Chapter 2

Consumer Theory Basics

Recall that the goal of economic theory is to account for behavior based on the assumption that actors have stable preferences, attempt to do as well as possible given those preferences and the constraints placed on their resources, and that changes in behavior are due to changes in these constraints. In this section, we use this approach to develop a theory of consumer behavior based on the simplest assumptions possible. Along the way, we develop the tool of **comparative statics analysis**, which attempts to characterize how economic agents (i.e. consumers, firms, governments, etc.) react to changes in the constraints they face.

2.1 Commodities and Budget Sets

To begin, we need a description of the goods and services that a consumer may consume. We call any such good or service a **commodity**. We number the commodities in the world 1 through L (assuming there is a finite number of them). We will refer to a "generic" commodity as l (that is, l can stand for any of the L commodities) and denote the quantity of good l by x_l. A **commodity bundle** (i.e. a description of the quantity of each commodity) in this economy is therefore a vector $x = (x_1, x_2, ..., x_L)$. Thus if the consumer is given bundle $x = (x_1, x_2, ..., x_L)$, she is given x_1 units of good 1, x_2 units of good 2, and so on.[1] We will refer to the set of all possible allocations as the **commodity space**, and it will contain all possible combinations of the L possible commodities.[2]

Notice that the commodity space includes some bundles that don't really make sense, at least

[1] For simplicity of terminology - but not because consumers are more or less likely to be female than male - we will call our consumer "she," rather than "he/she."

[2] That is, the commodity space is the L-dimensional real space R^L.

economically. For example, the commodity space includes bundles with negative components. And, it includes bundles with components that are extremely large (i.e., so large that there simply aren't enough units of the relevant commodities for a consumer to actually consume that bundle). Because of this, it is useful to have a (slightly) more limited concept than the commodity space that captures the set of all realistic consumption bundles. We call the set of all reasonable bundles the **consumption set**, denoted by X. What exactly goes into the consumption set depends on the exact situation under consideration. In most cases, it is important that we eliminate the possibility of consumption bundles containing negative components. But, because consumers usually have limited resources with which to purchase commodity bundles, we don't have to worry as much about very large bundles. Consequently, we will, for the most part, take the consumption set to be the L dimensional non-negative real orthant, denoted R^L_+. That is, the possible bundles available for the consumer to choose from include all vectors of the L commodities such that every component is non-negative.

The consumption set eliminates the bundles that are "unreasonable" in all circumstances. We are also interested in considering the set of bundles that are available to a consumer at a particular time. In many cases, this corresponds to the set of bundles the consumer can afford given her wealth and the prices of the various commodities. We call such sets (**Walrasian) budget sets**.[3]

Let w stand for the consumer's wealth and p_l stand for the price of commodity l. Without any exceptions that I can think of, we assume that $p_l \geq 0$ for all l and that $w \geq 0$. That is, prices and wealth are either positive or zero, but not negative.[4] We will let $p = (p_1, ..., p_L)$ stand for the vector of prices of each of the goods. Hence if the consumer purchases consumption bundle x and the price vector is p, the consumer will spend

$$p \cdot x = \sum_{l=1}^{L} p_l x_l$$

on commodities.[5] Since the consumer's total income is w, the consumer's Walrasian budget set is

[3]We call the budget set Walrasian after economist Leon Walras (1834-1910), one of the founders of this type of analysis.

[4]What do you imagine would happen if there were goods with negative prices?

[5]A few words about notation: In the above equation, x and p are both vectors, but they lack the usual notation $\overrightarrow{x}$ and $\overrightarrow{p}$. Since economists almost never use the formal vector notation, you will need to use the context to judge whether an "x" is a single variable or actually a vector. Frequently we'll write someting with subscript l to denote a particular commodity. Then, when we want to talk about all commodities, we put them together into a vector, which has no subscript. For example, p_l is the price of commodity l, and $p = (p_1, ..., p_L)$ is the vector containing the prices of all commodities.

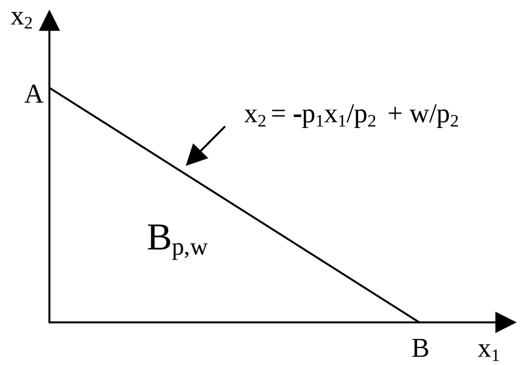

Figure 2.1: The Budget Set

defined as all bundles x such that $p \cdot x \leq w$ - in other words, all affordable bundles given prices and wealth. More formally, we can write the budget set as:

$$B_{p,w} = \left\{ x \subset R_+^L : p \cdot x \leq w \right\}.$$

The term Walrasian is appended to the budget set to remind us that we are implicitly speaking of an environment where people can buy as much as they want of any commodity at the same price. In particular, this rules out the situations where there are limits on the amount of a good that a person can buy (rationing) or where the price of a good depends on how much you buy. Thus the Walrasian budget set corresponds to the opportunities available to an individual consumer whose consumption is small relative to the size of the total market for each good. This is just the standard "price taking" assumption that is made in models of competitive markets.

In order to understand budget sets, it is useful to assume that there are two commodities. In this case, the budget set can be written as

$$B_{p,w} = \left\{ x \subset R_+^2 : p_1 x_1 + p_2 x_2 \leq w \right\}.$$

Or, if you plot x_2 on the vertical axis of a graph and x_1 on the horizontal axis, $B_{p,w}$ is defined by the set of points below the line $x_2 = \frac{-p_1 x_1}{p_2} + \frac{w}{p_2}$. See Figure 2.1.

How does the budget set change as the prices or income change? If income increases, budget line AB shifts outward, since the consumer can purchase more units of the goods when she has more wealth. If the price of good 1 increases, when the consumer purchases only good 1 she can afford fewer units. Hence if p_1 increases, point B moves in toward the origin. Similarly, if p_2 increases, point A moves in toward the origin.

