283$$

for the period recalculated as shown.

Jan 31	\$1,000,000	Initial Calculations	Recalculations (Part G)
Feb 28	\$1,020,000	1.020000 = 1 + 2.000%	
Mar 31	\$1,099,600	1.078039 = 1 + 7.804%	
Apr 30	\$1,074,750	0.977401 = 1 - 2.260%	
May 31	\$1,730,000	1.609677 = 1 + 60.968%	1.099607 = 1 + 9.961%
Holding Period Return		1.730000 = 1 + 73.000%	
Average Return		17.128%	4.376%
Time Weighted	Four Month	1.730000 = 1 + 73.000%	1.181803 = 1 + 18.180%
	Monthly	$4\sqrt{1.730000} = 1.14686$ = 1 + 14.686%	$4\sqrt{1.181803} = 1.042645$ = 1 + 4.264%
	Annualized	$(4\sqrt{1.73})^{12}$ = 5.177717 = 1 + 417.772%	$(4\sqrt{1.180709})^{12}$ = 1.650575 = 1 + 65.057%

2-10:

A. $R = \frac{\$0.275}{\$0.415} = 0.662651 = 1 - 0.33734940$
 $r = - 33.73\%$

B. $R = 0.662651^{12} = 0.007168 = 1 - 0.992832$
 $r = - 99.28\%$

C. $R = -33.734940 * 12 = -404.819277$
 $r = - 404.82\%$

Since the most you can lose is 100%, the statistical rate of return makes no sense. Losses of 99.28% calculated as the time-weighted rate of return may be painful, but at least it's reasonable.

2-11:

- A. The actual rate of return is 8.077998% rather than 8.1% but you could get away with rounding to 8.078%
- B. 7.107% Change the cash flow from 5,000 to 4,000 and the terminal cash flow from 115,000 to 114,000
- C. 6.000%

Cash Flow:	enter	calculate
CF0 =	- 100,000.	
C01 =	5,000.	
F01 =	2.	
C02 =	115,000.	
F02 =	1.	
P/Y	1.	
IRR =		8.077998