Exercise 1 *Here is a task to show that you understand budget sets: Show that the effect on a budget set of doubling p_1 and p_2 is the same as the effect of cutting w in half. This is an illustration of the key economic concept that only relative prices matter to a consumer, which we will see over and over again.*[6]

Now that we have defined the set of consumption bundles that the consumer can afford, the next step is to try to figure out which point the consumer will choose from the budget set. In order to determine which point from the budget set the consumer will choose, we need to know something about the consumer's preferences over the commodities. For example, if x_1 is onions and x_2 is chocolate, the consumer may prefer points with relatively high values of x_2 and low values of x_1 (unless, of course, p_2 is very large relative to p_1). If we knew exactly the trade-offs that the consumer is willing to make between the commodities, their prices, and the consumer's income, we would be able to say exactly which consumption bundle the consumer prefers. However, at this point we do not want to put this much structure on preferences.

2.2 Demand Functions

Now we need to develop a notation for the consumption bundle that a consumer chooses from a particular budget set. Let $p = (p_1, ..., p_L)$ be the vector of prices of the L commodities. We will assume that all prices are non-negative. When prices are p and wealth is w, the set of bundles that the consumer can afford is given by the Walrasian budget set $B_{p,w}$. Assume that for any price vector and wealth (p, w) there is a single bundle in the budget set that the consumer chooses. Let $x_i (p, w)$ denote the quantity of commodity i that the consumer chooses at these prices and wealth. Let $x (p, w) = (x_1 (p, w), ..., x_L (p, w)) \in B_{p,w}$ denote the bundle (vector of commodities) that the consumer chooses when prices are p and income is w. That is, it gives the optimal consumption bundle as a function of the price vector and wealth. To make things easier, we will assume that $x_l (p, w)$ is single-valued (i.e. a function) and differentiable in each of its arguments.

Exercise 2 *How many arguments does $x_l (p, w)$ have? Answer: $L + 1 : L$ prices and wealth.*

Functions $x_l (p, w)$ represent the consumer's choice of commodity bundle at a particular price and wealth. Because of this, they are often called **choice functions**. They are also called **demand**

[6] The idea that only relative prices matter goes by the mathematical name "homogeneity of degree zero", but we'll return to that later.

functions, although sometimes that name is reserved for choice functions that are derived from the utility-maximization framework we'll look at later. Generally, I use the terms interchangeably, except when I want to emphasize that we are not talking about utility maximization, in which case I'll use the term "choice function."

At this point, we should introduce an important distinction, the distinction between **endogenous** and **exogenous** variables. An endogenous variable in an economic problem is a variable that takes its value as a result of the behavior of one of the economic agents within the model. So, the consumption bundle the consumer chooses $x(p, w)$ is endogenous. An exogenous variable takes its value from outside the model. Exogenous variables determine the constraints on the consumer's behavior. Thus in the consumer's problem, the exogenous variables are prices and wealth. The consumer cannot choose prices or wealth. But, prices and wealth determine the budget set, and from the budget set the consumer chooses a consumption bundle. Hence the consumption bundle is endogenous, and prices and wealth are exogenous. The consumer's demand function $x(p, w)$ therefore gives the consumer's choice as a function of the exogenous variables.

One of the main activities that economists do is try to figure out how endogenous variables depend on exogenous variables, i.e., how consumers' behavior depends on the constraints placed on them (see principles 1-4 above).

2.3 Three Restrictions on Consumer Choices

So, let's begin with the following question: What are the bare minimum requirements we can put on behavior in order for them to be considered "reasonable," and what can we say about consumers' choices based on this? It turns out that relatively weak assumptions about consumer behavior can generate strong requirements for how consumers should behave.[7] We will start by enumerating three requirements.

- **Requirement 1:** The consumer always spends her entire budget (Walras' Law).

Requirement 1 is reasonable only if we are willing to make the assumption that "more is better." That is, for any commodity bundle x, the consumer would rather have a bundle with at least as much of all commodities and strictly more of at least one commodity. Actually, we can get away

[7] A "weak assumption" imposes less restriction on the behavior of an economic agent than a "strong assumption" does, so when designing a model, we prefer to use weaker assumptions if possible.

with a weaker assumption: Given any bundle x, there is always a bundle that has more of at least one commodity that the consumer strictly prefers to x. We'll return to this later. For now, just remember that the consumer spends all of her budget.

- **Requirement 2:** Only real opportunities matter (demand is homogeneous of degree zero).

The essence of requirement 2 is that consumers care about wealth and prices only inasmuch as they affect the set of allocations in the budget set. Or, to put it another way, changes in the environment that do not affect the budget set should not affect the consumer's choices. So, for example, if you double each price and wealth, the budget set is unchanged. Hence the consumer can afford the same commodity bundles as before and should choose the same bundle as before.

- **Requirement 3:** Choices reveal information about (stable) preferences.

So, suppose I offer you a choice between an apple and a banana, and you choose an apple. Then if tomorrow I see you eating a banana, I can infer that you weren't offered an apple (remember we assume that your preferences stay constant). Requirement 3 is known as the Weak Axiom of Revealed Preference (WARP). The essence is this. Suppose that on occasion 1 you chose bundle x when you could have chosen y. If I observe that on occasion 2 you choose bundle y, it must be because bundle x was not available. Put slightly more mathematically, suppose two bundles x and y are in the budget set $B_{p,w}$ and the consumer chooses bundle x. Then if at some other prices and wealth (p', w') the consumer chooses y, it must be that x was not in the budget set $B_{p',w'}$. We'll return to WARP later, but you can think of it in this way. If the consumer's preferences remain constant over time, then if x is preferred to y once, it should always be preferred to y. Thus if you observe the consumer choose y, you can infer from this choice that x must not have been available. Or, to put it another way, if you observe the consumer choosing x when x and y were available on one day and y when x and y were available on the next day, then your model had better have something in it to account for why this is so (i.e., a reason why the two days were different).

2.4 A First Analysis of Consumer Choices

In the rest of this chapter, we'll develop formal notation for talking about consumer choices, show how the three requirements on consumer behavior can be represented using this notation, and determine what imposing these restrictions on consumer choices implies about the way consumers should behave when prices or wealth change. Thus it is our first pass at the four-step process of economics: Assume consumers make choices that satisfy certain properties (the three requirements), subject to some constraints (the budget set); assume further that any changes in choices are due to changes in the constraints; and then derive testable predictions about consumer's behavior.

2.4.1 Comparative Statics

The analytic method we will use to develop testable predictions is what economists call **comparative statics.** A comparative statics analysis consists of coming up with a relationship between the exogenous variables and the endogenous variables in a problem and then using calculus to determine how the endogenous variables (i.e., the consumer's choices) respond to changes in the exogenous variables. Then, hopefully, we can tell if this response is positive, negative, or zero.[8] We'll see comparative statics analysis used over and over again. The important thing to remember for now is that even though "comparative statics" as a phrase doesn't mean anything, it refers to figuring out how the endogenous variables depend on the exogenous variables.[9]

2.5 Requirement 1 Revisited: Walras' Law

Requirement 1 for consumer choices is that consumers spend all of their wealth (Walras Law). The implication of this is that given a budget set $B_{p,w}$, the consumer will choose a bundle on the boundary of the budget set, sometimes called the budget frontier. The equation for the budget frontier is the set of all commodity bundles that cost exactly w. Thus, Walras' Law implies:

$$p \cdot x(p, w) \equiv w.$$

When a consumer's demand function $x(p, w)$ satisfies this identity for all values of p and w, we say that the consumer's demand satisfies Walras' Law. Thus the formal statement for "consumers always spend all of their wealth" is that "demand functions satisfy Walras' Law."

[8] Although it would be nice to get a more precise measurement of the effects of changes in the exogenous parameters, often we are only able to draw implications about the sign of the effect, unless we are willing to impose additional restrictions on consumer demand.

[9] The term "comparative statics" is meant to convey the idea that, while you analyze what happens before and after the change in the exogenous parameter, you don't analyze the process by which the change takes place.

2.5.1 What's the Funny Equals Sign All About?

Notice that in the expression of Walras' Law, I wrote a funny, three-lined equals sign. Contrary to popular belief, this doesn't mean "really, really equal." What it means is that, no matter what values of p and w you choose, this relationship holds. For example, consider the equality:

$$2z = 1.$$

This is true for exactly one value of z, namely $z = \frac{1}{2}$. However, think about the following equality:

$$2z = a.$$

Suppose I were to ask you, for any value of a, tell me a value of z that makes this equality hold. You could easily do this: $z = \frac{a}{2}$. Suppose I denote this by $z(a) = \frac{a}{2}$. That is, $z(a)$ is the value of z that makes $2z = a$ true, given any value of a. If I substitute the function $z(a)$ into the expression $2z = a$, I get the following equation:

$$2z(a) = a.$$

Note that this expression is no longer a function of z. If I tell you a, you tell me $z(a)$ (which is $\frac{a}{2}$), and no matter what value of a I choose, when I plug $z(a)$ in on the left side of the equals, the equality relation holds. Thus

$$2z(a) = a$$

holds for any value of a. We call an expression that is true for any value of the variable (in this case a) an **identity**, and we write it with the fancy, three-lined equals sign in order to emphasize this.

$$2z(a) \equiv a.$$

Why should we care if something is an equality or an identity? In a nut-shell, you can differentiate both sides of an identity and the two sides remain equal. You can't do this with an equality. In fact, it doesn't even make sense to differentiate both sides of an equality. To illustrate this point, think again about the equality: $2x = 1$. What happens if you increase x by a small amount (i.e. differentiate with respect to x)? If you differentiate both sides with respect to x, you get $2 = 0$, which is not true.

On the other hand, think about $2z(a) \equiv a$. We can ask the question what happens to z if you increase a. We can answer this by differentiating both sides of the identity with respect to a. If you do this, you get

$$\begin{aligned} 2\frac{dz(a)}{da} &= 1 \\ \frac{dz}{da} &= \frac{1}{2} \end{aligned}$$

That is, if you increase a by 1, z increases by $\frac{1}{2}$. (If you don't believe me, plug in some numbers and confirm.)

It may seem to you like I'm making a big deal out of nothing, but this is really a critical point. We are interested in determining how endogenous variables change in response to changes in exogenous variables. In this case, z is our endogenous variable and a is our exogenous variable. Thus, we are interested in things like $\frac{dz(a)}{da}$. The only way we can determine these things is to get identities that depend only on the exogenous variables and then differentiate them. Even if you don't quite believe me, you should keep this in mind. Eventually, it will become clear.

2.5.2 Back to Walras' Law: Choice Response to a Change in Wealth

As we said, Walras' Law is defined by the identity:

$$p \cdot x(p, w) \equiv w$$

or

$$\sum_{l=1}^{L} p_l x_l(p, w) \equiv w.$$

where the vector $x(p, w)$ describes the bundle chosen:

$$x(p, w) = (x_1(p, w), ..., x_L(p, w))$$

Suppose we are interested in what happens to the bundle chosen if w increases a little bit. In other words, how does the bundle the consumer chooses change if the consumer's income increases by a small amount? Since we have an identity defined in terms of the exogenous variables p and w, we can differentiate both sides with respect to w:

$$\begin{aligned} \frac{d}{dw}\left(\sum_{l=1}^{L} p_l x_l(p, w)\right) &\equiv \frac{d}{dw} w \\ \sum_{l} p_l \frac{\partial x_l(p, w)}{\partial w} &\equiv 1. \end{aligned} \tag{2.1}$$

So, now we have an expression relating the changes in the amount of commodities demanded in response to a change in wealth. What does it say? The left hand side is the change in expenditure due to the increase in wealth, and the right-hand side is the increase in wealth. Thus this expression says that if wealth increases by 1 unit, total expenditure on commodities increases by 1 unit as well. Thus the latter expression just restates Walras' Law in terms of responses to changes in wealth. Any change in wealth is accompanied by an equal change in expenditure. If you think about it, this is really the only way that the consumer could satisfy Walras' Law (i.e. spend all of her money) both before and after the increase in wealth.

Based only on this expression, $\sum_i p_i \frac{\partial x_i(p,w)}{\partial w} \equiv 1$, what else can we say about the behavior of the consumer's choices in response to income changes? Well, first, think about $\frac{\partial x_i(p,w)}{\partial w}$. Is this expression going to be positive or negative? The answer depends on what kind of commodity this is. Ordinarily, we think that if your wealth increases you will want to consume more of a good. This is certainly true of goods like trips to the movies, meals at fancy restaurants, and other "normal goods." In fact, this is so much the normal case that we just go ahead and call such goods - which have $\frac{\partial x_i(p,w)}{\partial w} > 0$ - "normal goods." But, you can also think about goods you want to consume less of as your wealth goes up - cheap cuts of meat, cross-country bus trips, nights in cheap motels, etc. All of these are things that, the richer you get, the less you want to consume them. We call goods for which $\frac{\partial x_i(p,w)}{\partial w} < 0$ "inferior goods." Since $x(p,w)$ depends on w, $\frac{\partial x_l(p,w)}{\partial w}$ depends on w as well, which means that a good may be inferior at some levels of wealth but normal at others.

So, what can we say based on $\sum_i p_i \frac{\partial x_i(p,w)}{\partial w} \equiv 1$? Well, this identity tells us that there is always at least one normal good. Why? If all goods are inferior, then the terms on the left hand side are all negative, and no matter how many negative terms you add together, they'll never sum to 1.

2.5.3 Testable Implications

We can use this observation about normal goods to derive a testable implication of our theory. Put simply, we have assumed that consumers spend all of the money they have on commodities. Based on this, we conclude that following any change in wealth, total expenditure on goods should increase by the same amount as wealth. If we knew prices and how much of the commodities the consumer buys before and after the wealth change, we could directly test this. But, suppose that we don't observe prices. However, we believe that prices do not change when wealth changes. What should we conclude if we observe that consumption of all commodities decreases following an increase in wealth? Unfortunately, the only thing we can conclude is that our theory is wrong. People aren't spending all of their wealth on commodities 1 through L.

Based on this observation, there are a number of possible directions to go. One possible explanation is that there is another commodity, $L+1$, that we left out of our model, and if we had accounted for that then we would see that consumption increased in response to the wealth increase and everything would be right in the world. Another possible explanation is that in the world we are considering, it is not the case that there is always something that the consumer would like more of (which, you'll recall, is the implicit assumption behind Walras' Law). This would be the case, for example, if the consumer could become satiated with the commodities, meaning that there is a level of consumption beyond which you wouldn't want to consume more even if you could. A final possibility is that there is something wrong with the data and that if consumption had been properly measured we would see that consumption of one of the commodities did, in fact, increase. In any case, the next task of the intrepid economist is to determine which possible explanation caused the failure of the theory and, if possible, develop a theory that agrees with the data.

2.5.4 Summary: How Did We Get Where We Are?

Let's review the comparative statics methodology. First, we develop an identity that expresses a relationship between the endogenous variables (consumption bundle) and the exogenous variable of interest (wealth). The identity is true for all values of the exogenous variables, so we can differentiate both sides with respect to the exogenous variables. Next, we totally differentiate the identity with respect to a particular exogenous variable of interest (wealth). By rearranging, we derive the effect of a change in wealth on the consumption bundle, and we try to say what we can about it. In the previous example, we were able to make inferences about the sign of this relationship. This is all there is to comparative statics.

2.5.5 Walras' Law: Choice Response to a Change in Price

What are other examples of comparative statics analysis? Well, in the consumer model, the endogenous variables are the amounts of the various commodities that the consumer chooses, $x_i(p, w)$. We want to know how these things change as the restrictions placed on the consumer's choices change. The restriction put on the consumer's choice by Walras' Law takes the form of the budget constraint, and the budget constraint is in turn defined by the exogenous variables – the prices of the various commodities and wealth. We already looked at the comparative statics of wealth changes. How about the comparative statics of a price change?

Return to the Walras' Law identity:

$$\sum p_i x_i (p, w) \equiv w.$$

Since this is an identity, we can differentiate with respect to the price of one of the commodities, p_j:

$$x_j (p, w) + \sum_{i=1}^{L} p_i \frac{\partial x_i (p, w)}{\partial p_j} = 0. \tag{2.2}$$

How does spending change in response to a price change? Well, if p_j increases, spending on good j increases, assuming that you continue to consume the same amount. This is captured by the first term in (2.2). Of course, in response to the price change, you will also rearrange the products you consume, purchasing more or less of the other products depending on whether they are gross substitutes for good j or gross complements to good j.[10] The effect of this rearrangement on total expenditure is captured by the terms after the summation. Thus the meaning of (2.2) is that once you take into account the increased spending in good j and the changes in spending associated with rearranging the consumption bundle, total expenditure does not change. This is just another way of saying that the consumer's demand satisfies Walras' Law.

2.5.6 Comparative Statics in Terms of Elasticities

The goal of comparative statics analysis is to determine the change in the endogenous variable that results from a change in an exogenous variable. Sometimes it is more useful to think about the percentage change in the endogenous variable that results from a percentage change in the exogenous variable. Economists refer to the ratio of percentage changes as elasticities. Equations (2.1) and (2.2), which are somewhat difficult to interpret in their current state, become much more meaningful when written in terms of elasticities.

A price elasticity of demand gives the percentage change in quantity demanded associated with a 1% change in price. Mathematically, price elasticity elasticity is defined as:

$$\varepsilon_{ip_j} = \frac{\% \Delta x_i}{\% \Delta p_j} = \frac{\partial x_i}{\partial p_j} \cdot \frac{p_j}{x_i}$$

Read ε_{ip_j} as "the elasticity of demand for good i with respect to the price of good j."[11]

[10] The term 'gross' refers to the fact that wealth is held constant. It contrasts with the situation where utility is held constant, where we drop the gross. All will become clear eventually.

[11] Technically, the second equals sign in the equation above should be a limit, as $\% \Delta \longrightarrow 0$.

Now recall equation (2.2):

$$x_j(p,w) + \sum_{i=1}^{L} p_i \frac{\partial x_i(p,w)}{\partial p_j} = 0.$$

The terms that are summed look almost like elasticities, except that they need to be multiplied by $\frac{p_j}{x_i}$. Perform the following sneaky trick. Multiply everything by $\frac{p_j}{w}$, and multiply each term in the summation by $\frac{x_i}{x_i}$ (we can do this because $\frac{x_i}{x_i} = 1$ as long as $x_i \neq 0$).

$$\begin{aligned}
\frac{p_j x_j(p,w)}{w} + \sum_{i=1}^{L} p_i \frac{p_j}{w} \frac{x_i}{x_i} \frac{\partial x_i(p,w)}{\partial p_j} &= 0 \\
\frac{p_j x_j(p,w)}{w} + \sum_{i=1}^{L} \frac{p_i x_i}{w} \frac{p_j}{x_i} \frac{\partial x_i(p,w)}{\partial p_j} &= 0 \\
b_j(p,w) + \sum_{i=1}^{L} b_i(p,w)\, \varepsilon_{ip_j} &= 0 \qquad (2.3)
\end{aligned}$$

where $b_j(p,w)$ is the share of total wealth the consumer spends on good j, known as the budget share.

What does (2.3) mean? Consider raising the price of good j, p_j, a little bit. If the consumer did not change the bundle she consumes, this price change would increase the consumer's total spending by the proportion of her wealth she spends on good x_j. This is known as a "wealth effect" since it is as if the consumer has become poorer, assuming she does not change behavior. The wealth effect is the first term, $b_j(p,w)$. However, if good j becomes more expensive, the consumer will choose to rearrange her consumption bundle. The effect of this rearrangement on total spending will have to do with how much is spent on each of the goods, $b_i(p,w)$, and how responsive that good is to changes in p_j, as measured by ε_{ip_j}. Thus the terms after the sum represent the effect of rearranging the consumption bundle on total consumption - these are known as substitution effects. Hence the meaning of (2.3) is that when you combine the wealth effect and the substitution effects, total expenditure cannot change. This, of course, is exactly what Walras' Law says.

2.5.7 Why Bother?

In the previous section, we rearranged Walras' Law by differentiating it and then manipulating the resulting equation in order to get something that means exactly the same thing as Walras' Law. Why, then, did we bother? Hopefully, seeing Walras' Law in other equations forms offers some insight into what our model predicts for consumer behavior. Furthermore, many times it is easier for economists to measure things like budget shares and elasticities than it is to measure actual quantities and prices. In particular, budget shares and elasticities do not depend on price levels, but only on relative prices. Consequently it can be much easier to apply Walras' Law when it is written as (2.3) than when it is written as (2.2).

2.5.8 Walras' Law and Changes in Wealth: Elasticity Form

Not to belabor the point, but we can also write (2.1) in terms of elasticities, this time using the wealth elasticity, $\varepsilon_{iw} = \frac{\partial x_i}{\partial w} \cdot \frac{w}{x_i}$. Multiplying (2.1) by $\frac{x_i w}{x_i w}$ yields:

$$\begin{aligned} \sum_i \frac{p_i x_i}{w} \frac{w}{x_i} \frac{\partial x_i (p, w)}{\partial w} &\equiv 1 \qquad (2.4) \\ \sum_i b_i (p, w) \varepsilon_{iw} &= 1. \end{aligned}$$

The wealth elasticity ε_{iw} gives the percentage change in consumption of good i induced by a 1% increase in wealth. Thus, in response to an increase in wealth, total spending changes by ε_{iw} weighted by the budget share $b_i (p, w)$ and summed over all goods. In other words, if wealth increases by 1, total expenditure must also increase by 1. Thus, equation (2.4) is yet another statement of the fact that the consumer always spends all of her money.

2.6 Requirement 2 Revisited: Demand is Homogeneous of Degree Zero.

The second requirement for consumer choices is that "only real opportunities matter." In mathematical terms this means that "demand is homogeneous of degree zero," or:

$$x (\alpha p, \alpha w) \equiv x (p, w)$$

Note that this is an identity. Thus it holds for any values of p and w. In words what it says is that if the consumer chooses bundle $x (p, w)$ when prices are p and income is w, and you multiply all prices and income by a factor, $\alpha > 0$, the consumer will choose the same bundle after the multiplication as before, $x (\alpha p, \alpha w) = x (p, w)$. The reason for this is straightforward. If you multiply all prices and income by the same factor, the budget set is unchanged. $B_{p,w} = \{x : p \cdot x \leq w\} = \{x : \alpha p \cdot x \leq \alpha w\} = B_{\alpha p, \alpha w}$. And, since the set of bundles that the consumer could choose is not changed, the consumer should choose the same bundle.

There are two important points that come out of this:

1. This is an expression of the belief that changes in behavior should come from changes in the set of available alternatives. Since the rescaling of prices and income do not affect the budget set, they should not affect the consumer's choice.

2. The second point is that nominal prices are meaningless in consumer theory. If you tell me that a loaf of bread costs \$10, I need to know what other goods cost before I can interpret the first statement. And, in terms of analysis, this means that we can always "normalize" prices by arbitrarily setting one of them to whatever we like (often it is easiest to set it equal to 1), since only the real prices matter and fixing one commodity's nominal price will not affect the relative values of the other prices.

Exercise 3 *If you don't believe me that this change doesn't affect the budget set, you should go back to the two-commodity example, plug in the numbers and check it for yourself. If you can't do it with the general scaling factor α, you should let $\alpha = 2$ and try it for that. Most of the time, things that are hard to understand with general parameter values like α, p, w are simple once you plug in actual numbers for them and churn through the algebra.*

2.6.1 Comparative Statics of Homogeneity of Degree Zero

We can also perform a comparative statics analysis of the requirement that demand be homogeneous of degree zero, i.e. only real opportunities matter. What does this imply for choice behavior?

The homogeneity assumption applies to proportional changes in all prices and wealth:

$$x_i(\alpha p, \alpha w) \equiv x_i(p, w) \text{ for all } i,\ \alpha > 0.$$

To make things clear, let the initial price vector be denoted $p^0 = \left(p_1^0, ..., p_L^0\right)$ and let w^0 original wealth, and (for the time being) assume that $L = 2$. For example, $\left(p^0, w^0\right)$ could be $p^0 = (3, 2)$ and $w^0 = 7$. Before we differentiate, I want to make sure that we're clear on what is going on. So, rewrite the above expression as:

$$x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right) \equiv x_i\left(p_1^0, p_2^0, w^0\right). \tag{2.5}$$

Now, notice that on the left-hand side for any $\alpha > 0$ the price of good 1 is $p_1 = \alpha p_1^0$, the price of good 2 is $p_2 = \alpha p_2^0$, and wealth is $w = \alpha w^0$. That is, given α. We are interested in what happens to demand as α changes, so it is important to recognize that the prices and wealth are functions of α.

We are interested in what happens to demand when, beginning at original prices p^0 and wealth w^0, we scale up all prices and wealth proportionately. To do this, we want to see what happens when we increase α, starting at $\alpha = 1$. Because the prices and wealth are functions of α, we have to use the Chain Rule in evaluating the derivative of (2.5) with respect to α. Differentiating (2.5) with respect to α yields:

$$\begin{aligned} \frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial p_1}\frac{\partial p_1}{\partial \alpha} + \frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial p_2}\frac{\partial p_2}{\partial \alpha} & \\ + \frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial w}\frac{\partial w}{\partial \alpha} &\equiv 0. \end{aligned}$$

Since $p_1 = \alpha p_1^0$, $\frac{\partial p_1}{\partial \alpha} = p_1^0$, and similarly $\frac{\partial p_2}{\partial \alpha} = p_2^0$, and $\frac{\partial w}{\partial a} = w^0$, so this expression becomes:

$$\frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial p_1}p_1^0 + \frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial p_2}p_2^0 + \frac{\partial x_i\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)}{\partial w}w^0 \equiv 0. \tag{2.6}$$

Notice that the first line takes the standard Chain Rule form: for each argument (p_1, p_2, and w), take the partial derivative of the function with respect to that argument and multiply it by the derivative with respect to α of "what's inside" the argument.[12]

Finally, notice that (2.6) has prices and wealth $\left(\alpha p_1^0, \alpha p_2^0, \alpha w^0\right)$. We are asking the question "what happens to x_i when prices and wealth begin at $\left(p_1^0, p_2^0, w^0\right)$ and are all increased slightly by the same proportion?" In order to make sure we are answering this question, we need to set $\alpha = 1$, so that the partial derivatives are evaluated at the original prices and wealth. Evaluating the last expression at $\alpha = 1$ yields the following expression in terms of the original price-wealth vector (s_1, s_2, v) :

$$\frac{\partial x_i\left(p_1^0, p_2^0, w^0\right)}{\partial p_1}p_1^0 + \frac{\partial x_i\left(p_1^0, p_2^0, w^0\right)}{\partial p_2}p_2^0 + \frac{\partial x_i\left(p_1^0, p_2^0, w^0\right)}{\partial w}w^0 \equiv 0. \tag{2.7}$$

Generalizing the previous argument to the case where L is any positive number, expression (2.7) becomes:

$$\frac{\partial x_i\left(p^0, w^0\right)}{\partial w}w^0 + \sum_{j=1}^{L}\frac{\partial x_i\left(p^0, w^0\right)}{\partial p_j}p_j^0 = 0 \text{ for all } i. \tag{2.8}$$

This is where we need to face an ugly fact. Economists are terrible about notation, which makes this stuff harder to learn than it needs to be. When you see (2.8) written in a textbook, it will look like this:

$$\frac{\partial x_i\left(p, w\right)}{\partial w}w + \sum_{j=1}^{L}\frac{\partial x_i\left(p, w\right)}{\partial p_j}p_j = 0 \text{ for all } i.$$

[12] If you are confused, see the next subsection for further explanation.

That is, they drop the superscript "0" that denotes the original price vector. But, notice that the symbol "w" in this expression has two different meanings. The "w" in "∂w" in the denominator of the first term says "we're differentiating with respect to the wealth argument," while the "w" in "$\partial x_i(p,w)$" and the "w" multiplying this term refer to the original wealth level, i.e., the wealth level at which the expression is being evaluated. Similarly, "p_j" also has two different meanings in this expression. To make things worse, economists frequently skip steps in derivations.[13]

It is straightforward to get an elasticity version of (2.8). Just divide through by $x_i(p,w)$:

$$\varepsilon_{iw} + \sum_{j=1}^{L} \varepsilon_{ip_j} = 0. \tag{2.9}$$

Elasticities ε_{iw} and ε_{ip_j} give the elasticity of the consumer's demand response to changes in wealth and the price of good j, respectively. The total percentage change in consumption of good i is given by summing the percentage changes due to changes in wealth and in each of the prices. Homogeneity of degree zero says that in response to proportional changes in all prices and wealth the total change in demand for each commodity should not change. This is exactly what (2.9) says.

2.6.2 A Mathematical Aside ...

If this is unfamiliar to you, the computation may seem strange. If it doesn't seem strange, then skip on to the next section.

If you're still here, let's try it one more time. This time, we'll let $L = 2$, and choose specific values for the prices and wealth. Let good 1's price be 5, good 2's price be 3, and wealth be 10 initially. Then, (2.5) writes as:

$$x_i(5\alpha, 3\alpha, 10\alpha) \equiv x_i(5, 3, 10).$$

Now, starting at prices $(5,3)$ and wealth 10, we are interested in what happens to demand for x_i as we increase all prices and wealth proportionately. To do this, we will first increase α by a small amount (i.e., differentiate with respect to α), and then we'll evaluate the resulting expression at $\alpha = 1$. This will give us an expression for the effect of a small increase in α. So, totally differentiate both sides with respect to α :

$$\begin{aligned}
\frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_1}\frac{d(5\alpha)}{d\alpha} + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_2}\frac{d(3\alpha)}{d\alpha} + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial w}\frac{d(10\alpha)}{d\alpha} &\equiv 0 \\
\frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_1}5 + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial p_2}3 + \frac{\partial x_i(5\alpha,3\alpha,10\alpha)}{\partial w}10 &\equiv 0.
\end{aligned}$$

[13] These are a couple of the main reasons why documents such as these are needed.

Again, the partial derivatives $\frac{\partial x_i}{\partial p_j}$ denote the partial derivative of function $x_i(p,w)$ with respect to the "p_j slot," i.e., the j^{th} argument of the function. And, since we are interested in what happens when you increase all prices and wealth proportionately beginning from prices $(5,3)$ and wealth 10, we would like the left-hand side to be evaluated at $(5,3,10)$. To get this, set $\alpha = 1$:

$$\frac{\partial x_i(5,3,10)}{\partial p_1}5 + \frac{\partial x_i(5,3,10)}{\partial p_2}3 + \frac{\partial x_i(5,3,10)}{\partial w}10 \equiv 0 \tag{2.10}$$

Comparing this expression with (2.7) shows that the role of p_1^*, p_2^*, and w^* are played by 5, 3, and 10, respectively in (2.10), as is expected.

The source of confusion in understanding this derivation seems to lie in confusing the partial derivative of x_i with respect to the p_1 **argument** (for example) with the particular price of good 1, which is 5α in this example and αp_1^* in the more general derivation above. The key is to notice that, in applying the chain rule, you always differentiate the function (e.g., $x_i()$) with respect to its argument (e.g., p_1), and then differentiate the function that is in the argument's "slot" (e.g., 5α or αs_1 or αp_1 if you are an economist) with respect to α.

2.7 Requirement 3 Revisited: The Weak Axiom of Revealed Preference

The third requirement that we will place on consumer choices is that they satisfy the Weak Axiom of Revealed Preference (WARP). To remind you of the informal definition, WARP is a requirement of consistency in decision-making. It says that if a consumer chooses z when y was also affordable, this choice reveals that the consumer prefers z to y. Since we assume that consumer preferences are constant and we have modeled all of the relevant constraints on consumer behavior and preferences, if we ever observe the consumer choose y, it must be that z was not available (since if it were, the consumer would have chosen z over y since she had previously revealed her preference for z). We now turn to the formal definition.

Definition 4 *Consider any two distinct price-wealth vectors* (p,w) *and* $(p',w') \neq (p,w)$. *Let* $z = x(p,w)$ *and* $y = x(p',w')$. *The consumer's demand function satisfies WARP if whenever* $p \cdot y \leq w$, $p' \cdot z > w'$.

We can restate the last part of the definition as: if $y \in B_{p,w}$, then $z \notin B_{p',w'}$. If y could have been chosen when z was chosen, then the consumer has revealed that she prefers z to y. Therefore if you observe her choose y, it must be that z was not available. I apologize for repeating the same definition over and over, but a) it helps to attach words to the math, and b) if you wanted math without explanation you could read a textbook.

In its basic form, WARP does not generate any predictions that can immediately be taken to the data and tested. But, if we rearrange the statement a little bit, we can get an easily testable prediction. So, let me ask the WARP question a different way. Suppose the consumer chooses z when prices and wealth are (p, w) , and z is affordable when prices and wealth are (p', w'). What does WARP tell us about which bundles the consumer could choose when prices are (p', w')?

There are two choices to consider: either $x(p', w') = z$. This is perfectly admissible under WARP. The other choice is that $x(p', w') = y \neq z$. In this case, WARP will place restrictions on which bundles y can be chosen. What are they? By virtue of the fact that z was chosen when prices and wealth were (p, w), we know that $y \notin B_{p,w}$, since if it were there would be a violation of WARP. Thus it must be that if the consumer chooses a bundle y different than x at (p', w'), y must not have been affordable when prices and wealth were (p, w).

This is illustrated graphically in figure 2.F.1 in MWG (p. 30). In panel a, since $x(p', w')$ is chosen at (p', w'), when prices are (p'', w'') the consumer must either choose $x(p', w')$ again or a bundle $x(p'', w'')$ that is not in $B_{p',w'}$. If we assume that demand satisfies Walras' Law as well, $x(p'', w'')$ must lay on the frontier. Thus if $x(p', w')$ is as drawn, it cannot be chosen at prices (p'', w''). The chosen bundle must lay on the segment of $B_{p'',w''}$ below and to the right of the intersection of the two budget lines, as does $x(p'', w'')$. Similar reasoning holds in panel b. The chosen bundle cannot lay within $B_{p',w'}$ if WARP holds. Panel c depicts the case where $x(p', w')$ is affordable both before and after the change in prices and wealth. In this case, $x(p', w')$ could have been chosen after the price change. But, if it is not chosen at (p'', w''), then the chosen bundle must lay outside of $B_{p'',w''}$, as does $x(p'', w'')$. In panels d and e, $x(p'', w'') \in B_{p',w'}$, and thus this behavior does not satisfy WARP.

2.7.1 Compensated Changes and the Slutsky Equation

Panel c in MWG Figure 2.F.1 suggests a way in which WARP can be used to generate predictions about behavior. Imagine two different price-wealth vectors, (p, w) and (p', w'), such that bundle $z = x(p, w)$ lies on the frontier of both $B_{p,w}$ and $B_{p',w'}$. This corresponds to the following hypothetical situation. Suppose that originally prices are (p, w) and you choose bundle $z = x(p, w)$. I tell you that I am going to change the price vector to p'. But, I am fair, and so I tell you that in order to make sure that you are not made worse off by the price change, I am also going to change your wealth to w', where w' is chosen so that you can still just afford bundle z at the new prices and wealth (p', w'). Thus $w' = p' \cdot z$. We call this a **compensated change in price**, since I change your wealth to compensate you for the effects of the price change.

Since you can afford z before and after the price change, we know that:

$$p \cdot z = w \text{ and } p' \cdot z = w'.$$

Let $y = x(p', w') \neq z$ be the bundle chosen at (p', w'). Since you actually choose y at price-wealth (p', w'), assuming your demand satisfies Walras Law we know that $p' \cdot y = w'$ as well. Thus

$$\begin{aligned} 0 &= w' - w' = p' \cdot y - p' \cdot z \\ \text{so, } p' \cdot (y - z) &= 0. \end{aligned}$$

Further, since z is affordable at (p', w'), by WARP it must be that y was not affordable at (p, w) :

$$\begin{aligned} p \cdot y &> w \\ p \cdot y - p \cdot z &> 0 \\ p \cdot (y - z) &> 0. \end{aligned}$$

Finally, subtracting $p \cdot (y - z) > 0$ from $p' \cdot (y - z) = 0$ yields:

$$\left(p' - p\right) \cdot (y - z) < 0 \tag{2.11}$$

Equation (2.11) captures the idea that, following a compensated price change, prices and demand move in opposite directions. Although this takes a little latitude since prices and bundles are vectors, you can interpret (2.11) as saying that if prices increase, demand decreases.[14] To put it another way, let $\Delta p = p' - p$ denote the vector of price changes and $\Delta x = x(p', w') - x(p, w)$ denote the vector of quantity changes. (2.11) can be rewritten as

$$\Delta p \cdot \Delta x^c \leq 0$$

where we have replaced the strict inequality with a weak inequality in recognition that it may be the case that $y = z$. Note that the superscript c on Δx^c is to remind us that this is the compensated change in x. This is a statement of the **Compensated Law of Demand** (CLD): If the price of

[14] This is especially true in the case where p and p' differ only in the price of good j, which changes by an amount dp_j. In this case, $p' - p = (0, 0, ..., dp_j, 0, ..., 0)$, and $(p' - p) \bullet (y - z') = dp_j dx_j$.

a commodity goes up, you demand less of it. If we take a calculus view of things, we can rewrite this in terms of differentials: $dp \cdot dx^c \leq 0$.

We're almost there. Now, what does it mean to give the consumer a compensated price change? Let $\hat{x}$ be the initial consumption bundle, i.e., $\hat{x} = x(p, w)$, where p and w are the original prices and wealth. A compensated price change means that at any price, p, bundle $\hat{x}$ is still affordable. Hence, after the price change, wealth is changed to $\hat{w} = p \cdot \hat{x}$. Note that the $\hat{x}$ in this expression is the original consumption bundle, not the choice function $x(p, w)$. Consider the consumer's demand for good i

$$x_i^c = x_i(p, p \cdot \hat{x})$$

following a compensated change in the price of good j:

$$\begin{aligned} \frac{d}{dp_j}(x_i(p, p \cdot \hat{x})) &= \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}\frac{\partial (p \cdot \hat{x})}{\partial p_j} \\ \frac{dx_i^c}{dp_j} &= \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}\hat{x}_j. \end{aligned}$$

Since $\hat{x}_j = x(p, w)$, we'll just drop the "hat" from now on. If we write the previous equation as a differential, this is simply:

$$dx_i^c = \left(\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}x_j\right)dp_j = s_{ij}dp_j$$

where $s_{ij} = \left(\frac{dx_i}{dp_j} + \frac{dx_i}{dw}x_j\right)$. If we change more than one p_j, the change in x_i^c would simply be the sum of the changes due to the different price changes:

$$dx_i^c = \sum_{j=1}^{L}\left(\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}x_j\right)dp_j = s_i \cdot dp$$

where $s_i = (s_{i1}, ..., s_{ij}, ..., s_{iL})$ and $dp = (dp_1, ..., dp_L)$ is the vector of price changes. Finally, we can arrange the dx_i^c into a vector by stacking these equations vertically. This gives us:

$$dx^c = Sdp$$

where S is an $L \times L$ matrix with the element in the ith the row and jth column being s_{ij}.

Now, return to the statement of WARP:

$$dp \cdot dx^c \leq 0$$

Substituting in $dx^c = Sdp$ yields

$$dp \cdot Sdp \leq 0. \tag{2.12}$$

Inequality (2.12) has a mathematical significance: It implies that matrix S, which we will call the substitution matrix, is negative semi-definite. What this means is that if you pre- and post-multiply S by the same vector, the result is always a non-positive number. This is important because mathematicians have figured out a bunch of nice properties of negative semi-definite matrices. Among them are:

1. The principal-minor determinants of S follow a known pattern.
2. The diagonal elements s_{ii} are non-positive. (Generally, they will be strictly negative, but we can't show that based on what we've done so far).
3. Note that WARP does not imply that S is symmetric. This is the chief difference between the choice-based approach and the preference-based approach we will consider later.

All I want to say about #1 is this. Basically, it amounts to knowing that the second-order conditions for a certain maximization problem are satisfied. But, in this course we aren't going to worry about second-order conditions. So, file it away that if you ever need to know anything about the principal minors of S, you can look it up in a book.

Item #2 is a fundamental result in economics, because it says that the change in demand for a good in response to a compensated price increase is negative. In other words, if price goes up, demand goes down. This is the **Compensated Law of Demand** (CLD). You may be thinking that it was a lot of work to derive something so obvious, but the fact that the CLD is derived from WARP and Walras' Law is actually quite important. If these were not sufficient for the CLD, which we know from observation to be true, then that would be a strong indicator that we have left something out of our model.

The fact that $s_{ii} \leq 0$ can be used to help explain an anomaly of economic theory, the **Giffen good**. Ordinarily, we think that if the price of a good increases, holding wealth constant, the demand for that good will decrease. This is probably what you thought of as the "Law of Demand," even though it isn't always true. Theoretically, it is possible that when the price of a good increases, a consumer actually chooses to consume more of it. By way of motivation, think of the following story. A consumer spends all of her money on two things: food and trips to Hawaii. Suppose the price of food increases. It may be that after the increase, the consumer can no longer afford the trip to Hawaii and therefore spends all of her money on food. The result is that the consumer actually buys more food than she did before the price increase.

How does this story manifest itself in the theory we have learned up until now? We know that:

$$s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$$

Rearranging it:

$$\frac{\partial x_i (p,w)}{\partial p_i} = s_{ii} - \frac{\partial x_i}{\partial w} x_i.$$

We know that $s_{ii} \leq 0$ since S is negative semi-definite. Clearly, $x_i \geq 0$. But, what happens if x_i is a strongly inferior good? In this case, $\frac{\partial x_i}{\partial w} < 0$, meaning $-\frac{\partial x_i}{\partial w} x_i > 0$. And, if the magnitude of $-\frac{\partial x_i}{\partial w} x_i$ is greater than s_{ii}, it can be that $\frac{\partial x_i}{\partial p_i} > 0$, which is what it means to be a Giffen good.

What does the theory tell us? Well, it tells us that in order for a good to be a Giffen good, it must be a strongly inferior good. Or, to put it the other way, a normal good cannot be a Giffen good.[15]

Before going on, let me give one more aside on why we bother with all of this stuff. Remember when I started talking about increasing prices, and I said that I was *fair*, so I was going to also change your wealth? Well, it turns out that a good measure of the impact of a price change on a consumer is given by the change in wealth it would take to compensate you for a price change. So, if we could observe the s_{ii} terms, this would help us to measure the impact of price changes on consumers. But, the problem is that we never observe *compensated* price changes, we only observe the uncompensated ones, $\frac{dx_i(p,w)}{dp_i}$. But, the relationship above gives us a way to recover s_{ii} from observations on uncompensated price changes $\frac{\partial x_i}{\partial p_i}$, wealth changes, $\frac{\partial x_i}{\partial w}$, and actual consumption, x_i. Thus the importance of the relationship $s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$ is that it allows us to recover an unobservable quantity that we are interested it, s_{ii}, from observables.

2.7.2 Other Properties of the Substitution Matrix

Based on what we know about demand functions, we can also determine a couple of additional properties of the Substitution matrix. They are:

$$\begin{aligned} p \cdot S(p,w) &= 0 \\ S(p,w)\, p &= 0 \end{aligned}$$

[15] Technically, it is not the goods that are Giffen. Rather, the consumer's behavior at a particular price-wealth combination is Giffen. For example, it has been shown that very poor consumers in China exhibit Giffen behavior: their demand curve for rice slopes upward in the price of rice. But, non-poor consumers do not exhibit Giffen behavior: their demand curve slopes downward. See R. Jensen and N. Miller (2001).

These can be derived from the comparative statics implications of Walras' Law and homogeneity of degree zero. Their effect is to impose additional restrictions on the set of admissible demand functions. So, suppose you get some estimates of $\frac{\partial x_i}{\partial p_j}$, p, w, and $\frac{\partial x_i}{\partial w}$, which can all be computed from data, and you are concerned with whether you have a good model. One thing you can do is compute S from the data, and check to see if the two equations above hold. If they do, you're doing okay. If they don't, this is a sign that your data do not match up with your theory. This could be due to data problems or to theory problems, but in either case it means that you have work to do.[16]

[16] The usual statistical procedure in this instance is to impose these conditions as restrictions on the econometric model and then test to see if they are valid. I leave it to people who know more econometrics than I do to explain how.